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Vectorial Charge Displacement.

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Summary. — The consequences are examined of recognizing an extreme symmetry between the nucleon and cascade (Ξ) particle. The simplest formalism for expressing this symmetry is to take the charge displacement number $a = q - I_z$ for heavy particles as the z-component of a second, independent vector $\boldsymbol{A}$ in charge space. This scheme makes evident the possibility of two independent charge conjugation operators C and C' for heavy particles: it also has suggestive applications to the θ_1, θ_2 scheme and the π-e decay problem. Experimental tests for vectorial $\boldsymbol{A}$ appear remote; the scheme appears mainly as an ideal limiting case, which may be useful for comparison and contrast with the real situation.

1. – Introduction.

A considerable degree of symmetry is apparent among the heavy particles now known. The π is a charge triplet as is the Σ; the Λ is a singlet that could be regarded as the $I = 0$ combination of $(\Sigma + \pi)$. The K is a charge doublet; the $\overline{\mathrm{K}}$ is exactly analogous, so that the K and $\overline{\mathrm{K}}$ together form a compound doublet. The only remaining particles are the charge doublets N and presumably Ξ, the cascade particle. These two doublets resemble the K, $\overline{\mathrm{K}}$ compound doublet except for their substantial mass difference. Even this mass difference is approximately symmetric with respect to the Λ. It may therefore be of interest, as a sort of zeroth approximation, to explore the implications of a Ξ, N symmetry exactly similar to that of K, $\overline{\mathrm{K}}$. This is the purpose of

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the present note; a more realistic situation would then be approached by allowing for a failure of complete N, Ξ symmetry as indicated by their 35% mass difference. The various limitations imposed by complete N, Ξ symmetry would also have to be relaxed, but the extent of their relaxation might in some cases be small, since it would depend specifically upon the way in which the N, Ξ asymmetry was introduced. It thus may be worth while at least to catalogue the features of the ideal case.

The simplest formal device for expressing complete symmetry between Ξ and N is to take the heavy particle charge displacement number $a = q - I_z$ to be the z-component of a charge space vector $\boldsymbol{A}$ that is additional to and independent of $\boldsymbol{I}$. This is mathematically equivalent to assuming a charge space of at least four dimensions ([1-3]) but it is algebraically simpler to avoid the four-dimensional formalism and speak entirely in terms of two independent three-vectors, whose properties are more familiar.

The assignment of vectorial $\boldsymbol{A}$ to various particles ([4]) is straightforward in this ideal case: (K, $\overline{\text{K}}$) and (N, Ξ) are each compound doublets with $A = I = \frac{1}{2}$; π and Σ have $A = 0$, $I = 1$; and Λ has $A = I = 0$. The assignment for light particles follows from considering γ-radiation: the photon presumably has $A = I = 0$, but its interaction with heavy particles is proportional to $q = A_z + I_z$. The selection rules ΔA, $\Delta I = 0, \pm 1$ stem from these explicit operators and not from intrinsic properties of the photon. Extension of this procedure ([5])

([1]) A. Pais: *Proc. Nat. Acad. Sci. U. S.*, **40**, 484 (1954).

([2]) A. Salam and J. C. Polkinghorne: *Nuovo Cimento*, **2**, 685 (1955). A direct way of making connection with the four-dimensional formalism is by analogy with relativistic particles of arbitrary spin (e.g., K. J. Le Couteur: *Proc. Roy. Soc.* (*London*), **202**, 284 (1950)). In the simplest non-trivial case where $A = I = \frac{1}{2}$, the real space analogues can be taken as $\boldsymbol{A} \sim (\boldsymbol{\sigma} + \boldsymbol{\alpha})/4$, $\boldsymbol{I} \sim (\boldsymbol{\sigma} - \boldsymbol{\alpha})/4$, where $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}$ are Dirac matrices. It is apparent that these representations for $\boldsymbol{A}$ and $\boldsymbol{I}$ commute and have eigenvalues $\frac{1}{2}$. Upon reflection of an odd number of axes $A \leftrightarrow I$, so that only in case $A = I$ can a single $\psi(A, I)$ be an eigenfunction of charge reflection.

([3]) J. Schwinger: *Phys. Rev.*, **104**, 1164 (1956). A recent suggestion by H. Yukawa: *Progr. Theor. Phys. Japan* (to be publshed) on the internal co-ordinates of strange particles seems very close to the notion of four-dimensional isotopic spin.

4 ([4]) It does not seem feasible to use the antiparticles $\overline{\text{N}}$ in a way to make (N, $\overline{\text{N}}$) the two members of an $A = \frac{1}{2}$ doublet. In order to conserve heavy fermions absolutely, all interaction operators would have to be strict scalars in A-space, which would seem to make superfluous the introduction of a vector formalism. The present type of A-doublet is also considered by F. Duimio: *Nuovo Cimento*, **3**, 595 (1956).

5 ([5]) One might also contemplate an extension to the π-N and π-Ξ interaction, by writing it in the form $(0_1 + A_z 0_2)$: here the π has intrinsic $A = 0$, but can induce $\Delta A = 0, \pm 1$ in the fermions because of the explicit appearance of the operator A_z. States of $A = \frac{3}{2}$ could then appear in π-N scattering. This type of interaction would also give a Ξ-N mass difference; the relative signs of 0_1 and 0_2 would have to be opposite for fermion and antifermion in order that both have the same mass.

1862

to light fermions is already customary in nuclear β-decay: the leptons have $A = I = 0$, but explicit operators like $(I_x \pm iI_y)$ appear in the interaction to account for the heavy-particle transitions.

Since the charge number q is always an integer, under vectorial $\boldsymbol{A}$ all particles will behave as tensors and not as spinors under rotations and reflections of charge space. This allows the possibility of assigning an additional quantum number, the « charge reflection » to each particle; it has the simple eigenvalues ± 1 upon reflection of an odd number of axes in charge space and is exactly analogous to parity in ordinary space ([6]). There is no present evidence for assuming each particle to be an eigenfunction of charge reflection; but this possibility does not even exist for scalar, non-integer a, when the particle would be a spinor in charge space. Charge reflection could, for example, serve as a parameter of distinction among various types of K-mesons; it would be conserved by the operator $(A_z + I_z)$ for γ-emission ([3]).

It seems natural to extend the procedure for β-decay to V-decays like $\Lambda \to \pi + \mathrm{N}$ and assume the interaction to contain explicit operators for changing I_z and a of the system. Under vectorial $\boldsymbol{A}$ these operators are all tensors under rotations of the total charge space, so that V-decays appear more closely analogous to ordinary β-decay than is the case if a is a scalar.

2. – Symmetry properties.

2·1. *Charge conjugation.* – The most interesting of the symmetries connected with Ξ, N relates to charge conjugation. This is usually defined in direct analogy with the lepton case as $C\mathrm{N} = \overline{\mathrm{N}}$, $C\Xi = \overline{\Xi}$, etc. Under any scheme completely symmetrical in N and Ξ, it is possible to define a « second charge conjugation » C' such that $C'\mathrm{N} = \Xi$, etc., without any change of fermion for antifermion. For vectorial $\boldsymbol{A}$ this second charge conjugation is particularly easy to express ([7]):

$$C' = \exp\left[i\pi(A_x + I_x)\right]. \tag{1}$$

One can, of course, formally define an operator $X = CC'$, which effects exchanges of the type $\overline{\mathrm{N}} \leftrightarrow \Xi$, etc. Then by Eq. (1)

$$X\Lambda = -C\Lambda\,, \qquad X\Sigma^0 = C\Sigma^0 \tag{2}$$

so that an alternation of sign is possible between X and C for fermions.

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6 ( ) G. C. Wick, A. S. Wightman and E. P. Wigner: *Phys. Rev.*, **88**, 101 (1952): a particle of integer spin (tensor) can have absolute (intrinsic) parity ± 1; a particle of half-integer spin (spinor) can have only relative parity ± 1 as compared with another spinor. The absolute parity of a spinor (i.e., relative to a tensor) is indeterminate.

7 ( ) This is identical in form with the operator B of d'Espagnat and Prentki: *Phys. Rev.*, **102**, 1684 (1956) if we define a vector $\boldsymbol{Q} = \boldsymbol{A} + \boldsymbol{I}$ so that $C' = \exp\left[i\pi Q_x\right]$.

1863

For applications involving strong forces there is generally no ambiguity between C and C'. An N-$\bar{\text{N}}$ system, for example, is not an eigenfunction of C' but may be one of C. In any case of apparent contradiction between C and C', it must be remembered that C is a physically realistic operation while C' is not, so that the contradiction is always to be resolved by violation of C' instead of C. For example, the π^0 is an eigenfunction of C with eigenvalue $+1$ leading to two-photon decay; but if the π^0 is a pure state with $A=0$, $I=1$, then the eigenvalue of C' is -1, precluding two-photon decay. One must therefore conclude that the π^0 is not a strict eigenfunction of C'; it is not clear at present whether this deviation can be entirely ascribed to the $T=0$ component associated with the charged neutral pion mass difference, or whether a substantial admixture of $A=1$, $A_z=0$ is necessary for the pion.

An instance of electromagnetic interaction in which C and C' are not contradictory but complementary is provided by the neutral fermions Λ^0 and Σ^0. There are not eigenfunctions of C, which therefore provides no information about their magnetic moments; if they are eigenfunctions of C', however, their magnetic and all other charge moments must vanish, along with the sum $(\mu^+ + \mu^-)$ of Σ^+ and Σ^- magnetic moments and hence the $\Sigma^- - \Sigma^+$ mass difference. On the present view this mass difference has an interesting dual aspect: although its immediate location is in electromagnetic interactions, the difference in these interactions stems basically from the non-electromagnetic forces associated with the Ξ, N mass difference. These non-electromagnetic forces are likewise responsible for any non-vanishing electromagnetic moments possessed by Λ^0 and Σ^0.

2·2. *The θ_1, θ_2 scheme* ([8]). – Although originally formulated on the basis of charge conjugation C, this scheme is readily recast in terms of C' and vectorial $\boldsymbol{A}$, since for K-particles C and C' appear to be equivalent. Thus θ^+, θ^- and $\theta_1 = 2^{-\frac{1}{2}}(\bar{\theta}^0 + \theta^0)$ are symmetric with respect to interchange of the vectors $\boldsymbol{A}$ and $\boldsymbol{I}$, while $\theta_2 = i\,2^{-\frac{1}{2}}(\bar{\theta}^0 - \theta^0)$ is antisymmetric. This is just analogous to the well-known combination of two spins $\frac{1}{2}$ to form a total spin of 1 or 0. The θ_1 and θ_2 are thus eigenfunctions of a total charge vector $\boldsymbol{Q} = \boldsymbol{A} + \boldsymbol{I}$, corresponding to respective eigenvalues $Q=1$ and $Q=0$. Since $\boldsymbol{Q} = \boldsymbol{I}$ for pions, Bose statistics would allow two-pion decay for only one of the pair θ_1, θ_2.

2·3. *π-e decay.* – The possibility of a second charge conjugation C' reflects the fact that the charge co-ordinates of heavy particles are somehow more complicated than for leptons. This additional complexity provides oppor-

([8]) M. Gell-Mann and A. Pais: *Phys. Rev.*, **97**, 1387 (1955); A. Pais and O. Piccioni: *Phys. Rev.*, **100**, 1487 (1955).

tunity for selection rules on lepton decay based on the charge co-ordinates themselves. This type of selection rule can readily be exhibited in the case of ideal vector A; although this may be unrealistic, it at least provides the simplest illustration of the principle.

Since the leptons have no intrinsic A or I spin, the decay interactions themselves must contain operators of this type: one conventionally assumes a factor $(I_x \pm iI_y)$ in the (e+ν) and (μ+ν) decay interactions. For transitions involving nucleons with $A_z = +\frac{1}{2}$, however, this factor might as well be $A_z(I_x \pm iI_y)$. Suppose in particular that the operator containing A_z holds for (e+ν), and that without A_z for (μ+ν). Then if the pion is an eigenfunction with $A = 0$, only (μ+ν) decay will occur to first order, since A_z has a vanishing matrix element for $\Delta A = 0$. The same principle could be extended to K-decay by introducing two different decay operators: a $\frac{1}{2}$-spinor in both A and I for (μ+ν), but a $\frac{1}{2}$-spinor and a $\frac{3}{2}$-spinor for (e+ν). In fact, the ambiguity of parity associated with the θ and τ modes of K-decay is a further argument for basing the absence of K_{e2} on interaction symmetries in charge space rather than in real space. One must of course abandon the simplest idea of a universal Fermi interaction, at least in charge space.

Even if the π is not an eigenfunction of A, it would be possible to eliminate (e+ν) in favor of (μ+ν) by taking a suitable linear combination of operators with and without A_z, although this appears rather artificial. In any case the present procedure does nothing to eliminate $\pi \to e+\nu+\gamma$, but the matrix element for this may be small ([9]).

3. – Experimental tests.

A prerequisite of any symmetry between Ξ and N is identity of their spins and parities. Experimental evidence on this point seems at present remote; in any case it would not determine whether a is essentially vectorial in character. The β-decay $\Xi \to N+e+\nu$ with selection rules $\Delta A = \pm 1$, $\Delta I = 0$ seems to be required (as pointed out in reference ([4])), if one assumes vector A symmetry for the weak interactions as well as for the heavy particles themselves. The exact branching ratio with the cascade mode cannot be predicted

([9]) For example, suppose that instead of the usual perturbation-type procedure one adopts a compound state approach, suitable to strongly coupled systems: then the transition proceeds through an excited (virtual) state of the pion: $\pi \to \gamma+\pi^*$, $\pi^* \to e+\nu$, where π^* is the lowest energy boson system with $A = I = 1$. Then with an electric dipole matrix element M_γ for γ-emission of uninhibited nuclear magnitude ($\tau \approx 10^{-14}$ s for $E \approx 1$ MeV), one obtains the reduction factor $f = \tau(e\nu)/\tau(e\nu\gamma) \sim (10\Delta E)^{-2}$. Here ΔE is $E_{\pi^*} - E_\pi$ in units of MeV; it seems not unreasonable to take ΔE of order $Mc^2 \sim 10^3$ MeV, in which case $f \sim 10^{-5}$. Here $\tau(e\nu)$ is supposed computed with no selection rules.

because of the obvious failure of perfect vector A symmetry; it seems reasonable to expect a branching ratio of at least several per cent, however. The competing decay mode $\Xi \to \pi + N$ can be inhibited by assuming that the weak decay operator for pion emission satisfies the same selection rules as for lepton decay in this case ($\Delta A = \pm 1$, $\Delta I = 0$). Then $\Delta I_z = 0$ and $\Xi \to \pi + N$ is forbidden.

Observation of the reaction ([10]) $\pi^{\pm} \to \pi^0 + e^{\pm} + \nu$ would contradict the π-e mechanism discussed above and thus provide a negative test. Some weak restrictions are placed on high energy K-nucleon scattering if A behaves as a vector; namely,

$$(3) \qquad d\sigma_- + d\sigma_{ex} \geqslant \tfrac{1}{2} d\sigma_+ ,$$

where $d\sigma$ is the differential cross-section at any angle. The subscripts + and — refer to elastic K^+ and K^- scattering, while « exchange » is the process $K^- + p \to K^+ + \Xi^-$.

It is perhaps an exaggeration to speak of experimental tests for vectorial A, since it appears to represent an unrealistic idealization. In any ultimate scheme vor V-particles, however, one of the primary difficulties will be to exhibit in a simple and self-consistent fashion both the symmetry between Ξ and N, and the failure of this symmetry. As a guidepost for such an ultimate scheme it may be useful to keep in mind the properties of the limiting case where the symmetry is complete: this case is most compactly described by vectorial A.

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The author wishes to thank all the many people who have clarified these questions by discussion.

([10]) E. Feenberg and H. Primakoff: private communication.

RIASSUNTO (*)

Si esaminano le conseguenze della constatazione di una estrema simmetria esistente tra nucleone e particella (Ξ) di una cascata. Il formalismo più semplice per esprimere tale simmetria è prendere il numero dello spostamento della carica $a = q - I_z$ per le particelle pesanti come componente z di un secondo vettore A indipendente nello spazio delle cariche. Questo schema rende evidente la possibilità di due operatori di coniugazione delle cariche, C e C', indipendenti per le particelle pesanti: ha anche interessanti applicazioni allo schema θ_1, θ_2 del problema del decadimento π-e. Le prove sperimentali dell'esistenza di A non appaiono di prossima realizzazione; lo schema è principalmente un ideale caso limite, da servire per confrontarlo e metterlo a contrasto con la realtà.

(*) *Traduzione a cura della Redazione.*

On the Radiative Contributions to the Van der Waals Force.

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Summary. — The effect of the electromagnetic interaction between two neutral atoms is important for large separations where the London potential proportional to R^{-6} is altered and becomes the Casimir-Polder potential varying as R^{-7}. It is shown that the interaction energy, for all separations, may be computed in a relatively simple manner by quantum electrodynamics with the aid of a reduced Hamiltonian involving transverse quanta alone. This Hamiltonian function is justified by an appeal to the dipole approximation in the Lagrangian.

1. – Introduction.

The retarded interaction energy between two identical neutral atoms in their ground state was computed some time ago by Casimir and Polder (1)). These authors used the quantum theory of radiation in interaction with non-relativistic sources and carried through the calculation to lowest order in the coupling constant by an elegant but unsymmetrical method. The same result may be obtained in a more straight-forward fashion by the standard prescription of stationary state perturbation theory; this has been reaffirmed recently (Aub *et al.* (2)). The energy of interaction is found to differ from the London-Van der Waals energy for large separations, where it shows an R^{-7} dependence, but reduces to it at small distances; it is attractive throughout. That both methods of computation should agree can be seen quite generally. The calculations necessary to obtain the interaction energy in

(1) H. B. G. Casimir and D. Polder: *Phys. Rev.*, **73**, 360 (1948).

(2) M. R. Aub, E. A. Power and S. Zienau: *Phil. Mag.*, **2** 571 (1957); cf. also J. E. Dsjaloshinsky *J.E.T.P* (*USSR*) **30**, 1112 (1956).

closed form or to obtain the leading term at large separation are, however, extensive in both cases.

Interesting considerations on the perturbation of the quantum mechanical zero point energy of the radiation field in a cavity within which the two interacting atoms may be thought placed, have been published by CASIMIR [3] in an endeavour to reproduce the retarded law of interaction by simpler means. In this way one obtains indeed the mutual energy

$$\Delta E(R) = -\frac{23\alpha(1)\alpha(2)\hbar c}{4\pi R^7}, \tag{1}$$

which, to lowest order, is valid at large separation. Here $\alpha(l)$ is the approximate static polarizability of atom l, given by

$$\alpha(l) = 2\sum_{xyz} |q_x(l)|^2/3E = 2(q(l))^2/E\,. \tag{2}$$

An idealized spectrum is assumed for the atom, namely, above the ground state one triply degenerate p-state of energy E. In (2), $q_x(l)$ is the matrix element between the ground state and the excited state of the x-component of the total electric moment of atom l. CASIMIR did not succeed in recovering by this method his interaction energy law which is valid for all separation and of which (1) is the leading term in an expansion in powers of $\lambda\!\!\!^{-}/R$ where $\lambda\!\!\!^{-} = \hbar c/E$.

In this paper, we obtain the complete energy of interaction to lowest order in the coupling constant ($\sim e^4$) using symmetric perturbation theory (symmetric between the two atoms) on a reduced Hamiltonian thereby simplifying the calculations considerably. The essential step is a careful introduction of the dipole approximation into the Lagrangian for the electro-magnetic field in interaction with two neutral atoms before the transformation to the Hamiltonian and subsequent quantization is carried through. The reduced Hamiltonian reached in this manner has been used in older work on dispersion problems (BREIT [4]), but it is not always realised that in dipole approximation one may eliminate both the electrostatic potential *and* the longitudinal waves from the formalism. For this reason we have explained our method of calculation in somewhat greater detail.

We adhere closely to Casimir's model for the two neutral atoms. As previously stated, each atom has a spherically symmetric ground state and one excited p state. In addition all recoil is neglected and the transport of cano-

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[3] H. B. G. CASIMIR: *Journ. Chim. Phys.*, **46**, 407 (1950).
[4] G. BREIT: *Rev. Mod. Phys.*, **4**, 504 (1932); **5**, 91 (1933).

1868

nical field momentum into the charges of the atom is completely ignored, so that only electric dipole waves are coupled to the atom.

2. – General theory.

The Hamiltonian for two neutral atoms interacting with the electromagnetic field is usually written in Coulomb gauge as

$$H = \sum_{l=1,2} H(l) + H^{\mathrm{RAD}} + q_i(1)q_j(2)\frac{R^2\delta_{ij} - 3x_ix_j}{R^5} + \\ + \sum_{l=1,2}\left\{-\frac{e}{mc}p_i(l)A_i(l) + \frac{e^2}{2mc^2}A^2(l)\right\}. \tag{3}$$

Here $H(l)$ is the non-relativistic Hamiltonian of atom l and H^{RAD} is the Hamiltonian of the free transverse electromagnetic field. The electric moment $q_i(l)$ of atom l is related to $p_i(l)$, the canonical momentum, by the equation $ep_i(l) = m(\mathrm{d}/\mathrm{d}t)q_i(l)$. In this gauge the static interaction between the dipoles is separated from their interaction via the transverse electromagnetic field. In dipole approximation, the solenoidal vector $\boldsymbol{A}$ is constant over each atom. We shall show in paragraph (3) that, except for unimportant contact and self-energy interactions, this is equivalent to the more convenient Hamiltonian

$$H = \sum_{l=1,2} H(l) + H^{\mathrm{RAD}} - \sum_{l} q_i(l)E_i^t(l)\,, \tag{4}$$

where E_i^t is the *transverse* electric field (or more precisely the transverse vector canonically conjugate to A_i).

In Hamiltonian (4) all the interaction between the atoms is carried by transverse waves whereas the more usual expression (3) contains the electrostatic interaction explicitly. One sees in (3) that if retardation is entirely neglected the interatomic force reduces to the London-Van der Waals potential, while this requires calculation if it is extracted from (4). Nevertheless, where retardation becomes important, i.e. where $R \sim \lambda$, this feature is a cause of complication when (3) is used. In this limit the use of (4) is a considerable simplification. To see this physically it may be recalled that the electric field of an oscillating dipole is entirely retarded and transverse outside the charge-current distribution (cf. MORSE and FESHBACH [5]; Chapter 13). If this field is computed in Coulomb gauge from (3), there are terms in the vector potential which are non-retarded and serve to subtract out the contribution to the electromagnetic field from the instantaneous Coulomb potential.

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[5] P. MORSE and H. FESHBACH: *Methods of Theoretical Physics* (New York, 1953).

This aspect of the calculation is mirrored in the analytical behaviour of the propagator for the transverse vector potential in Coulomb gauge, which has the four dimensional Fourier transform

$$\frac{4\pi}{(k^2 - k_0^2)}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right); \qquad k = |\boldsymbol{k}|,\ i, j = 1, 2, 3. \tag{5}$$

If one computes the three dimensional Fourier transform with retarded boundary condition, a singularity at the origin of the k-integral is encountered, in addition to those at $k = \pm k_0$. So the retarded transverse Green's dyadic for frequency ck_0 is (*)

$$\begin{aligned} \mathcal{G}_{ij}^{\mathrm{ret}}(\boldsymbol{x}; k_0) &= \frac{4\pi}{(2\pi)^3}\int \frac{\exp[i\boldsymbol{k}\boldsymbol{x}]}{k^2 - k_0^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \mathrm{d}^3 k = \\ &= \frac{\exp[ik_0 R]}{R}\left[-\delta_{ij}\left(\frac{1 - ik_0 R + k_0^2 R^2}{k_0^2 R^2}\right) + \frac{x_i x_j}{R^2}\left(\frac{3 - 3ik_0 R - k_0^2 R^2}{k_0 R^2}\right)\right] + \\ &+ \frac{1}{k_0^2}\frac{\delta_{ij} R^2 - 3x_i x_j}{R^5}, \qquad R = |\boldsymbol{x}|. \end{aligned} \tag{6}$$

The last non-retarded summand in (6) arises from the pole at $k = 0$ and is that term which affects the cancellation with the instantaneous Coulomb field of the dipole. A rather similar situation obtains in higher orders of the perturbation theory involving Hamiltonian (3). Thus in the fourth order calculation to find the interaction energy between two neutral atoms due to the last term in (3), the final complicated integral permits the extraction of a residue from the poles at the origin of k-space, which cancels against the London energy which is itself obtained in second order from the third summand of (3). This occurrence of poles which merely serve to subtract out instantaneous contributions is completely avoided if Hamiltonian (4) is used.

The quantization of Hamiltonian (4) is carried out in the standard way. The normal mode expansion in unit normalization volume of $\boldsymbol{E}(\boldsymbol{r})$ in Schrödinger representation is

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{k,\lambda} \boldsymbol{e}(\boldsymbol{k}\lambda) i\sqrt{2\pi\kappa}\left\{a_{\boldsymbol{k}\lambda}^{\dagger} \exp[-i\boldsymbol{k}\boldsymbol{r}] - a_{\boldsymbol{k}\lambda} \exp[i\boldsymbol{k}\boldsymbol{r}]\right\}, \tag{7}$$

where

$$[a_{\boldsymbol{k}\lambda},\ a_{\boldsymbol{k}'\beta}^{\dagger}] = \delta_{\lambda\beta}\delta_{\boldsymbol{k}\boldsymbol{k}'}, \tag{8}$$

(*) This differs from formula (13.1.31) in MORSE and FESHBACH [5] by a δ-function. Their dyadic $\mathcal{L}$, (13.1.32), is inconsistent with its definition (13.1.27) as is seen by computing the trace of both expressions.

and $\boldsymbol{e}(\boldsymbol{k}\lambda) = \boldsymbol{e}(-\boldsymbol{k}\lambda)$, $\lambda = 1, 2$ are two linearly independent unit vectors perpendicular to $\boldsymbol{k}$. Units are chosen so that $\hbar = c = 1$.

To bring out clearly the difference between Hamiltonians (3) and (4), it is of interest to compute the expectation value of the Heisenberg electric field strength operator at a point $\boldsymbol{r}$ in the presence of a neutral atom at the co-ordinate origin. We restrict ourselves to the lowest order in the coupling constant ($\sim e$). The calculation may proceed in two different ways. The coupled equations of motion in Heisenberg representation may be solved to order e for the operator $\boldsymbol{E}(\boldsymbol{r})$ which requires the propagator for the transverse electric field. Alternatively the computation may be carried out in Schrödinger representation, using the eigenstates which diagonalize H correct to order e. To enable a comparison with Casimir and Polder's paper we use the latter alternative.

The eigenstates of H, formula (4), for one atom at $\boldsymbol{r} = 0$ needed here, are to order e

$$(9)\qquad \begin{cases} \Phi(0;0) = |0\rangle|0) + \sum_{n}\sum_{\boldsymbol{k}\lambda} i\sqrt{2\pi k}\,\dfrac{(e_i(\boldsymbol{k}\lambda)q_{in/0})}{-(E+k)}\,|n\rangle|\boldsymbol{k}\lambda)\,, \\ \Phi(n;0) = |n\rangle|0) + \sum_{\boldsymbol{k}\lambda} i\sqrt{2\pi k}\,\dfrac{(e_i(\boldsymbol{k}\lambda)q_{in/0})}{E-k}\,|0\rangle|\boldsymbol{k}\lambda)\,, \end{cases}$$

where $|0\rangle$ is the ground state, $|n\rangle$ one of the excited p-states of the free atom and $|0)$, $|\boldsymbol{k}\lambda)$ are eigenstates of H^{RAD} corresponding to the vacuum and presence of the photon $\boldsymbol{k}\lambda$ respectively. Hence, to order e,

$$(10)\qquad \langle\Phi(n;0)|\boldsymbol{E}\cdot\hat{\boldsymbol{r}}|\Phi(0;0)\rangle = \\ = \frac{(\boldsymbol{q}_{n/0}\cdot\hat{\boldsymbol{r}})}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{k+E}\left\{\frac{\exp[ikr]}{ir}\left(-\frac{2ik}{r}+\frac{2}{r^2}\right)+\text{c.c.}\right\}.$$

The integral on the right hand side is oscillating at infinity, which is a general feature of the dipole approximation, and should be limited by a convergence factor. It is also ambiguous at $k = -E$, corresponding to the open choice of boundary condition. The retarded solution implies a detour into the complex k-plane below the pole at $k = -E$, and is given by

$$(11)\qquad \langle\Phi(n;0)|\boldsymbol{E}\cdot\hat{\boldsymbol{r}}|\Phi(0;0)\rangle = (\boldsymbol{q}_{n/0}\cdot\hat{\boldsymbol{r}})\frac{\exp[-iEr]}{r}\left(\frac{2iE}{r}+\frac{2}{r^2}\right) = \\ = \boldsymbol{J}_{n/0}\cdot\hat{\boldsymbol{r}}\exp[-iEr]\left\{\frac{2}{r^2}+\frac{2}{iEr^3}\right\}.$$

The spatial factor in (11) agrees with the well known formula for the radial component of the retarded electric field of a dipole oscillator at the origin.

The occurrence of the matrix element $\langle n|(\mathrm{d}/\mathrm{d}t)\boldsymbol{q}|0\rangle = iE\boldsymbol{q}_{n0}$ of the unperturbed current operator of the atom is due to our restriction to first order in the coupling constant, so that the current operator appears in zero order. The noteworthy analytical feature of the integrand in (12) is the regularity of both summands, containing $\exp[ikr]$ and $\exp[-ikr]$ respectively, at the origin in k-space. The only singularity met arises from the real decay possibility of the excited atom by emission of a photon with energy E. Such real processes are not encountered in the course of calculating to any order the interaction energy between two atoms initially in their ground state.

The validity of Casimir's zero point energy subtraction method involving transverse vibrations alone follows naturally from Hamiltonian (4). If in first approximation we neglect the reaction of the atoms on the field which is then treated classically we can obtain the well known perturbation energy of a dielectric inclusion in a cavity by computing the second order perturbation energy in the atomic variables.

$$(12) \qquad \sum_n \langle 0|\boldsymbol{q}(1)\cdot\boldsymbol{E}^t(\boldsymbol{x}_1)|n\rangle \frac{1}{-E}\langle n|\boldsymbol{q}(1)\cdot\boldsymbol{E}^t(\boldsymbol{x}_1)|0\rangle = -2\alpha(1)(\boldsymbol{E}^t(\boldsymbol{x}_1))^2 .$$

Using (11) the interaction (1) at the limit $R \gg \lambda\!\!\!^{-}$ follows at once.

3. – Equivalence proof.

To show that (4) is an exact consequence of the radiation theory in dipole approximation we define the polarization vector field

$$\boldsymbol{P}(\boldsymbol{x}) = \boldsymbol{q}(1)\delta(\boldsymbol{x}-\boldsymbol{x}_1) + \boldsymbol{q}(2)\delta(\boldsymbol{x}-\boldsymbol{x}_2) ,$$

and the charge-current densities

$$\varrho(\boldsymbol{x}) = -\operatorname{div}\boldsymbol{P}(\boldsymbol{x}) , \qquad \boldsymbol{J}(\boldsymbol{x}) = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{P}(\boldsymbol{x}) ,$$

which act as sources for Maxwell's equations. The Langrangian L^M of the two atoms need not be specified closely, except that it is independent of $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ which are the position vectors of the two atoms and contains $\boldsymbol{q}(1)$, $\boldsymbol{q}(2)$ in the form $\frac{1}{2}m\dot{\boldsymbol{q}}^2(1)$, $\frac{1}{2}m\dot{\boldsymbol{q}}^2(2)$. Using Coulomb gauge the Lagrangian of the total system is

$$(13) \qquad L = \frac{1}{8\pi}\int(\boldsymbol{E}^2 - \boldsymbol{H}^2)\,\mathrm{d}V + L^M + \frac{1}{c}\int\dot{\boldsymbol{P}}(\boldsymbol{x})\cdot\boldsymbol{A}(\boldsymbol{x})\,\mathrm{d}V - \int\boldsymbol{P}(\boldsymbol{x})\cdot\nabla\varphi\,\mathrm{d}V ,$$

where

$$\operatorname{div}\boldsymbol{A} = 0 , \qquad \boldsymbol{E} = -\operatorname{grad}\varphi - \frac{1}{c}\dot{\boldsymbol{A}} , \qquad \boldsymbol{H} = \operatorname{curl}\boldsymbol{A} .$$

To obtain Hamiltonian (3) is standard and is not given here. On the other hand we may subtract from (13)

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \boldsymbol{P}(\boldsymbol{x})\cdot\boldsymbol{A}(\boldsymbol{x})\,\mathrm{d}V = \int \frac{\partial}{\partial t}\left(\boldsymbol{P}(\boldsymbol{x})\cdot\boldsymbol{A}(\boldsymbol{x})\right)\mathrm{d}V\ , \tag{14}$$

where we can neglect the convective derivative since $\boldsymbol{P}(\boldsymbol{x})$ is localized at $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ which are fixed points. This leaves an equivalent Lagrangian

$$L = \frac{1}{8\pi}\int(\boldsymbol{E}^2 - \boldsymbol{H}^2)\,\mathrm{d}V + L^M - \frac{1}{c}\int \boldsymbol{P}(\boldsymbol{x})\cdot\dot{\boldsymbol{A}}\,\mathrm{d}V - \int \boldsymbol{P}(\boldsymbol{x})\cdot\nabla\varphi\,\mathrm{d}V\ . \tag{15}$$

It is well known that the Maxwell equation $\operatorname{div}\boldsymbol{E} = 4\pi\varrho$ has to be imposed as a subsidiary condition. Here this implies

$$\boldsymbol{E}^l = -\,4\pi\boldsymbol{P}^l(\boldsymbol{x}) = -\,\nabla\varphi\ , \tag{16}$$

where the affix l denotes the longitudinal (irrotational) part of a vector. The longitudinal terms in (11) therefore reduce to

$$\frac{1}{8\pi}\int|\nabla\varphi|^2\,\mathrm{d}V - \int \boldsymbol{P}(\boldsymbol{x})\cdot\nabla\varphi\,\mathrm{d}V = -\,2\pi\int \boldsymbol{P}^{l2}(\boldsymbol{x})\,\mathrm{d}V\ , \tag{17}$$

which, apart from self energy contributions, is just the instantaneous electrostatic interaction between two dipoles $\boldsymbol{q}(1)$, $\boldsymbol{q}(2)$. Equation (11) now becomes

$$L = \frac{1}{8\pi}\int\left(\frac{1}{c^2}\dot{\boldsymbol{A}}^2(\boldsymbol{x}) - (\operatorname{curl}\boldsymbol{A})^2\right)\mathrm{d}V - \frac{1}{c}\int \boldsymbol{P}^t(\boldsymbol{x})\cdot\dot{\boldsymbol{A}}(\boldsymbol{x})\,\mathrm{d}V + \tag{18}$$

$$+\,L^M - 2\pi\int \boldsymbol{P}^{l2}(\boldsymbol{x})\,\mathrm{d}V,$$

where the first two terms contain only transverse vectors. The Hamiltonian is easily obtained as

$$H = \int\left(2\pi c^2\boldsymbol{p}^2 + \frac{\boldsymbol{H}^2}{8\pi}\right)\mathrm{d}V + H^M + 4\pi c\int \boldsymbol{p}\cdot\boldsymbol{P}^t\,\mathrm{d}V + \tag{19}$$

$$+\,2\pi\int\left(|\boldsymbol{P}^l(\boldsymbol{x})|^2 + |\boldsymbol{P}^t(\boldsymbol{x})|^2\right)\mathrm{d}V,$$

where $\boldsymbol{p}(\boldsymbol{x})$ is a transverse vector field, canonically conjugate to $\boldsymbol{A}(\boldsymbol{x})$,

$$\boldsymbol{p}(\boldsymbol{x}) = \frac{1}{4\pi c^2}\dot{\boldsymbol{A}}(\boldsymbol{x}) - \frac{1}{c}\boldsymbol{P}^t(\boldsymbol{x})\ . \tag{20}$$

The last integral in (19) contains not only self energy terms but also contact interaction energies. It is of interest to analyse these contact terms wherever they arise since the longitudinal part of a δ-function is closely related to the static Coulomb interaction between two dipoles (BELINFANTE [6]). However genuine contact terms, i.e. the sum of longitudinal and transverse δ-functions, are unimportant and may be neglected. Thus in (3) we may and have subtracted from $-\nabla_i\nabla_j(1/R)$ the factor $(4\pi/3)\delta_{ij}\delta(\boldsymbol{R})$ leaving $(R^2\delta_{ij}-3x_ix_j)/R^5$. Contact terms arise also in solving the perturbation theory involving the $\boldsymbol{A}$ terms in Hamiltonian (3): these too are not considered. Nevertheless, if none of these interactions were ignored at this level the final contact energies, which would have finite range if recoil was not neglected, arising from (3) and (4) are equal to order e^4. Defining

$$4\pi c\boldsymbol{p}(\boldsymbol{x}) = -\boldsymbol{E}^t(\boldsymbol{x}) ,$$

the reduced Hamiltonian becomes

$$H = \frac{1}{8\pi}\int(\boldsymbol{E}^{t^2}+\boldsymbol{H}^2)\,\mathrm{d}V + H^M - \int \boldsymbol{E}^t\cdot\boldsymbol{P}\,\mathrm{d}V , \tag{21}$$

which is (4).

It is of interest to note that (4) can also be obtained easily by starting from the Lagrangian in the gauge $\varphi = 0$, $\operatorname{div}\dot{\boldsymbol{A}} = -4\pi c \operatorname{div}\varrho$ and carrying out the dipole approximation.

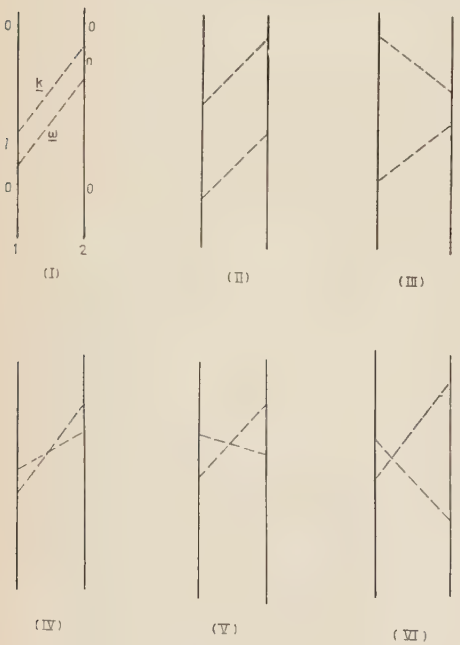

Fig. 1.

4. – Calculation.

We now turn to our main problem. We use Hamiltonian (4) and treat $\sum_i q_i(l)E_i(l)$ as a perturbation. Since the dipole moments $q_i(l)$ have only non-diagonal matrix elements we obtain a first non-vanishing contribution to the mutual energy of the two neutral atoms in order e^4, that is after two transverse photons have been exchanged. To facilitate classification of all possible processes which may occur virtually in this order we write down the set of time ordered Feynman

(6) F. J. BELINFANTE: *Physica*, **12**, 1 (1946).

graphs. There are six different graphs, not allowing for interchange of the two atoms (Fig. 1). At each vertex in any graph the interaction $q_i(l)E_i(l)$, where $l = 1, 2$, is active, and gives a factor of type $\pm i\sqrt{2\pi k}(e(\boldsymbol{k}\lambda)_i q_{in/0}(l))\cdot$ $\cdot \exp[\pm i\boldsymbol{k}\boldsymbol{x}(l)]$. The energy denominators follow by inspection. Furthermore, there is no contribution to the mutual energy from the boundary renormalization term which occurs in the perturbation theory in fourth order. Thus the six graphs exhaust the entire fourth order mutual energy shift.

We write down in detail the contribution of graph (I) (Fig. 1). This is

$$\Delta E(\mathrm{I}) = 2\langle 0|q_i(1)E_i(1)\frac{1}{-H_0}q_j(1)E_j(1)\frac{1}{-H_0}q_l(2)E_l(2)\frac{1}{-H_0}q_k(2)E_k(2)|0\rangle - \tag{22}$$

$$= -2(2\pi)^2\sum_{k\lambda}\sum_{w\mu}\sum_{ln}\frac{kw\exp[i(\boldsymbol{k}+\boldsymbol{w},\boldsymbol{R})}{(E+k)(E+w)(k+w)}(\boldsymbol{e}(\boldsymbol{w}_l\mu)\cdot\boldsymbol{q}_{0/l}(1))(\boldsymbol{e}(\boldsymbol{k}\lambda)\cdot\boldsymbol{q}_{l/0}(1))\cdot$$

$$\cdot(\boldsymbol{e}(\boldsymbol{w}_l\mu)q_{0/n}(2))(\boldsymbol{e}(\boldsymbol{k}\lambda)\cdot\boldsymbol{q}_{n/0}(2))\,,$$

where $\boldsymbol{R}$ is the vector distance between atoms 1 and 2, and where the factor 2 accounts for interchange of the two atoms. The summation over polarization directions and the excited p-states is carried out making use of

$$\sum(q_{i0/l}(1)q_{j0/l}(1)) = \delta_{ij}\langle q(1)\rangle^2\,, \tag{23}$$

and we obtain,

$$\Delta E(\mathrm{I}) = -8\pi^2\langle q(1)\rangle^2\langle q(2)\rangle^2\sum_{k,w}\frac{kw\exp[i(\boldsymbol{k}+\boldsymbol{w},\boldsymbol{R})][1+(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{w}})^2]}{(E+k)(E+w)(k+w)}\,. \tag{24}$$

The entire energy shift due to photon exchange is now written down by inspection as

$$\Delta E^M = -2(2\pi)^2\langle q(1)\rangle^2\langle q(2)\rangle^2\sum_{kw}\frac{kw[1+(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{w}})^2]\exp[i(\boldsymbol{k}+\boldsymbol{w},\boldsymbol{R})]}{(E+k)}\cdot \tag{25}$$

$$\cdot\left[\frac{1}{(E+w)(k+w)}+\frac{1}{E(E+w)}+\frac{1}{(E+k)(k+w)}+\frac{1}{(E+k)(E+w)}\right]\cdot$$

Here we have already combined a number of energy denominators. The angular integration introduces a rather complex factor and it appears desirable to convert at least one of the vector length integrations, which run from 0 to ∞, into one running from $-\infty$ to ∞. This can be achieved as follows: we have e.g.

$$\frac{1}{(E+k)}\left\{\frac{1}{(E+k)(k+w)}+\frac{1}{(E+k)(E+w)}\right\} =$$

$$= \frac{1}{(E+k)^2}\left[\frac{1}{k+w}+\frac{1}{w-k}\right]+\frac{1}{(E+k)(E+w)(k-w)}\,,$$

where the last summand is seen to vanish under the summation sign in (25). It is easily verified that this splitting up of the energy denominators does not introduce any new singularity. In this way we find, after executing the angular integrations,

$$(26)\quad \begin{cases} \Delta E^M = -\dfrac{4}{\pi^2}\dfrac{1}{R^6 E}\langle q(1)\rangle^2\langle q(2)\rangle^2 \displaystyle\int_0^\infty x^2\,\mathrm{d}x \int_0^\infty y^2\,\mathrm{d}y\, \frac{2(ER)+x}{(ER+x)^2}\left(\frac{1}{x+y}+\frac{1}{y-x}\right)\cdot \\ \qquad\qquad \cdot\,(2N(x)N(y)+M(x)M(y))\,, \\ = -\dfrac{4}{\pi^2}\dfrac{\langle q(1)\rangle^2\langle q(2)\rangle^2}{ER^3}\displaystyle\int_0^\infty \frac{x^2(2ER+x)}{(ER+x)^2}\,\mathrm{d}x\,\mathcal{P}\int_{-\infty}^{\infty}\frac{y^2\,\mathrm{d}y}{x+y}\,(2N(x)N(y)+M(x)M(y)). \end{cases}$$

Here $M(x) = \sin x + N(x) = \sin x + (\cos x)/x - (\sin x)/x^2$. It should be noted that the integral over y is a principal value. Furthermore a convergence factor at infinity is understood. The only singularity of the y-integrand is at $y = -x$ and this persists if the y-integrand is split up into complex factors involving $\exp[iy]$, $\exp[-iy]$ respectively for each summand separately. Hence, after contour integration in the complex y-plane

$$(27)\quad \Delta E^M = -\frac{4}{\pi^2}\,\frac{\langle q(1)\rangle^2\langle q(2)\rangle^2}{ER^6}\,\frac{\pi}{4i}\int_0^\infty \frac{x^4(2ER+x)\,\mathrm{d}x}{(ER+x)^2}\cdot \left[\exp[2ix]\left(1-\frac{2}{ix}-\frac{5}{x^2}+\frac{6}{ix^3}+\frac{3}{x^4}\right)-\text{c.c.}\right].$$

Here again we find that the two complex summands in the integrand are regular at the origin, so that the path of integration can be deformed without difficulty into the positive and negative imaginary axes respectively. After straight forward calculation we find

$$(28)\quad \Delta E^M = -\frac{4}{\pi}\,\frac{\langle q(1)\rangle^2\langle q(2)\rangle^2 E^2}{hcR^2}\int_0^\infty \frac{u^4\exp[-2uR]\,\mathrm{d}u}{(E^2+u^2)^2}\cdot \left(1+\frac{2}{uR}+\frac{5}{u^2R^2}+\frac{6}{u^3R^3}+\frac{3}{u^4R^4}\right),$$

which is the result of Casimir and Polder.

A physical discussion of this very interesting formula need not be repeated here and we refer to Casimir's publications. We emphasize, however, the relative simplicity of our derivation through Hamiltonian (4) involving as it does transverse waves alone. In the limit $R \gg \lambda\!\!\!^{-}$ we obtain the leading term (1)

1878

in an expansion of (28) in powers of $\hat{\lambda}/R$ directly from graphs (I) and (IV) systematically neglecting k, w against E. (The other four graphs have in this limit leading terms proportional to R^{-8}). The non-retarded London energy is, of course, more directly computed from Hamiltonian (3), but is also contained in (23) in the limit $R \ll \hat{\lambda}$. This corresponds to neglecting E against k, w in the energy denominators, a situation also encountered in deducing Breit's formula ([4]).

* * *

We wish to thank our friend L. Castillejo for most valuable discussions. We also acknowledge the assistance of M. R. Aub, and the encouragement of Professor H. S. W. Massey.

RIASSUNTO (*)

L'effetto dell'interazione elettromagnetica tra due atomi neutri è importante nelle grandi separazioni, nelle quali il potenziale di London proporzionale a R^{-6} si trasforma nel potenziale di Casimir-Polder che varia come R^{-7}. Si dimostra che per qualsiasi separazione l'energia d'interazione si può calcolare in modo relativamente semplice con l'elettrodinamica quantistica servendosi di un'hamiltoniana ridotta costruita con soli quanti trasversali. Tale hamiltoniana è giustificata dal ricorso ad un'approssimazione di dipolo fatto nel lagrangiano.

(*) *Traduzione a cura della Redazione.*

The Decay of ^{185}W.

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(ricevuto il 12 Marzo 1957)

Summary. — The decay of ^{185}W has been studied with a magnetic β-ray spectrometer. The β spectrum and the electron spectrum from a cadmium converter have been examined down to electron energies of 30 keV. The ground state β spectrum has an allowed shape and an end-point of (430 ± 4) keV. Since no evidence is found for either internal conversion electrons or γ-rays associated with 55.6 or 125 keV transitions in ^{185}Re, it is concluded that β transitions to such levels do not exist with branching ratios greater than $1.5 \cdot 10^{-3}$ and $9 \cdot 10^{-4}$, respectively.

Although there have been many investigations of the decay of 74-day ^{185}W ([1-4]), there still seems to be disagreement as to whether the decay process is simple or complex. In particular, Bisi *et al.* ([3]) have proposed a decay scheme involving a $(7/2)+$ level at 55.6 keV in ^{185}Re which is fed by 10% of the β transitions of ^{185}W and is de-excited by an M1 transition to the $d\ 5/2$ ground state of ^{185}Re. Support for their proposal has been supplied by the coincidence experiments of Bhattacherjee and Raman ([5]) and of Mijotovic ([6]). From Coulomb excitation experiments ([7]) and from studies of

([1]) F. B. Shull: *Phys. Rev.*, **74**, 917 (1948).

([2]) N. Lazar, R. J. D. Moffat and L. M. Lenzer: *Phys. Rev.*, **91**, 498 A (1953).

([3]) A. Bisi, S. Terrani and L. Zappa: *Nuovo Cimento*, **1**, 291 (1955).

([4]) V. S. Dubey, C. E. Mandeville, A. Mukerji and V. R. Potnis: *Bull. Am. Phys. Soc.*, **2**, 24 A (1957)

([5]) S. K. Bhattacherjee and Shee Raman: *Nuovo Cimento*, **3**, 1131 (1956).

([6]) A. M. Mijatovic: *Bull. Inst. Nuc. Sci.* (*Boris Kidric*), **4**, 75 (1954).

([7]) R. H. Davis, A. S. Divatia, D. A. Lind and R. D. Moffat: *Phys. Rev.*, **103**, 1801 (1956).

the decay of ^{185}Os ([8,9]) there is good evidence for a (7/2)+ level at 125 keV in ^{185}Re which is also de-excited by an almost pure M1 transition ([7]). Since both of these γ-rays should be strongly converted, the authors have undertaken an examination of the low internal conversion spectrum of ^{185}W in order to provide a sensitive test of the intensity of these transitions.

Tungsten metal was irradiated for a period of 47 days in the Chalk River reactor. The metal was dissolved in a solution of hydrofluoric and nitric acid and a β source prepared by deposition on a thin film of VYNS. The β spectrum was examined with a large double focusing β-ray spectrometer ([10]) set at a resolution of 0.87 % on the 36.1 keV *B* line of Thorium C (see insert *C* of Fig. 1). The efficiency of the anthracene detector at this energy was very nearly 100 %. A Fermi analysis of the β spectrum gave a straight line from

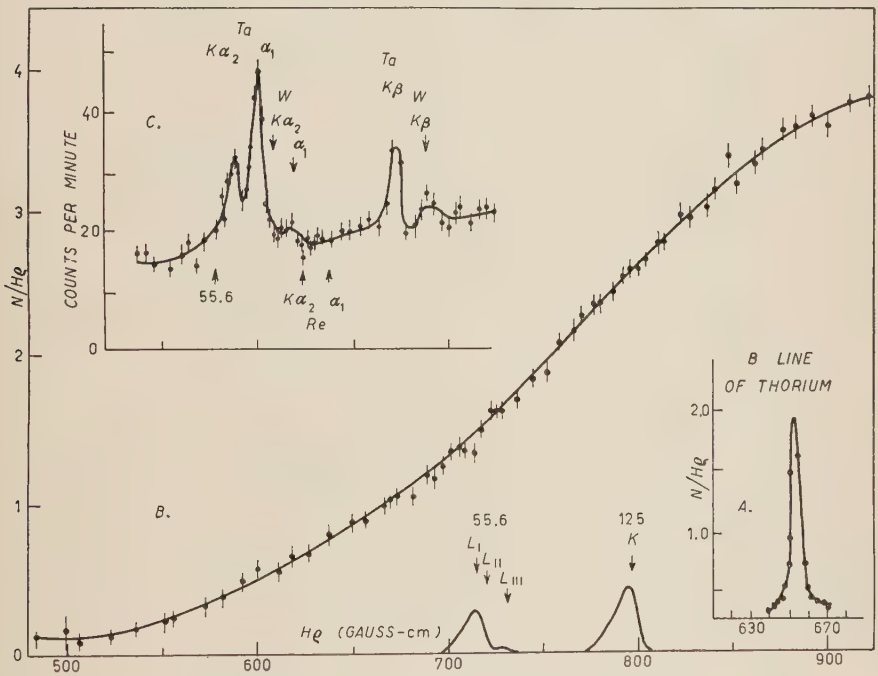

Fig. 1. – The low energy electron spectrum of ^{185}W and ^{181}W. Curve A: the low energy β spectrum; Curve B: the external conversion spectrum with a 0.3 mg/cm^2 cadmium radiator; Curve C: 36.1 keV calibration line of Thorium C.

([8]) C. H. Pruett and R. G. Wilkinson: *Phys. Rev.*, **100**, 1237 A (1955).

([9]) S. V. Nablo: *Neutron induced activities in* Ir *and* Os: Ph.D. thesis (McMaster University, Hamilton, Ontario, Canada, 1956).

([10]) M. W. Johns, H. Waterman, D. MacAskill and C. D. Cox: *Can. Journ. Phys.*, **31**, 225 (1953).

the measured end-point of (430 ± 4) keV down to 120 keV where effects of source thickness began to be apparent. Fig. 1 presents the low energy portion of the β spectrum in the neighbourhood of the L-conversion lines of the 55.6 keV and the K-conversion line of the 125 keV transition. From this data there is no evidence for the existence of the either of these radiations.

An extrapolation of the linear Fermi plot to zero energy indicated that the effect of source thickness had reduced the continuum counting rate at 42 keV to 44% and at 54 keV to 70% of the value for a source of zero thickness. The small peaks shown at the bottom of Fig. 1 indicate the line profiles expected for the $(L_1+L_2+L_3)$-conversion peak of the 55.6 keV and the K-conversion peak of the 125 keV transition. Using the efficiencies given above, the peaks have been adjusted to the area expected for a conversion line intensity of $1\cdot10^{-3}$ electrons per β-particle. The peaks shown have been given a width at half maximum of 1.5%, a value characteristic of lines in this region from osmium sources of comparable thickness examined under identical conditions.

Assuming that an $(L_1+L_2+L_3)$-conversion electron intensity of $1\cdot10^{-3}$ per β particle would easily have been detected, the branching ratio to the proposed 55.6 keV level cannot be greater than $1.5\cdot10^{-3}$ if the 55.6 keV transition is M1 in character. For other multipole assignments, the limits are slightly but not significantly different.

Assuming that the K-conversion line of the 125 keV M1 transition would be evident in Fig. 1, if its intensity were greater than $6\cdot10^{-4}$ electrons per β particle, one can similarly set an upper limit of $9\cdot10^{-4}$ for the intensity of the β group feeding this level.

The low energy external conversion spectrum of the irradiated tungsten was examined with a Cd radiator of thickness 0.3 mg/cm². The spectrum presented in curve B of Fig. 1 shows the K_α and K_β X-ray peaks from tantalum converted in the K-shell of cadmium. There is also a slight indication of the presence of tungsten X-rays which could have been caused by electron bombardment of tungsten in the source material. There is, however, no suggestion of the presence of peaks corresponding to the K-conversion of a 55.6 keV γ-ray. It can be reasonably assumed that if either peak were 0.1 of the height of the $K\alpha_1$ peak in tantalum it would have been detected.

The atomic activation cross-sections of ^{180}W and ^{184}W are given by Pomerance ([11]) as 0.03 and 0.60 barns. Although apparently the ^{185}W activity is favoured by a factor of 20 in the activation process, the errors quoted for the ^{180}W cross-section are so large that one can only be certain that the factor favouring ^{185}W is larger than 4. From this factor, and the upper limits placed

([11]) H. Pomerance: *Phys. Rev.*, **88**, 412 (1952).

1880

on the intensities of the external conversion peaks in the last paragraph, it is a straight-forward matter to show that the branching ratio to the proposed 55.6 keV level cannot be greater than $4 \cdot 10^{-2}$, and that the branching ratio to the 125 keV level cannot be greater than $1 \cdot 10^{-2}$. Although these limits are not as low as those obtained from the internal conversion data, they lead to the same general conclusions.

There is no evidence from these experiments for the existence of a level at 55.6 keV in ^{185}Re. If such a level does exist, the branching ratio to it is less than $1.5 \cdot 10^{-3}$, in marked disagreement with the 10% value given by Bisi and his co-workers. It is difficult to escape the conclusion that their results can be explained on the assumption that they were dealing with tantalum X-rays following the decay of ^{181}W which, on the basis of Pomerance's measurements, should be roughly 5% as intense as the ^{185}W β-rays. With the resolution available to them, they would be unable to differentiate between a 55.6 keV γ-ray and the 56.32 keV tantalum $K\alpha_2$ radiation.

The evidence from the present experiments for the existence of a level at 125 keV is equally negative. However, since such a level is well established by other types of experiments, the value of $9 \cdot 10^{-4}$ may be used as an upper limit for the branching ratio to this level in order to consider other features of the decay scheme. The ground state of ^{185}Re is a (5/2)+ level and the 125 keV state is the (7/2)+ first rotational level. The log f_0t value for the ground state transition from ^{185}W is 7.5 characteristic of a first forbidden transition. The log f_0t value for the 125 keV transition is greater than 10.0. These facts, coupled with the allowed shape of the ground state transition, suggest very strongly that the ground state of ^{185}W is a (3/2)− level. If this is so, the transition to the (7/2)+ level in rhenium is first forbidden unique; the lower limit of 8.7 for its log (f_1t) values ([12]) is consistent with such a view.

From these experiments we conclude that the main β decay proceeds by a first forbidden transition of allowed shape to the ground state of ^{185}Re, and that no other mode of decay with intensity greater than 0.15% exists. These conclusions are in complete agreement with the findings of Lazar ([2]) and Dubey ([4]).

* * *

This research has been supported by a grant from the National Research Council of Canada and through a scholarship awarded to one of the authors (I.R.W.) by the Ontario Research Foundation.

([12]) J. P. Davidson: *Phys. Rev.*, **82**, 48 (1951).

RIASSUNTO (*)

Con uno spettrometro magnetico a raggi β è stato studiato il decadimento del ^{185}W. Lo spettro β e lo spettro elettronico di un convertitore a cadmio sono stati esaminati fino ad energia degli elettroni di 30 keV. Lo spettro β dello stato fondamentale ha andamento permesso e un punto finale di (430 ± 4) keV. Dato che non si trovano prove della creazione di elettroni per conversione interna o di raggi γ associati a transizioni di 55.6 o 125 keV nel ^{185}Re, si conclude che le transizioni β a tali livelli non esistono con rapporti superiori rispettivamente a $1.5 \cdot 10^{-3}$ e $9 \cdot 10^{-4}$.

(*) *Traduzione a cura della Redazione.*

Ein Störungstheoretischer Zugang zu der Bruecknerschen Theorie des Atomkerns und der Watsonschen Streunäherung (*).

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(ricevuto il 14 Marzo 1957)

Zusammenfassung. — Die Bruecknersche Theorie kann man durch eine geeignete Störungsrechnung zwischen einem Modellproblem und dem exakten Problem erhalten, indem man das Auftreten sog. linked-cluster-Terme vermeidet. Es gelingt den Zusammenhang zwischen Modell und Kern exakt anzugeben. Durch Vernachlässigung von Gliedern der Größenordnung $1/A$ gelangt man dann zu den Bruecknerschen Näherungen. Das Modell wird festgelegt durch die Forderung, daß die Korrelationen zwischen zwei Teilchen so gut wie möglich berücksichtigt (d.h. die Vernachlässigungen möglichst klein) werden und daß das Modell. der Hartree-Fockschen Methode zugänglich ist. Die Watsonsche Streutheorie erhält man, indem man außerdem noch die Transformation von dem Problem freier Nukleonen in das Modell betrachtet. Eine geeignete Wahl des Modells entspricht der Watsonschen Näherung.

1. – Einleitung.

In letzter Zeit ist die Bruecknersche Theorie des Atomkerns mehr und mehr in Mode gekommen. Es besteht eben die Hoffnung, daß sie den lange gesuchten Nachweis der Brauchbarkeit des Schalenmodelles liefert. Jedenfalls ist sie derzeit die einzige Theorie, die verständlich macht, wie das Schalenmodell herauskommen *könnte*. Wir wollen uns nicht mit der Nutzanwendung der Theorie bezw. mit Prognosen hierüber beschäftigen. Darüber ist von Brueckner und Bethe bereits viel Wichtiges gesagt.

(*) Vorläufige Mitteilung in *Zeits. f. Naturf.*, **12***a*, 85 (1957). Dort ist die Brillouin-Wignersche Störungstheorie zugrunde gelegt. Hier wird etwas anders verfahren.

Uns interessiert hier mehr die Begründung der Methode, die bisher immer als « deux ex machina » erschien. Von BETHE [1], BRUECKNER [2] und EDEN [3] ist zwar der Nachweis erbracht worden, daß die Methode für einen Kern brauchbar ist, weil die Kernkräfte Absättigung zur Folge haben. Was bisher fehlte, war eine systematische Herleitung aus der exakten Theorie des Vielkörperproblems.

In letzter Zeit haben BRENIG [4] und der Verf. [5] zwei scheinbar unabhängige Herleitungen der Theorie gegeben (*). BRENIG geht aus von einer Untersuchung der Korrelationen zwischen den Teilchen: Er definiert ein System von nicht notwendig orthogonalen Enteilchenfunktionen φ^{ν}, die das Modell repräsentieren (Wellenfunktion = Slaterdeterminate aus den φ^{ν}); die Korrelationen zwischen einem, zwei usw. Teilchen beschreibt er durch die Wellenfunktionen

$$\varphi_1(1) \quad = \int \prod_{\varrho=2}^{A} \varphi^{\varrho *}(\varrho)\psi(1 \ldots A)\, \mathrm{d}x_2 \ldots \mathrm{d}x_A ,$$

$$\psi_{12}(1, 2) = \int \prod_{\varrho=3}^{A} \varphi^{\varrho *}(\varrho)\psi(1 \ldots A)\, \mathrm{d}x_3 \ldots \mathrm{d}x_A .$$

u.s.w. .

Die Wellengleichungen für φ_1, ψ_{12} usw. enthalten ein nichthermitesches Potential, also komplexe Einteilchen-, Zweiteilchen- usw. Energien. Sie stimmen (im wesentlichen) mit den Gleichungen für die « Korrelationsmatrix » $\Omega'_{i\varkappa}$ (s. unten) überein. Der Vorteil dieses Verfahrens ist die anschauliche physikalische und dabei doch systematische Formulierung, der Nachteil die unphysikalische Nichthermitizität des mittleren Potentials und der Verzicht auf eine Übersicht über die Größe der Vernachlässigungen.

Im folgenden soll eine Annäherung an die Brueckner'sche Theorie erläutert werden, die weniger anschaulich als die Brenig'sche ist, doch das Entstehen der « Fehler » und damit ihre Größe erkennen läßt. Wir wollen jetzt den physikalischen Hintergrund unserer Methode erläutern, um uns dann nur noch mit der formalen Seite beschäftigen zu müssen.

Die Idee ist etwa folgende: Man weiß nach CHEW und GOLDBERGER [6]

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[1] H. A. BETHE: *Phys. Rev.*, **103**, 1353 (1956). Dort weitere Literatur.

[2] K. A. BRUECKNER: *Phys. Rev.*, **100**, 36 (1955).

[3] R. J. EDEN: *Proc. Roy. Soc.*, **235**, 408 (1956).

[4] W. BRENIG: Manuskript. Der Verf. ist Herrn Dr. BRENIG für die Übersendung dieser besonders einleuchtenden Arbeit vor der Publikation zu Dank verpflichtet.

[5] H. KÜMMEL: *Zeits. f. Naturf.*, **12***a*, 85 (1957).

(*) Inzwischen erschien eine weitere Arbeit zu diesem Problem von Goldstone. Über den Zusammenhang seiner Theorie mit der unsrigen vgl. Anhang III.

[6] G. F. CHEW und M. L. GOLDBERGER: *Phys. Rev.*, **87**, 778 (1952).

[ASN]

daß die Matrix

$$(\alpha) \qquad \Omega'_{i\varkappa} = 1 + \frac{P}{E - \sum \overset{0}{H}_i} v_{i\varkappa} \Omega'_{i\varkappa} ,$$

die Wechselwirkung eines Teilchenpaares $(i, \varkappa)$ beschreibt (*), wenn die anderen Teilchen diese nicht beeinflussen, d.h. wenn i und $\varkappa$ sonst frei sind. Der Operator bestimmt dann wesentlich die Korrelation zwischen den Teilchen i und $\varkappa$, indem er die Wellenfunktion χ der freien Teilchen in eine solche umwandelt, in der die Zweiteilchenwechselwirkungen jeweils so eingehen, als ob immer nur zwei Teilchen aufeinander wirken. Entsprechend ist

$$(\beta) \qquad \Omega''_{i\varkappa} = 1 + \frac{P}{E - \sum \overset{0}{H}_i - \sum \bar{g}_{i\varkappa}} (v_{i\varkappa} - \bar{g}_{i\varkappa}) \Omega''_{i\varkappa} ,$$

der Operator, der die Korrelationen zwischen i und $\varkappa$ beschreibt, wenn die Teilchen sonst nicht frei sind, sondern mit den anderen Teilchen über das Potential $\bar{g}_{i\varkappa}$ in Wechselwirkung stehen. Will man nun von einer Modellwellengleichung

$$(\gamma) \qquad (\sum \overset{0}{H}_i + \sum \bar{g}_{i\varkappa}) \varphi = E \varphi$$

ausgehen, wobei φ die Zweiteilchenkorrelationen mit Einfluß aller Teilchen richtig beschreiben soll, so hat man die « Reaktionsmatrix » $g_{i\varkappa} = v_{i\varkappa} \Omega''_{i\varkappa}$ für $\bar{g}_{i\varkappa}$ einzusetzen, die gerade das richtige Potential für diese Korrelationen gibt. Dann hätte man also eine Theorie, in der die Zweiteilchenkorrelationen *exakt* berücksichtigt sind. Jedoch ist das Problem (γ) praktisch nicht lösbar. Man muß daher statt $\bar{g}_{i\varkappa} = g_{i\varkappa}$ eine schwächere Relation erfüllen, die einerseits die Lösbarkeit von (γ) (z.B. Anwendbarkeit der Hartree-Fockschen Näherung) garantiert, andererseits jedoch so weit wie möglich $\bar{g}_{i\varkappa} = g_{i\varkappa}$ macht. Was hier « so weit wie möglich » heißt, werden wir unten sehen. Jedenfalls führt dieser Gedanke zu einer ziemlich eindeutigen Festlegung von $\bar{g}_{i\varkappa}$.

Wenn man nun einmal die $\bar{g}_{i\varkappa}$ in (γ) kennt, kann man versuchen, (γ) zu lösen. Dann tritt die Frage auf, mit welcher Genauigkeit (γ) die exakte Wellengleichung für den Kern beschreibt. Um das zu untersuchen, muß man feststellen, wie φ mit der « richtigen » Wellenfunktion ψ zusammenhängt. Das macht man, indem man eine geeignete Störungsrechnung von φ zu dem exakten Problem einführt. Hier gehen wieder die Größen $g_{i\varkappa}$ entscheidend ein, nur treten jetzt Produkte $g_{i\varkappa} g_{em}$, $g_{i\varkappa} g_{em} g_{np}$, usw. auf, die dann die Korrelationen

(*) $\Omega'_{i\varkappa}$ bezw. $\Omega''_{i\varkappa}$ ist nicht mit $\Omega_{i\varkappa}$ bezw. $\widetilde{\Omega}_{i\varkappa}$ des 2. Abschnittes zu verwechseln! Letztere haben eine weniger anschauliche Bedeutung.

zwischen mehr als zwei Teilchen beschreiben. Man hat damit einen exakten Ausdruck für die dabei auftretende Energieverschiebung und die Transformationsmatrix von φ in ψ. Diese Störungstheorie wird allgemein im zweiten und speziell für das Modellproblem im dritten Abschnitt erläutert (*). Das gibt eine Theorie, aus der durch kleine Vernachlässigungen die Brueckner'sche Theorie (im vierten Abschnitt) abgeleitet werden kann. Im fünften Abschnitt werden dann die Konsequenzen aus der Forderung der « Lösbarkeit » der Modellwellengleichung gezogen. Das Resultat wird sein, daß die meisten Matrixelemente von $\bar{g}_{ix}$ Null gesetzt werden müssen, nur wenige in der Nähe der Diagonale liegende Elemente werden gleich solchen von g_{ix} gesetzt. Daß die Fehler der Brueckner'schen Theorie trotzdem klein werden, ist von BRUECKNER und BETHE gezeigt worden. Dabei muß darauf hingewiesen werden, daß dies nur dann gilt, wenn die Kernkräfte für Absättigung sorgen und das Pauliprinzip gilt. Bei der Betrachtung der allgemeinen Matrixelemente reproduziert man leicht ein von BETHE gefundenes Ergebnis, daß das mittlere Potential eines Teilchens in störender Weise von den Zuständen der anderen Teilchen abhängt. In dem siebenten Abschnitt wird gezeigt, daß die entwickelte allgemeine Störungstheorie leicht zu der Watson'schen Streutheorie ([7-9]), insbesondere seiner Näherung führt. Dabei schaltet man zwischen das Problem der freien Nukleonen und den exakten Kern ein Modellproblem ein, zerlegt also die Transformation Ω von χ zu ψ in zwei Schritte: $\Omega = F \cdot \Omega_c$. Wählt man nun das Modellproblem passend, so kann man die Watson'sche Näherung reproduzieren, ebenso aber andere Verfahren. Im Anhang wird die (für die Herleitung der Brueckner'schen Theorie nicht so geeignete) Brillouin-Wigner'sche Störungstheorie diskutiert und eine systematischere Festlegung des Modellpotentials angedeutet. Sie läßt aber noch viel zu wünschen übrig, weil es nicht gelingt, hierfür Vorschriften abzuleiten, die in jeder Näherung gültig sind.

Schließlich wird in einem dritten Anhang der interessante Zusammenhang mit der Methode von GOLDSTONE diskutiert. Wir haben dies im Anhang gebracht, weil seine Arbeit erst nach Fertigstellung des Manuskripts erschien.

Die Untersuchung der Kleinheit der Vernachlässigungen ist ausführlich von BRUECKNER und BETHE vorgenommen worden. Dem haben wir hier nichts hinzuzufügen. Mit der Lösung der Integralgleichung für g_{ix} haben sich neuerdings BRUECKNER und WADA ([10]) sowie BETHE und GOLDSTONE ([11]) be-

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(*) Sie ist so eingerichtet, daß die störenden « unlinked graphs » nicht auftreten. Über den Zusammenhang mit einer ähnlichen Theorie von Goldstone vgl. Anhang III.

([7]) K. M. WATSON: *Phys. Rev.*, **89**, 875 (1953).

([8]) K. M. WATSON and N. C. FRANCIS: *Phys. Rev.*, **92**, 291 (1956).

([9]) K. M. WATSON: *Phys. Rev.*, **103**, 489 (1956).

([10]) K. A. BRUECKNER und W. WADA: *Phys. Rev.*, **103**, 1010 (1956); dort weitere Literaturangaben.

([11]) H. A. BETHE und J. GOLDSTONE: *Proc. Roy. Soc.*, **238**, 551 (1957).

1886

schäftigt. Auf diese sehr interessanten und wichtigen Arbeiten weisen wir nur hin.

2. – Störungsrechnung.

Wir wollen in diesem Abschnitt eine allgemeine Störungstheorie für Mehrteilchensysteme zusammenstellen. Es handelt sich dabei im wesentlichen um eine passende Formulierung und geringe Abänderung schon bekannter Theorien. Dabei werden wir zwei Möglichkeiten diskutieren.

Es sei also ein Hamiltonoperator $H = \overset{0}{H} + \overset{1}{H}$ mit der «Störung» $\overset{1}{H}$ vorgegeben. Das Eigenwertproblem

$$H\psi = E\psi \tag{1}$$

soll gelöst werden, wenn

$$\overset{0}{H}\chi_\nu = \overset{0}{E}_\nu\chi_\nu \tag{2}$$

bereits gelöst ist.

a) Der übliche Weg (s. CHEW-GOLDBERGER [6]) ist der folgende: Wenn χ_ϱ eine Lösung zum Eigenwert $\overset{0}{E}_\varrho$ von (2) ist, dann ist

$$\psi_\varrho = \Omega\chi_\varrho \tag{3}$$

mit

$$\Omega = 1 + \frac{P_\varrho}{e_0}\overset{1}{H}\Omega\,, \qquad e_0 = E - \overset{0}{H}\,. \tag{4}$$

Eigenfunktion zu (1) mit der Normierung $(\psi_\varrho, \chi_\varrho) = 1$ und dem Eigenwert

$$E = \overset{0}{E}_\varrho + (\varrho\,|\,T\,|\,\varrho) \tag{5}$$

(wenn $\overset{0}{E}_\varrho$ nicht entartet ist). Dabei ist T durch die Integralgleichung

$$T = \overset{1}{H} + \overset{1}{H}\frac{P_\varrho}{e_0}T\,, \tag{6}$$

mit

$$P_\varrho\chi_\nu = \begin{cases} 0\,, & \text{wenn } \overset{0}{E}_\nu = \overset{0}{E}_\varrho \\ \chi_\nu & \text{sonst} \end{cases} \tag{7}$$

bestimmt. Für Ω kannn man auch

$$\Omega = 1 + \frac{P_\varrho}{e_0} T , \tag{8}$$

schreiben, weil

$$T = \overset{1}{H}\Omega \tag{9}$$

gilt. (6) kann man formal durch

$$T = \left(1 - \overset{1}{H}\frac{P_\varrho}{e_0}\right)^{-1} \overset{1}{H} . \tag{10}$$

lösen. T und Ω hängen von ϱ ab (*).

Es habe nun $\overset{1}{H}$ die Form

$$\overset{1}{H} = \sum_{i<\varkappa}^{A} \overset{1}{H}_{i\varkappa} . \tag{11}$$

Es sollen also mehrere Teilchen vorhanden sein, die durch Zweikörperkräfte aufeinander wirken. Die Gleichung (6) ist dann sehr kompliziert. In dem Bestreben, die Wechselwirkungen verschiedener Paare weitgehend zu entkoppeln oder wenigstens zu erkennen, wie sie zusammenhängen, wird man Größen

$$T_{i\varkappa} = \overset{1}{H}_{i\varkappa} + \overset{1}{H}_{i\varkappa}\frac{P}{e_0} T_{i\varkappa} , \tag{12}$$

einführen, die wenigstens im wesentlichen die Reaktionen zwischen einzelnen Paaren beschreiben. Ω kann man natürlich nicht in entsprechender Weise zerlegen, sondern muß

$$\Omega_{i\varkappa} = 1 + \frac{P}{e_0}\sum_{em}{}' T_{em}\Omega_{em} , \tag{13}$$

ansetzen (wobei die Summe über alle «anderen» Paare geht). Man zeigt nun leicht, daß dann

$$T = \sum_{i<\varkappa} T_{i\varkappa}\Omega_{i\varkappa} \tag{14}$$

wird. Wir geben den folgenden kurzen Beweis: Wegen

$$\sum_{em}{}' T_{em}\Omega_{em} = T - T_{i\varkappa}\Omega_{i\varkappa} ,$$

(*) Wir werden den Index ϱ im folgenden oft weglassen.

18881

folgt aus (13):

$$\Omega_{i\varkappa} = \left(1 + \frac{P}{e_0} T_{i\varkappa}\right)^{-1} \left(1 + \frac{P}{e_0} T\right).$$

Andererseits kann man (12) auch

$$T_{i\varkappa} = \overset{1}{H}_{i\varkappa} \left(1 + \frac{P}{e_0} T_{i\varkappa}\right),$$

schreiben. Daher ist

$$\sum_{i<\varkappa} T_{i\varkappa}\Omega_{i\varkappa} = \sum_{i<\varkappa} \overset{1}{H}_{i\varkappa} \left(1 + \frac{P}{e_0} T\right),$$

und das ist mit T nach (6) identisch.

Das ist der Formalismus, der der Tamm-Dancoff-Methode (*) bezw. der Brillouin-Wigner-schen Störungstheorie entspricht. Bevor wir eine abgeänderte Fassung behandeln, wollen wir eine Bemerkung über die Nachteile dieser Methode machen: Es kann leicht vorkommen, daß der Eigenwert E des gestörten Problems mit einem Eigenwert $\overset{0}{E}_\nu$ des ungestörten zusammenfällt. Das ist nicht so schlimm, weil man sich leicht überlegt, daß man dann bei der Summation über die Zwischenzustände $\overset{0}{E}_\mu$ diesen Term ganz weglassen muß. Jedoch wird die Sache dann problematisch, wenn ein $\overset{0}{E}_\nu$ nur in die Nähe von E kommt. Dann wird der Resonanznenner sehr klein und die störungstheoretische Entwicklung konvergiert sehr langsam, wird also praktisch unbrauchbar (+). Daher braucht man für die Begründung der Brueckner-schen Theorie noch eine zusätzliche Überlegung, deren Notwendigkeit nicht so gut einzusehen ist (vgl. den Anhang II).

b) Wir wollen daher ein etwas abgeändertes Verfahren wählen (−): Während oben die Energieverschiebung

$$E - \overset{0}{E}_\varrho \equiv \Delta, \tag{15}$$

nach (5) durch $(\varrho | T | \varrho)$ gegeben war, lassen wir Δ jetzt als eine prinzipiell bestimmbare Zahl in der Störungstheorie stehen. Wir führen jetzt also den Operator (×)

$$\widetilde{\Omega} = 1 + \frac{P_0}{\overset{0}{E}_\varrho - \overset{0}{H}} \overset{1}{\widetilde{H}} \widetilde{\Omega}, \tag{16}$$

(*) Vgl. z.B. W. Silin und W. J. Fainberg: *Fortschr. d. Phys.*, **4**, 233 (1956).

(+) Man kann auch sagen, daß in der Entwicklung « unlinked graphs » auftreten, die die Konvergenz stören; vgl. hierzu Anhang III.

(−) Vgl. hierzu Anhang III.

(×) Natürlich ist $\widetilde{\Omega} = \Omega$. Das neue Symbol soll nur das veränderte Iterationsverfahren andeuten.

ein mit

$$\overset{1}{\tilde{H}} = \overset{1}{H} - \Delta \,. \tag{17}$$

Wir können dann offensichtlich den ganzen Formalismus ohne Veränderung übernehmen. Wir haben überall statt $\overset{1}{H}\overset{1}{\tilde{H}}$ zu setzen. Als Energieverschiebung muß man setzen:

$$\Delta = \left(\varrho \; \overset{1}{H} + \overset{1}{\tilde{H}} \frac{P}{\tilde{e}_0} \overset{1}{\tilde{H}} \Omega \,\middle|\, \varrho \right) \qquad \text{bzw.} \qquad (\varrho \,|\, \tilde{T} \,|\, \varrho) = 0 \,. \tag{18}$$

Δ bestimmt sich also aus einer (nichtlinearen) Gleichung. Hier ist

$$\tilde{T} \;\; = \sum \tilde{T}_{\varkappa} \tilde{\Omega}_{\varkappa} \,, \tag{19}$$

$$\tilde{T}_{i\varkappa} = \overset{1}{\tilde{H}}_{i\varkappa} + \overset{1}{\tilde{H}}_{i\varkappa} \frac{P}{\tilde{e}_0} \tilde{T}_{i\varkappa} \,, \qquad \tilde{e}_0 = \overset{0}{E}_{\varrho} - H \,, \tag{20}$$

$$\tilde{\Omega}_{i\varkappa} = 1 + \frac{P}{\tilde{e}_0} \sum_{e\,m}{}' \; \tilde{T}_{em} \tilde{\Omega}_{em} \,, \tag{21}$$

$$\overset{1}{\tilde{H}}_{\varkappa} = \overset{1}{H}_{i\varkappa} - \Delta_{i\varkappa} \,, \tag{22}$$

$$\Delta_{i\varkappa} = \Delta \binom{A}{2}^{-1} , \tag{23}$$

um wenigstens einige der Ausdrücke unserer Theorie explizit hinzuschreiben.

Der Vorteil unseres Verfahrens liegt auf der Hand (*): Während bei dem Chew-Goldberger-Formalismus die Güte der Störungstheorie allein davon abhängt, ob zufällig E in die Nähe von $\overset{0}{E}_{\nu}$ kommt, ist jetzt ihre Güte allein von dem Abstand der ungestörten Terme abhängig, d.h. es geht hier der zunächst unbekannte Einfluß der Störung nicht in die Theorie ein. Wenn der Abstand hinreichend groß ist, ist die Störungstheorie *immer* brauchbar. Wir werden auf diesen Sachverhalt noch einmal zurückkommen, wollen daher die Begründung unserer Rechnung zunächst nicht weiter verfolgen.

Zum Abschluß dieser allgemeinen Erörterungen wollen wir noch auf folgende Punkte eingehen:

1. Wir haben vorausgesetzt, daß $\overset{0}{E}_{\varrho}$ nicht entartet ist. Die Zulassung der Entartung bringt keine grundsätzliche Abänderung mit sich. Bei der Diskussion der exakten Brueckner'schen Gleichungen werden wir diese Verallgemeinerung noch einmal aufgreifen.

(*) S. auch Anhang III.

1850

2. Im allgemeinen wird die Termordnung der E nicht mit denen der $\overset{0}{E}_\nu$ übereinstimmen. Wegen des bekannten Wignerschen Theorems, wonach sich Terme gleicher Rasse nicht überschneiden, wird man allerdings erwarten dürfen, daß der Grundzustand von $\overset{0}{H}$ in den Grundzustand von $H = \overset{0}{H} + \overset{1}{H}$ übergeht.

3. Einige Schwierigkeit macht der wichtige Fall, daß $\overset{0}{H}$ ein kontinuierliches Spektrum hat. Bei unseren Überlegungen muß man aus Symmetriegründen den Operator P_ϱ durch den « Hauptwert » P ersetzen. Über den Zusammenhang zwischen gebundenen Zuständen und Streuzuständen vgl. ([12-14]).

4. Die Berücksichtigung der Pauliprinzips ist kein Problem (und natürlich von entscheidender Bedeutung). Man denke sich alle Operatoren im antisymmetrischen Teilraum des Hilbertraumes gebildet. Wir werden dies später explizit durchführen.

3. – Die « exakte Brueckner sche Theorie ».

Statt etwa von einem Problem freier Teilchen vom Typus (2) auszugehen, versucht man es mit einem « Modellproblem »

$$H_M \varphi_\varrho = E_\varrho \varphi_\varrho \quad \text{mit} \quad H_M = \sum \overset{0}{H}_i + \sum \bar{g}_\varkappa \tag{24}$$

mit zunächst *völlig willkürlichem Modellpotential* $\bar{g}_{i\varkappa}$. Der Grund liegt auf der Hand: Die freien Nukleonen sind für den Atomkern eine sehr viel schlechtere Näherung als beispielsweise das Schalenmodell. Wir können dann den Formalismus des vorigen Abschnittes übernehmen. Wir wollen uns dabei auf die dortige zweite Formulierung beschränken, weil sie etwas konsequenter zu den Brueckner schen Näherungen führt. Die erste Methode behandeln wir kurz im Anhang II.

Wir brauchen nur

$$\overset{0}{H} \to H_M, \qquad \overset{1}{H}_\varkappa \to v_{i\varkappa} - \bar{g}_{i\varkappa} \tag{25}$$

zu ersetzen. Jedoch ändern wir aus Konventionsgründen die Bezeichnungen (*).

Es wird

$$\psi_\varrho = F\varphi_\varrho \tag{26}$$

([12]) A. Reifman, B. S. De Witt and R. G. Newton: *Phys. Rev.*, **101**, 877 (1956).

([13]) N. Fukuda und R. G. Newton: *Phys. Rev.*, **103**, 1585 (1956).

([14]) B. S. De Witt, *Phys. Rev.*, **103**, 1565 (1956).

(*) D.h. es ist $\tilde{\Omega} \to F$, $\tilde{T} \to g - \bar{g}$ und $\overset{1}{H}_{i\varkappa} \to v_{i\varkappa}$ ersetzt.

mit

$$F = 1 + \frac{P}{e} \sum_{i<\varkappa} (\tilde{g}_{i\varkappa} - \bar{g}_{i\varkappa}) F_{i\varkappa} , \tag{27}$$

$$F_{i\varkappa} = 1 + \frac{P}{e} \sum_{em}{}' (\tilde{g}_{em} - \bar{g}_{em}) F_{em} , \tag{28}$$

$$\tilde{g}_{i\varkappa} = \tilde{v}_{i\varkappa} + (\tilde{v}_{i\varkappa} - \bar{g}_{\varkappa}) \frac{P}{e} (\tilde{g}_{i\varkappa} - \bar{g}_{i\varkappa}) . \quad (*) \tag{29}$$

$$e = E_\varrho - H_M \tag{30}$$

$$\bar{v}_{\varkappa} = v_{i\varkappa} - \Delta_{i\varkappa} . \tag{31}$$

Die « Energieverschiebung » Δ wird (ohne Entartung)

$$E - E_\varrho = \Delta = (\varrho | \sum_{i<\varkappa} (g_{i\varkappa} - \bar{g}_{i\varkappa}) F_{i\varkappa} | \varrho) . \tag{32}$$

mit

$$g_{i\varkappa} = \tilde{g}_{i\varkappa} + \Delta_{i\varkappa} .$$

An dieser Stelle wollen wir den Fall der Entartung von E_ϱ untersuchen. P macht alle Zustände mit der Energie E_ϱ zu Null. Der Zusammenhang zwischen der Schrödingergleichung des exakten Problems und des Modellproblems ist einfach

$$(E - \overset{0}{H})\psi = (E - H_M)\varphi_\varrho - w_1 \varphi_\varrho \tag{33}$$

mit

$$w_1 = (1 - P) \sum_{i<\varkappa} (g_{i\varkappa} - \bar{g}_{\varkappa}) F_{i\varkappa} . \tag{34}$$

Darin ist der Fall der Nichtentartung enthalten mit $\Delta = (\varrho | w_1 | \varrho)$ als Energieverschiebung (32). Dann wird die rechte Seite von (33) $(E_\varrho - H_M)\varphi_\varrho = 0$. Bei Entartung bildet man passende Linearkombinationen $\varphi_{\varrho\tau}$ in dem interessierenden Teilraum, in denen w_1 diagonal ist. Dann sind die

$$\Delta_\tau = (\varrho\tau | \sum_{i<\varkappa} (g_{i\varkappa} - \bar{g}_{i\varkappa}) F_{i\varkappa} | \varrho\tau) \tag{35}$$

(*) Man kann statt $g_{i\varkappa}$ auch die Matrix $\Omega'_{i\varkappa}$ (mit $g_{i\varkappa} = v_{ik}\Omega'_{i\varkappa}$) benutzen. Das hat manche Vorteile ($\Omega'_{i\varkappa}$ erfüllt eine einfache *Differential*gleichung s. [4] ; es beschreibt direkt die Korrelation der Teilchen i und k). Das soll hier jedoch nicht durchgeführt werden, weil wir uns eng an die übliche Formulierung anschließen wollen.

18092

die verschiedenen möglichen Energieverschiebungen $\Delta_\tau = E_\tau - E_\varrho$ und die $\varphi_{\varrho\tau}$ sind die den ψ_τ zugeordneten Lösungen. Praktisch wird man allerdings anders vorgehen (s. später).

Die Gleichungen (1) (24) und (26) bis (32) sind die Vorläufer der Brueckner-schen Theorie. Sie gelten immer noch exakt.

Die Willkür der $\bar{g}_{i\varkappa}$ ist ein ziemliches Hindernis, das man nie ganz los wird (vgl. hierzu BETHE [1] Abschn. III). Man kann sich aber von zwei naheliegenden Prinzipien leiten lassen:

1. Der « Unterschied » zwischen dem wirklichen Kern und dem Modell soll möglichst gering sein. Man kann also z.B. verlangen, daß möglichst alle Matrixelemente

$$(\varrho' | F - 1 | \varrho) \tag{36}$$

verschwinden oder klein werden sollen. Würden sie exakt verschwinden, dann wäre $\psi = \varphi$; das erreicht man natürlich nur, wenn man $g_{i\varkappa} = v_{i\varkappa}$ setzt, d.h. man kann das Verschwinden nur durch die Gleichheit von Modell und Kern erfüllen. Einfacher ist es, zu fordern, daß die Energieverschiebung

$$\Delta = (\varrho | w_1 | \varrho) = (\varrho | \sum_{i<\varkappa} (g_{i\varkappa} - \bar{g}_{i\varkappa}) F_{i\varkappa} | \varrho) \tag{37}$$

möglichst klein oder Null wird. Daß beides ziemlich auf dasselbe hinausläuft, kann man mit den im Anhang I benutzten Methoden leicht einsehen. Der Nutzen dieser Forderungen ist einleuchtend. Wenn $w_1 \varphi = 0$ ist, dann haben Modell und Kern dieselbe Energie: Das würde eine Rechtfertigung des Schalenmodells möglich machen, wenn es gelänge, E als Funktion der Nukleonenanzahlen anzugeben. Außerdem ist natürlich im Interesse einer schnellen Konvergenz der Integralgleichung für F die Forderung (36) naheliegend.

2. $\bar{g}_{i\varkappa}$ soll so gewählt werden, daß das Modellproblem hinreichend einfach ist. Z.B. kann man $\bar{g}_{i\varkappa}$ diagonal in der Energiedarstellung wählen. Das entspricht einem unendlich ausgedehnten Modellkern und wird nicht immer eine brauchbare Grundlage sein. Man wird dann besser $\bar{g}_{i\varkappa}$ « beinahe diagonal » wählen, so daß wenigstens die Hartree-Focksche Näherung zur Lösung von (24) herangezogen werden darf. Da noch andere Gesichtspunkte zu berücksichtigen sind, werden wir im übernächsten Abschnitt darauf zurückkommen.

Die Forderungen 1 und 2 widersprechen sich: Einerseits will man w_1 möglichst klein machen. Dann wird man also etwa $\bar{g}_{i\varkappa} = g_{i\varkappa}$ setzen wollen, was andererseits ein sehr kompliziertes $\bar{g}_{i\varkappa}$ liefert. Es bleibt also nichts anderes übrig, als einen Kompromiss zu schließen: Die wenigen erlaubten Matrixelemente von $\bar{g}_{\varkappa}$ sollen wenigstens (so weit wie möglich) den entsprechenden

Matrixelementen von $g_{i\varkappa}$ gleichgesetzt werden. Wir schreiben das symbolisch

$$\bar{g}_{i\varkappa} \sim g_{i\varkappa} \,. \tag{38}$$

Man kann natürlich im Zweifel sein, ob die Gleichsetzung (38) w_1 wirklich merkbar verkleinert, weil die Zahl der Matrixelemente von $\bar{g}_{i\varkappa}$ sehr klein im Vergleich zu der Zahl der Elemente von $g_{i\varkappa}$ sein wird. Das ist im allgemeinen ein berechtigter Zweifel. Jedoch dann, wenn der Kern sich absättigt, d.h. bei geeigneten Kräften $v_{i\varkappa}$, wird der « Fehler » w_1 sehr klein, während der « Gewinn » bei $(r|F-1|\varrho)$ für große Kerne klein ist. Der Beweis hierfür ist mit dem entsprechenden Betheschen Beweis identisch (vgl. auch Anhang I).

4. – Die Bruecknerschen Näherungen.

Wir haben im letzten Abschnitt den exakten Vorläufer der Bruecknerschen Theorie hingeschrieben. Ausgangspunkt für jede Näherung wird die exakte Gleichung (29) für die Reaktionsmatrix $\tilde{g}_{\varkappa}$ sein. Dies ist wegen (38) jetzt eine hoffnungslos komplizierte nichtlineare Integralgleichung geworden. Es liegt nahe, sie weitgehend zu linearisieren, also z.B. das erste $\bar{g}_{i\varkappa}$ der rechten Seite zu streichen:

$$g_{i\varkappa} = \tilde{v}_{i\varkappa} + \tilde{v}_{i\varkappa} \frac{P}{e} (\tilde{g}_{i\varkappa} - \bar{g}_{i\lambda}) \,. \tag{29a}$$

Dann verändert sich natürlich der Zusammenhang (33) zwischen Modellwellengleichung und exakter Gleichung. Es tritt dabei zusätzlich zu der Energieverschiebung w_1 der « Fehler » auf:

$$w_2\varphi_\varrho \qquad \text{mit} \qquad w_2 = -\sum_{j<\varkappa} \bar{g}_{i\varkappa} \frac{P}{e} (\tilde{g}_{i\varkappa} - \bar{g}_{i\varkappa}) F_{i\varkappa} \,. \tag{39}$$

Streicht man nun noch das zweite $\bar{g}_{\varkappa}$ in (29), setzt also

$$g_{i\varkappa} = \tilde{v}_{i\varkappa} + \tilde{v}_{i\varkappa} \frac{P}{e} \tilde{g}_{i\varkappa} \,, \tag{29b}$$

so tritt ein weiterer Fehler

$$w_3\varphi \qquad \text{mit} \qquad w_3 = -\sum v_{i\varkappa} \frac{P}{e} \bar{g}_{i\varkappa} F_{i\varkappa} \,, \tag{40}$$

auf. D.h. in (33) ist w_1 durch $\sum^{3} w_i$ zu ersetzen.

1954

Allgemein läßt sich die Wirkung unserer bisherigen Vernachlässigungen nicht abschätzen. Das wird im einzelnen von dem Potential $v_{i\varkappa}$ abhängen. Doch kann man qualitativ folgendes sagen: Wenn der Kern einen *großen endlichen* Radius hat, kann man die Summation über die Zwischenzustände zwischen den Operatoren aus (29) näherungsweise durch eine Integration über die « Impulse » ersetzen:

$$\sum_k \to \frac{v}{(2\pi)^3}\int \mathrm{d}^3k\,. \tag{41}$$

Wenn nun wegen der Absättigung das Kernvolumen proportional der (großen) Zahl A der Nukleonen ist, geben also die Summationen über die Zwischenzustände den Faktor A. In (29) gehen dabei nur die Impulse der Teilchen i und $\varkappa$ ein. Wegen des Impulserhaltungssatzes (der für $v_{i\varkappa}$ und damit für $g_{i\varkappa}$ und $\bar{g}_{i\varkappa}$ gilt, vgl. BETHE [1]), II), geht effektiv nur ein Faktor A ein. Hat nun $\bar{g}_{i\varkappa}$ keine oder nur wenig Nicht-Diagonalelemente, so ist effektiv $\bar{g}_{\varkappa}$ um den Faktor $1/A$ kleiner als $\tilde{g}_{i\varkappa}$ und $\tilde{v}_{i\varkappa}$ (*). Damit hat man eine gewisse Rechtfertigung für das obige Vorgehen. Eine andere erhält man durch die Abschätzung der Größen w_2 und w_3 (z.B. bei BETHE [1]), Abschn. XIV). Diese Abschätzung beruht praktisch auf denselben Argumenten wie die eben angestellte Überlegung.

Wir haben (29*b*) also aus der exakten Gl. (29) durch die Vernachlässigung von Größen $1/A$ erhalten. An dieser Stelle wollen wir anmerken, daß die Weglassung von $\sum \bar{g}_{i\varkappa}$ im Resonanznenner $e = E_\varrho - \sum \overset{0}{H}_i - \sum \bar{g}_{i\varkappa}$ sehr viel bedenklicher wäre: Hier handelt es sich ja um die Weglassung von A^2 Gliedern, deren jedes von der relativen Größenordnung $1/A$ ist. Einen Term der Größenordnung A dürfen wir natürlich nicht unterschlagen. Insofern ist die Brueckner-sche Näherung sehr konsequent, und es ist kein Wunder, daß die Fehler w_2 und w_3 klein werden. Der Term $\sum \bar{g}_{i\varkappa}$ ist sogar entscheidend für die Brueckner-sche Theorie. BRUECKNER selber hat versucht, ihn durch ein qualitatives Argument in die Watsonsche Streutheorie [7-9]) einzubauen und so seine Theorie zu begründen (vgl. hierzu Abschn. 7).

Schließlich stört bei der Lösung der Integralgleichung (29) bzw. (29*b*) für $g_{i\varkappa}$ die unbekannte Größe $\Delta_{i\varkappa}$. Man wird sie daher vernachlässigen wollen. Dazu muß man eine Abschätzung von $\Delta_{i\varkappa}$ gewinnen. Im Prinzip müßte man die Gleichung (32) für Δ lösen. Das kann man nun mit dem *Ansatz*

$$\Delta \sim 0.1 \cdot A \ [\mathrm{MeV}] \tag{42}$$

(vgl. BETHE) tun. Wegen

$$\Delta_{i\varkappa} = \binom{A}{2}^{-1} \Delta \sim A^{-1}\,, \tag{43}$$

(*) Die Zahl der möglichen Zustände ist proportional A; vgl. (41)!

ist dann nämlich auch $\Delta_{i\varkappa}$ hinreichend klein, um in (29*b*) vernachlässigt zu werden. Dann aber kann man mit BETHE den Ansatz (42) als richtig erkennen, indem man nun Δ nach (32) berechnet. So erhält man als Integralgleichung für die « Bruecknersche Reaktionsmatrix ».

$$G_{i\varkappa} = v_{i\varkappa} + v_{i\varkappa}\frac{P}{e}G_{i\varkappa}\,. \tag{44}$$

Damit sind wir bei der Bruecknerschen Theorie angelangt. Es ist überall $g_{i\varkappa}$ und $\tilde{g}_{i\varkappa}$ durch $G_{i\varkappa}$ und $\bar{g}_{i\varkappa}$ durch $\overline{G}_{i\varkappa}$ zu ersetzen. Wir haben uns jetzt der Betheschen Terminologie angeschlossen. Insbesondere wird

$$\Delta = (\varrho\,|\sum_{i<\varkappa}(G_{i\varkappa} - \overline{G}_{i\varkappa})F_{i\varkappa}\,|\,\varrho)\,. \tag{45}$$

Die lezte Vernachlässigung hat die Umwandlung von (33) in

$$(E - H)\psi = (E_\varrho - H_M)\varphi - \sum_{i=1}^{4} w_i\varphi \tag{46}$$

mit w_1, w_2, w_3 nach (34) (39) (40) und

$$w_4 = -\Delta F = (E_\varrho - E)F \tag{47}$$

zur Folge. Es tritt also w_1 und w_4 als zusätzlicher « Fehler » auf (w_1 war in (33) die Energieverschiebung, keine durch eine Vernachlässigung hereingekommene Größe).

Wir fassen also zusammen: Die Gleichungen (45) und (26) bis (30) mit $\Delta_{i\varkappa} = 0$ sowie $g_{i\varkappa} \to G_{i\varkappa}$ usw. stellen die Bruecknersche Theorie dar. Wir haben sie in einfacher Weise aus der exakten Theorie durch eine Reihe von Vernachlässigungen der Größenordnung $1/A$ gewonnen.

5. Modellwellengleichung und Festlegung des Modellpotentials (*).

Wir haben im 3. Abschnitt gefordert, daß die Modellwellengleichung praktisch lösbar sein muß. Da es sich um ein Problem aus vielen Teilchen handelt, ist diese Forderung damit identisch, daß sich die Wellengleichung durch die Hartree-Focksche Methode lösen lassen muß. Daher müssen die Wellenfunktionen sich als Produkte bezw. (mit Pauliprinzip) als Determi-

(*) Zu diesem Abschnitt vgl. besonders die Darstellung von BETHE [1].

1898

nanten aus Einteilchenwellenfunktionen schreiben lassen. Seien

$$\varphi_n(x) = (x\,|\,n) \tag{48}$$

die Einteilchenwellenfunktionen; dann muß also

$$\varphi = \frac{1}{\sqrt{A!}}\,\mathrm{Det}\,\{(x\,|\,n)\} \equiv \big((x_1 \dots x_A \| n_1 \dots n_A)\big)\,, \tag{49}$$

sein, und nach (24)

$$\big((n_1 \dots n_A \| \sum \overset{0}{H}_i + \sum \overline{G}_{ij} \| n_1' \dots n_A')\big) = E(n_1 \dots n_A)\,\delta_{n_1 n_1'} \dots \delta_{n_A n_A'} \tag{50}$$

gelten. Die Einteilchenfunktionen müssen Lösungen von Einteilchenwellengleichungen sein, deren allgemeinste Form

$$\overset{0}{H}_i \varphi_n(x_i) + \int \mathrm{d}^3x'(x\,|\,V\,|\,x')\varphi_n(x') = E_n \varphi_n(x)\,, \tag{51}$$

ist, mit einer Potentialmatrix $(x\,|\,V\,|\,x')$. Man kann leicht — BETHE folgend — notwendige Bedingungen für V angeben, damit (49) (50) (51) miteinander verträglich sind. Zunächst folgt aus (51)

$$(n_i\,|\,n_\varkappa) = \delta_{i\varkappa}\,, \tag{52}$$

wenn V hermitesch ist und (indem man $(n_1 n_2\,|\,\overset{0}{H}_1 + V\,|\,n_1' n_2')$ bildet):

$$\begin{cases} (n_1 n_2\,|\,\overset{0}{H}_1\,|\,n_1' n_2') = 0 & \text{für } n_2 \neq n_2' \\ (n_1 n_2\,|\,V\,|\,n_1' n_2') = 0 & \text{für } n_1 \neq n_1' \text{ und zugleich } n_2 \neq n_2'. \end{cases} \tag{53}$$

Damit enthalten sowohl $\overset{0}{H}$ als auch V nur Diagonalelemente oder Elemente, die hinsichtlich eines Teilchens nicht diagonal sind. Man könnte noch auf die Hermitizität von V und damit aber auch auf die Orthogonalitätsbeziehung (52) verzichten (vgl. BRENING [4]). Das bedeutet grundsätzlich eine Komplikation (praktisch spielt sie vermutlich keine erhebliche Rolle).

Weiterhin wird man $\overline{G}_{i\varkappa}$ so wählen, daß es möglichst die Wechselwirkung von zwei Teilchen beschreibt. Also muß

$$\overline{G}_{12} \text{ diagonal hinsichtlich der Zustände } n_3 \dots n_A \text{ sein (*).} \tag{54}$$

(*) Genauer: $\overline{G}_{12}$ ist diagonal hinsichtlich $A-2$ der A Nukleonen. Deren Zustände haben wir mit $n_3 \dots n_A$ bezeichnet.

Wünschenswert wäre vielleicht noch das Postulat, daß die Wechselwirkung eines Teilchenpaares von den Zuständen der anderen Teilchen nicht abhängt.

Wir ziehen nun die mathematischen Folgerungen aus unseren Postulaten. Dazu setzen wir in (50) $n_1 \neq n_1'$, $n_2 \neq n_2'$, $n_3 = n_3'$, ..., $n_A = n_A'$ und erhalten:

$$(55) \qquad (n_1 n_2 | \overset{0}{H}_1 | n_1' n_2') + (n_1 n_2 | \overset{0}{H}_2 | n_1' n_2') + \sum_{i<j} \left((n_1 n_2 \dots n_A \| \overline{G}_{12} \| n_1' n_2' \dots n_A) \right) = 0 .$$

Nun gilt für einen nur auf die Teilchen i und $\varkappa$ wirkenden Operator $A_{i\varkappa}$ der Satz

$$(56) \quad \left((n_1 n_2 n_3 \dots n_A \| A_{12} \| n_1' n_2' n_3 \dots n_A) \right) = \begin{cases} \dfrac{1}{A(A-1)} (n_1 n_2 n_3 \dots n_A | A_{12} | n_1' n_2' n_3 \dots n_A)_N \\ 0, \text{ wenn Pauliprinzip verletzt} \end{cases}$$

mit der Definition

$$(57) \qquad (n_1 n_2 \dots n_A | A_{12} | n_1 n_2 \dots n_A)_N = \\ = (n_1 n_2 \dots n_A | A_{12} | n_1' n_2' \dots n_A) - (n_1 n_2 \dots n_A | A_{12} | n_2' n_1' \dots n_A) .$$

jedoch nur, wenn $n_1 \neq n_1'$, $n_2 \neq n_2'$.

Aus (53) und (55) folgt dann:

$$(58a) \quad (n_1 n_2 n_3 \dots n_A | \overline{G}_{12} | n_1' n_2' n_3 \dots n_A)_N = 0 \quad \text{für } n_1 \neq n_1' \text{ und zugleich } n_2 \neq n_2' .$$

Also ist auch das Modellpotential notwendig diagonal hinsichtlich mindestens *eines* Teilchens: es « ändert » höchstens den Zustand eines der Teilchen. Wir wollen die Forderung (58*a*) durch die folgende ersetzen:

$$(58) \qquad (n_1 n_2 n_3 \dots n_A | \overline{G}_{12} | n_1' n_2' n_3 \dots n_A) = 0 \quad \text{für } n_1 \neq n_1' \text{ und zugleich } n_2 \neq n_2' .$$

Aus (58) folgt die vorhergehende Gl. (58*a*), wie sofort ersichtlich. Wir wollen auf die allgemeinere Formel (58*a*) verzichten, weil wir so das Auftreten von Austauschintegralen vermeiden. Grundsätzlich werden wir dazu nicht gezwungen. Die Ergebnisse werden kaum davon beeinflußt (*), insbesondere treten die weiter unten zu besprechenden charakteristischen Schwierigkeiten dieser Theorie bei der Wahl von (58) *und* (58*a*) auf.

Setzen wir nun zweitens in (50) $n_1 \neq n_1$, $n_2 = n_2'$, $n_3 = n_3'$, ... $n_A = n_A'$,

(*) Man braucht nur überall an die Matrizen $G_{i\varkappa}$ und $\overline{G}_{i\varkappa}$ das in (57) eingeführte Symbol N anzuschreiben. Manchmal ist auch eine zusätzliche Überlegung nötig, die wir uns hier sparen wollen.

so folgt sofort:

$$(59) \qquad (n_1|\overset{0}{H}_1|n_1') + \sum_{j=2} (n_1 n_2 \dots n_j \dots |\overline{G}_{1j}| n_1' n_2 \dots n_j \dots) = 0 \,.$$

Da $(n_1|\overset{0}{H}_1|m_1')$ unabhängig von $n_2 \dots n_A$ ist, muß, für $n_1 \neq n_1'$,

$$(60) \qquad (n_1 n_2 \dots |\overline{G}_{12}| n_1' n_2 \dots) \text{ unabhängig von } n_2 \dots n_A$$

sein.

Mit (58) und (60) haben wir die nach dem « Einfachheitsprinzip » (Anwendbarkeit der Hartree-Fockschen Näherung mit einem hermiteschen Einkörperpotential) notwendigen einschränkenden Bedingungen gewonnen. Die im dritten Abschnitt erhobene erste Forderung

$$(61) \qquad \overline{G}_{i\varkappa} \sim G_{i\varkappa}$$

(Kleinheit des Fehlers ε_1) ist nun noch zur Festlegung des Potentials zu verwerten.

Wir untersuchen zunächst den unendlichen Kern. Für diesen ist wegen des Impulssatzes (*)

$$(62) \qquad (n_1 n_2 n_3 \dots n_A | G_{12} | n_1' n_2 \dots n_A) = 0 \qquad \text{für } n_1 \neq n_1' .$$

Denn die Matrix G_{12} ist diagonal hinsichtlich mindestens $A - 2$ der Teilchen. Das gilt allgemein, wenn $v_{i\varkappa}$ ein Zweikörperpotential ist. Man sieht das direkt an Gl. (44) ein. (Im nächsten Abschnitt kommen wir hierauf noch einmal kurz zurück). Daher kann man für diesen Fall (61) durch (+)

$$(n_1 n_2 \dots n_A | \overline{G}_{12} | n_1 n_2 \dots n_A) = (n_1 n_2 \dots n_A | G_{12} | n_1 n_2 \dots n_A) \quad (\times)$$

erfüllen. Und es gibt auch keine andere (stetige) Wahl.

Für einen endlichen Kern gibt es einige Schwierigkeiten. Wenn der Kern jedoch groß ist, werden die Nichtdiagonalelemente von der Größenordnung $1/V \sim 1/A$ (Sättigung vorausgesetzt). Daher werden die im folgenden geschilderten Probleme nicht allzu gravierend sein.

(*) Die $n_i = k_i$ sind dann kontinuierliche Indizes (Impulse).

(+) Diese und alle folgenden Gleichungen sollen nur gelten, wenn das Pauliprinzip nicht verletzt ist $(n_i \neq n_\varkappa)$.

(×) $(n_1 n_2 \dots n_A | G_{12} | n_1 n_2 \dots n_A)$ ist in (75) genauer definiert.

Wir wiederholen: Einerseits sollen die Forderungen (54) (58) und (60) erfüllt werden, andererseits ist aber das konkurrierende Postulat (61) zu beachten (vgl. hierzu Anhang I).

Wir wollen drei Möglichkeiten für die Festlegung der $\overline{G}_{i\varkappa}$ diskutieren:

1. Methode: Die meisten Sorgen macht die Erfüllung der Gl. (60) für die « halb-diagonalen » Elemente. Irgendwie müssen die Indizes der Matrix $(n_1 n_2 n_3 \dots n_A | \overline{G}_{12} | n_1' n_2 n_3 \dots n_A)$ festgelegt werden, ohne daß sie von diesen « abhängt ». Der einzige Ausweg dürfte sein, daß man über diese Indizes verfügt, indem man dafür eine bestimmte festgewählte Basis $\overset{0}{n}_2 \dots \overset{0}{n}_A$ einsetzt, also

$$(63) \qquad (n_1 n_2 \dots n_A | \overline{G}_{12} | n_1' n_2 \dots n_A) = (n_1 \overset{0}{n}_2 \dots \overset{0}{n}_A | G_{12} | n_1' \overset{0}{n}_2 \dots \overset{0}{n}_A) \quad (*)$$

setzt. Alle « anderen » Teilchen sind also in diesem ausgezeichneten Zustand $|\overset{0}{n}_1 \dots \overset{0}{n}_A)$ (« chosen configuration » von BETHE). Für die Bestimmung der Diagonalelemente hat man mehr Freiheit. Man kann z.B.

$$(64a) \qquad (n_1 n_2 \dots n_A | \overline{G}_{12} | n_1 \dots n_A) = (n_1 n_2 \dots n_A | G_{12} | n_1 n_2 \dots n_A)$$

oder

$$(64b) \qquad (n_1 n_2 n_3 \dots n_A | \overline{G}_{12} | n_1 n_2 n_3 \dots n_A) = (n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A | G_{12} | n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A)$$

oder schließlich

$$(64c) \qquad (n_1 n_2 n_3 \dots n_A | \overline{G}_{12} | n_1 n_2 n_3 \dots n_A) = (\overset{0}{n}_1 \dots \overset{0}{n}_A | G_{12} | \overset{0}{n}_1 \dots \overset{0}{n}_A)$$

fordern. Gegen diese Wahl kann man folgende Einwendungen erheben: Die Fälle (64*b*) und (64*c*) erfüllen nicht die Forderung $\overline{G}_{i\varkappa} \sim G_{i\varkappa}$ (+). Denn fast alle der (sehr vielen!) Matrixelemente von $\overline{G}_{i\varkappa}$ werden nicht solchen von $G_{i\varkappa}$ gleichgesetzt. Der Fall (64*a*) läßt sich nicht stetig in den Fall (63) überführen, wenn man $n_1' \to n_1$ gehen läßt (bei großem Kern ist n_i praktisch eine kontinuierliche Variable). Außerdem haben die Matrixelemente (63) ebenfalls nicht die Eigenschaft (61). Doch muß man darauf verzichten, damit (60) gilt.

2. Methode: BETHE nimmt den Verzicht auf (60) für die meisten Matrixelemente in Kauf. Dafür muß er natürlich auf die Beschreibbarkeit durch Einteilchenwellengleichungen (51) (im Prinzip) verzichten und hat die von ihm diskutierten Schwierigkeiten, vor allem die, daß das Einteilchenpotential von den Zuständen aller Teilchen abhängt (s. unten). In unserer Schreibweise

(*) Siehe Anm. (×) Seite 39.

(+) D.h. die Fehler w_i werden nicht klein.

1999

lautet sein Ansatz (neben der immer gültigen Gl. (58)):

$$(65)\qquad (n_1 n_2 n_3 \dots n_A | \overline{G}_{12} | n_1 n_2 n_3 \dots n_A) = (n_1 n_2 n_3 \dots n_A | G_{12} | n_1 n_2 n_3 \dots n_A)$$

$$(66)\qquad (n_1 n_2 n_3 \dots n_A | \overline{G}_{12} | n_1' n_2 n_3 \dots n_A) = (\overset{0}{n_1} n_2 n_3 \dots n_A | G_{12} | \overset{0}{n_1'} n_2 n_3 \dots n_A)\,.$$

Dieser ist von der Hoffnung geleitet, daß G_{12} von $n_3 \dots n_A$ nur wenig abhängt. Offenbar erfüllt er die Forderung nach « Stetigkeit » als Funktion der fast-kontinuierlichen Variablen n_i (wegen des Impulssatzes) und die Forderung (61) außer für « wenige » Matrixelemente.

3. Methode: Die systematische Untersuchung der Zweiteilchenkorrelationen nach BRENIG [4] legt es nahe, auf die Hermitizität des Einteilchenpotentials zu verzichten. Dann sind die Eigenwerte E_n und $E(n_1 \dots n_A)$ nicht mehr reell und die Eigenfunktionen nicht mehr orthogonal. Man hat dann also einiges an « Einfachheit » aufgegeben. Auch wird man ungern komplexe Modellenergien in Kauf nehmen, obgleich gerade dieses Verfahren wegen seiner systematischen Herleitung große Vorzüge hat. Für einen großen Kern wird $E(n_1 \dots n_A)$ natürlich reell, und alle Methoden geben dieselben Resultate.

Zum Schluß dieses Abschnittes schreiben wir noch die Energie $E(n_1 \dots n_A)$ des Modelles hin. Wir beschränken uns dabei auf den Betheschen Fall. Die obige « erste Methode » gibt im wesentlichen dieselben Resultate. Für den besonders wichtigen Fall $n_3 = \overset{0}{n}_3 \dots n_A = \overset{0}{n}_A$ wird:

$$(67)\qquad E(n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) = (n_1 | \overset{0}{H}_1 | n_1) + \tfrac{1}{2} \sum_{j \neq 1} (n_1 n_2 n_3 \dots n_A | G_{1j} | n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A)$$
$$+ \text{dasselbe, 1 und 2 vertauscht}$$
$$+ \sum_{i>2} \{(\overset{0}{n}_i | \overset{0}{H}_i | n_i) + \tfrac{1}{2} \sum_j (n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A | G_{ij} | n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A)\}\,.$$

Man erkennt, daß man als Einteilchenpotential für das erste bezw. i-te Teilchen

$$(68)\qquad \begin{cases} (n_1 | V | n_1) = \sum_{j>1} (n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A | G_{1j} | n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) \\ \text{bzw.} \quad (\overset{0}{n}_i | V | \overset{0}{n}_i) = \sum_{j(\neq i)} (n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A | G_{ij} | n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) \end{cases}$$

ansetzen muß, damit $E(n_1 \dots n_A)$ entsprechend der in der Hartree-Fockschen Theorie bekannten Formel

$$(69)\qquad E(n_1 \dots n_A) = \sum_i (n_i | \overset{0}{H}_i | n_i) + \tfrac{1}{2} \sum (n_i | V | n_i) = \sum E_{n_i} - \tfrac{1}{2} \sum_i (n_i | V | n_i)$$

mit der Einteilchenenergie

$$E_{n_i} = (n_i|\overset{0}{H}_i|n_i) + (n_i|V|n_i) \tag{70}$$

geschrieben werden kann. Da jedoch $G_{i\varkappa}$ i.A., von den Zuständen der «anderen» Teilchen $\neq i, \varkappa$ abhängt, gilt dasselbe für V. Das hat aber zur Folge, daß sich z.B. $E(\overset{0}{n}_1 \dots \overset{0}{n}_A) - E(n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A)$ nicht als Differenz $E_{n_1^0} + E_{n_2^0} - E_{n_1} - E_{n_2}$ von Einteilchenergien der Teilchen 1 und 2 darstellen läßt, sondern daß

$$\begin{aligned} E(\overset{0}{n}_1 \dots \overset{0}{n}_A) - E(n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) = {} & (\overset{0}{n}_1|\overset{0}{H}_1|\overset{0}{n}_1) + \sum_{j>1} (\overset{0}{n}_1 \dots \overset{0}{n}_A|G_{1j}|\overset{0}{n}_1 \dots \overset{0}{n}_A) \\ & - (n_1|\overset{0}{H}_1|n_1) - \sum_{j>1} (n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A|G_{1j}|n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) \\ & + \text{dasselbe für Teilchen 2} \\ & + \Delta G_{12} - \delta G_{12} \end{aligned} \tag{71}$$

mit

$$\begin{aligned} \Delta G_{12} = {} & (n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A|G_{12}|n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) - (n_1 \overset{0}{n}_2 \dots \overset{0}{n}_A|G_{12}|n_1 \overset{0}{n}_2 \dots \overset{0}{n}_A) \\ & - (\overset{0}{n}_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A|G_{12}|\overset{0}{n}_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) + (\overset{0}{n}_1 \dots \overset{0}{n}_A|G_{12}|\overset{0}{n}_1 \dots \overset{0}{n}_A) \end{aligned} \tag{72}$$

und

$$\delta G_{12} = \sum_{ij(\neq 1,2)} \{(\overset{0}{n}_1 \dots \overset{0}{n}_A|G_{ij}|\overset{0}{n}_1 \dots \overset{0}{n}_A) - (n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A|G_{ij}|n_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) \tag{73}$$

gilt (*). Nur wenn $G_{i\varkappa}$ von den Zuständen der «anderen» Teilchen nicht (oder sehr wenig) abhängt, hat dies die erwünschte Form mit der Einteilchenenergie

$$E_{n_1} = (n_1|\overset{0}{H}_1|n_1) + \sum_{j(\neq 1)} (n_1 n_2 \dots \overset{0}{n}_A|G_{1j}|n_1 \overset{0}{n}_2 \dots \overset{0}{n}_A) . \tag{74}$$

Daß dieser Umstand bei der Lösung der Integralgleichung für $G_{i\varkappa}$ stört, hat BETHE erwähnt. Für die weitere Diskussion verweisen wir auf seine Arbeit. Wir wollten hier nur zeigen, daß man mit einer Betrachtung der vollständigen Matrixelemente zu etwa denselben Ergebnissen kommt wie BETHE.

6. – Eigenschaften der Reaktionsmatrix.

Der Integralgleichung (45) sieht man unmittelbar an, daß $G_{i\varkappa}$ hinsichtlich $A-2$ der Teilchen diagonal sein muß, weil $v_{i\varkappa}$ es ist. Nennen wir daher die

(*) Das letzte Glied ist von BETHE in Gl. (3.5) irrtümlich weggelassen worden. Es tritt nur bei unserer «ersten Methode» mit Gl. (64c) nicht auf.

Zustände dieser Teilchen $n_3 \dots n_A$, so wird für $n_1 \neq n_1'$, $n_2 \neq n_2'$

$$(75)\quad \big((n_1 n_2 \dots n_A \| G_{12} \| n_1' n_2' n_3 \dots n_A)\big) \equiv (n_1 n_2 n_3 \dots n_A \,|\, G_{12} \,|\, n_1' n_2' n_3 \dots n_A) =$$

$$= (n_1 n_2 \,|\, v_{12} \,|\, n_1' n_2')_N + \sum_{n_1'' n_2''} (n_1 n_2 \,|\, v_{12} \,|\, n_1'' n_2'')_N \frac{P}{E(\overset{0}{n}_1 \dots \overset{0}{n}_A) - E(n_1'' n_2'' \overset{0}{n}_3 \dots \overset{0}{n}_A)} \cdot$$

$$\cdot (n_1'' n_2'' n_3 \dots n_A \,|\, G_{12} \,|\, n_1' n_2' n_3 \dots n_A) \,.$$

Das macht deutlich, daß G_{12} von den Zuständen $n_3 \dots n_A$ abhängt, obwohl es auf diese wie eine Diagonalmatrix wirkt. Von dieser Tatsache haben wir im vorigen Abschnitt mehrmals Gebrauch gemacht. Wir haben den ausgezeichneten Zustand natürlich mit dem Ausgangszustand φ_ϱ der Störungsrechnung identifiziert:

$$(76)\quad \begin{cases} \big((x_1 \dots x_A \| \overset{0}{n}_1 \dots \overset{0}{n}_A)\big) = \varphi_\varrho \\ E(\overset{0}{n}_1 \dots \overset{0}{n}_A) = E_\varrho \,. \end{cases}$$

Der Operator P verhindert, daß die Zwischenzustände beide Teilchen in der « chosen configuration » beschreiben, d.h. es ist $n_1'' \neq \overset{0}{n}_1$ oder $n_2'' \neq \overset{0}{n}_2$.

An dieser Stelle kann man nun die folgenden weiteren naheliegenden Näherungen versuchen:

1. Man kann P durch einen anderen Operator P' ersetzen. Dann tritt in (46) ein zusätzlicher Fehler w_5 auf:

$$(77)\quad w_5 = \sum v_{i\varkappa} \frac{P - P'}{e} G_{i\varkappa} F_{i\varkappa} \,.$$

Seine Größe hängt von der Wahl von P' ab. Diese Abänderung ist wichtig, wenn man verhindern will, daß der Resonanznenner $e = E(\overset{0}{n}_i) - E(n_i)$ zu klein (und damit die rasche Konvergenz der Reihen in Frage gestellt) wird. Besonders bei der Berücksichtigung der miteinander entarteten oder fast-entarteten Zustände ist ein solches Vorgehen von Nutzen (vgl. BETHE, Abschn. XVI). Man muß dann natürlich die Kleinheit des Fehlers w_5 nachprüfen.

2. Man kann den Resonanznenner $E(\overset{0}{n}_i) - E(n_i)$ selber abändern. Z.B. kann man die Größen ΔG_{12} oder δG_{12} in (71) weglassen. Oder man ersetzt die Energiedifferenz durch eine Differenz von Einteilchenenergien (vgl. S. 42). Wir schreiben das so: Sei $e = E(\overset{0}{n}_i) - H_M$ durch e' ersetzt. Dann tritt der Fehler

$$(78)\quad w_5' = \sum v_{i\varkappa} P \left\{ \frac{1}{e} - \frac{1}{e'} \right\} G_{i\varkappa} F_{i\varkappa} \,,$$

in (46) auf. Auch diesen müßte man abschätzen. Das ist von praktischer Bedeutung, weil bei allen bisherigen Lösungsversuchen von (44) bezw. (75) e in $e' = E^0_{n_1} + E^0_{n_2} - E_{n_1} - E_{n_2}$ abgeändert worden ist.

7. – Die Streutheorie von Watson.

Wir wollen hier eine kurze Herleitung der Watsonschen Streutheorie (insbesondere seiner Näherung) geben. Sie hat einen gewissen formalen Zusammenhang mit der Brueckner'schen Theorie, der den Anstoß zu letzterer gegeben hat (vgl. hierzu auch WATSON und RIESENFELD [15]).

Wir beziehen uns dabei insbesondere auf die Watsonsche Arbeit [9], die sich mit der Anwendung auf statische Probleme (z.B. in der statistischen Mechanik) beschäftigt (*). Wir haben dabei ein ganz allgemeines Problem wechselwirkender Teilchen vor Augen, wie wir es bisher behandelt haben.

Dabei verwenden wir die im zweiten Abschnitt dargelegte « erste » (Wigner-Brillouinsche) Störungstheorie. Man kann es ebensogut mit der für die Brueckner'sche Theorie geeigneteren « zweiten » versuchen. Die Ergebnisse sind formal ähnlich, geben aber nicht genau das Watsonsche Resultat.

Im dritten Abschnitt hatten wir zwischen die Wellengleichung (2) freier Teilchen und die des Kernes (1) die Modellwellengleichung (24) eingeschoben. Wenn χ, φ bezw. ψ die Wellenfunktionen freier Nukleonen, des Modelles bezw. des Kernes sind, wird nach dem zweiten Abschnitt

$$\psi = \Omega\chi = F\varphi = F\Omega_c\varphi\,, \qquad \varphi = \Omega_c\chi\,, \qquad \Omega = F\cdot\Omega_c \tag{79}$$

mit F nach Gl. (27) und (28) mit

$$e = E - H_M \tag{80}$$

$$g_{i\varkappa} = v_{i\varkappa} + (v_{i\varkappa} - \bar{g}_{i\varkappa})\frac{P}{e}(g_{i\varkappa} - \bar{g}_{i\varkappa})\,, \tag{81}$$

und mit

$$\Omega_c = 1 + \frac{P}{E - \sum \overset{0}{H}_i}\sum \bar{g}_{i\varkappa}\Omega_c\,. \tag{82}$$

Wählt man nun speziell

$$\bar{g}_{\varkappa} = g_{c\,\varkappa} = \text{Diagonalteil von } g_{i\varkappa}\,, \tag{83}$$

[15] W. B. RIESENFELD und K. M. WATSON: *Phys. Rev.*, **104**, 492 (1956).

(*) Für Streuprobleme muß man im folgenden den Hauptwert P/e durch $1/(e + i\eta)$ ersetzen. Sonst ändert sich nichts.

hat also ein unendlich ausgedehntes Modell und denkt sich dies in (27) (28) und (79) bis (82) eingesetzt, so hat man den Zugang zur Watsonschen Näherung gewonnen. Dann wird nämlich

$$g_{i\varkappa} = v_{i\varkappa} + (v_{i\varkappa} - g_{ci\varkappa}) \frac{\acute{P}}{E - \sum \overset{0}{H}_i - \sum g_{\ i\varkappa}} (g_{i\varkappa} - g_{\ i\varkappa}) \,. \tag{84}$$

Die Ersetzung von $E - \overset{0}{H}$ durch $E - \overset{0}{H} - \sum g_{ci\varkappa}$ mußte von WATSON durch ein Plausibilitätsargument begründet werden. Hier haben wir dasselbe Resultat ganz zwanglos exakt *begründet*. Die von WATSON weiter durchgeführten Näherungen (Weglassung des ersten $g_{i\varkappa}$ bezw. beider $g_{ci\varkappa}$ in (84), können wir dann genau so begründen, wie die entsprechenden Brueckner'schen Näherungen (vgl. **4.** Abschnitt).

Damit ist auch der Zugang von der Watsonschen zur Brueckner'schen Theorie erleichtert. Auch dort treten Resonanznenner $E - \sum \overset{0}{H}_i - \sum g_{ci\varkappa}$ (für einen unendlichen Kern) auf, die man nicht leicht direkt versteht. Wie man sieht, ist die Brueckner'sche Theorie im wesentlichen die « zweite Hälfte » der Watsonschen. Nur die Transformation F vom Modell zum exakten Kern wird benutzt.

ANHANG

1. – Systematische Festlegung des Modellpotentials.

Wir wollen zeigen, daß man durch eine Untersuchung des Fehlers w_1 zu einer Bestimmung der Matrixelemente von $\bar{g}_{i\varkappa}$ kommt. Da die Rechnung ziemlich trivial ist und implizit in anderen Arbeiten zur Brueckner'schen Theorie enthalten ist, können wir uns sehr kurz fassen.

Wir versuchen, die erste Näherung

$$\overset{1}{w}_1 = \sum_{i\varkappa} \left((\overset{0}{n}_1 \dots \overset{0}{n}_A \| g_{i\varkappa} - \bar{g}_{i\varkappa} \| \overset{0}{n}_1 \dots \overset{0}{n}_A) \right), \tag{A.1}$$

von w_1 zu Null zu machen. Das gibt

$$(\overset{0}{n}_1 \dots \overset{0}{n}_A | \bar{g}_{i\varkappa} | \overset{0}{n}_1 \dots \overset{0}{n}_A) = (\overset{0}{n}_1 \dots \overset{0}{n}_A | g_{i\varkappa} | \overset{0}{n}_1 \dots \overset{0}{n}_A) \,, \tag{A.2}$$

also einen Spezialfall von (65). Die zweite Näherung

$$\overset{2}{w}_1 = \sum_{i\varkappa} \sum_{l m} \left((\overset{0}{n}_1 \dots \overset{0}{n}_A \| (g_{i\varkappa} - \bar{g}_{i\varkappa}) \frac{P}{e} (g_{\ m} - \bar{g}_{\ m}) \| n_1 \dots \overset{0}{n}_A) \right), \tag{A.3}$$

macht man zu Null, indem man

$$(n_1 \overset{0}{n}_2 \dots | \bar{g}_{12} | \overset{0}{n}_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) = (\overset{0}{n}_1 \overset{0}{n}_2 \dots | g_{12} | \overset{0}{n}_1 n_2 \overset{0}{n}_3 \dots \overset{0}{n}_A) \,, \tag{A.4}$$

setzt. Das ist ein Spezialfall von (66). In der dritten Näherung bleiben Terme (« linked cluster » - Terme von Brueckner) übrig, die man wegen der Bedingung (58) nicht zum Verschwinden bringen kann.

In ähnlicher Weise kommt man zu Bedingungen für das Potential $\bar{g}_{i\varkappa}$, wenn man das Postulat (36), d.h. « $F-1$ möglichst klein » zu erfüllen versucht. Nur ist, wie Bethe gezeigt hat, (vgl. dort Abschn. VI) der Erfolg des Verkleinerns (für einen großen Kern) nur « infinitesimal » (+).

Im ganzen muß zugegeben werden, daß die Bestimmung der $g_{i\varkappa}$ noch an Systematik zu wünschen übrig läßt. Die Bemühungen für einen endlichen Kern, $G_{i\varkappa}$ oder q_{μ} durch ein Variationsprinzip festzulegen, sind bisher fehlgeschlagen. Das hat wahrscheinlich seinen Grund darin, daß es sehr viel optimale Lösungen gibt, oder daß das Minimum von $F-1$ oder w_1 sehr breit ist. Das würde dann aber heißen, daß es auf die genaue Wahl von $G_{i\varkappa}$ oder q_{μ} nicht sehr ankommt (und daß die im 5. Abschnitt erläuterten Schwierigkeiten praktisch keine Rolle spielen).

2. – Die Brillouin-Wignersche Störungstheorie.

Diese unterscheidet sich von der oben entwickelten dadurch, daß in (27) bis (31) überall $\Delta_{i\varkappa}$ zu streichen ist, d.h. $\tilde{v}_{i\varkappa}$ ist durch $v_{i\varkappa}$ zu ersetzen. Außerdem tritt an die Stelle von e nach (30) die Größe

$$e = E - H_M \,. \tag{A.5}$$

Wenn man von dieser Theorie aus zu der Brueckner'schen gelangen will, muß man Gründe für die Ersetzung von E in (A.5) durch E_ϱ angeben. Z.B. kann man die schlechte Konvergenz anführen. Der entscheidende Grund ist jedoch der folgende: In (33) ist der Fehler w_1 nicht klein (*). Selbst dann, wenn man die von Bethe berechnete Größe $w_1 \sim 0.1 \cdot A$ MeV in den Resonanznenner der Integralgleichung einsetzt:

$$E - H_M = E_\varrho - w_1 - H_M \,, \tag{A.6}$$

versagt die von Bethe gefundene Gültigkeit der Bornschen Näherung. Die erste Näherung von G_{12} wird ca. 50% der nullten (×). In dem Resonanznenner $E - H_M$ wirkt die « kleine » Größe w_1 sehr störend, weil sie von der Größenordnung der Fermi-Energie ist, also vergleichbar mit den im Resonanznenner stehenden Einteilchenenergien. Das (mit diesem Ansatz $w_1 \sim 0.1 \cdot A$ MeV als nullte Näherung) nach (34) berechnete w_1 wird dann sehr groß.

Man *muß* also in (A.5) E durch E_ϱ ersetzen, damit der Fehler w_1 klein wird. Natürlich ist dies Vorgehen nicht so befriedigend wie das im dritten Abschnitt vorgezogene, weil man nur indirekt auf die Notwendigkeit dieser Abänderung schließen kann, und weil hier die Brueckner'schen Näherungen einen exakten

(+) φ_ϱ ist in ψ mit der « Intensität » $\exp[-\alpha A]$ enthalten. Dabei ist wie man abschätzen kann, $\alpha \gtrsim 10^{-1}$. Der Kern ist praktisch *nicht* im Modellzustand!

(*) Vgl. hierzu Anm. 50 bei Bethe (1) und unseren Anhang III.

(×) Statt 6% mit $w_1 = 0$ in (A.6).

Vorläufer haben, der selber unbrauchbar ist. Das heißt aber, daß man von der exakten Theorie nur durch Vernachlässigung großer Glieder zur Brueckner-schen Theorie kommt.

3. - Zusammenhang mit der Goldstoneschen Ableitung der Brueckner-schen Theorie.

Unsere im zweiten Abschnitt entwickelte « neue » Störungstheorie steht in einem engen Zusammenhang mit dem von GOLDSTONE ([16]) verwendeten durch zeitabhängige Störungsrechnung und zweite Quantelung gekennzeichneten Verfahren. Er zeigt durch die Untersuchung der S-Matrix, daß in ihr sog. « nicht kettenförmige » Graphen (unlinked graphs) nicht auftreten. Das ist sehr wichtig, weil diese Graphen mit A, A^2, ... usw. gehen. Sie führen daher zu einer Abhängigkeit des Fehlers w_1 von höheren Potenzen von A, so daß die Störungstheorie « große » Terme enthält, die die Konvergenz in Frage stellen.

Wir wollen zeigen, daß unsere Störungstheorie dies Ergebnis von selbst enthält: Sie ist so raffiniert gemacht, daß die *nicht kettenförmigen Graphen überhaupt nicht auftreten.*

Um dies einzusehen, betrachten wir die Vorgänger der Gleichungen (16) bis (23), bei denen also die Zerlegung nach den Zweiteilchenkorrelationen noch nicht durchgeführt ist:

$$\text{(A.8)} \qquad \begin{cases} \widetilde{\Omega} = 1 + \dfrac{P}{E - \overset{0}{H}} (\overset{1}{H} - \Delta)\widetilde{\Omega} , \\ \\ \psi = \widetilde{\Omega}\chi_\varrho . \end{cases}$$

Die Energieverschiebung ist durch

$$\text{(A.9)} \qquad \Delta = (\chi_\varrho, \overset{1}{H}\widetilde{\Omega}\chi_\varrho) ,$$

definiert, also durch eine komplizierte nichtlineare Gleichung. Wir untersuchen den Zusammenhang.

$$\text{(A.10)} \qquad \psi = \sum_\nu (\chi_\nu, \widetilde{\Omega}\chi_\varrho)\chi_\nu ,$$

zwischen Modell und Kern.

([16]) J. GOLDSTONE: *Proc. Roy. Soc.*, **293**, 267 (1957). Diese Ableitung ist besonders einfach und eindrucksvoll. Sie führt zu einer « exakten » Brueckner-schen Theorie, die mit der unsrigen identisch ist. Nur haben wir statt der Graphenschreibweise eine explizite mathematische Formulierung gefunden, die für Abschätzungen und praktische Rechnungen vor allem dann von Nutzen ist, wenn man die Integralgleichungen nicht nur durch Iteration lösen will. Auch für die Einsicht in den — oft als unverständlich empfundenen — « klassichen », Brueckner-Betheschen Formalismus is dies nützlich, wie wir gesehen haben. Man kann natürlich auch die in diesem Anhang geschilderte zeitunabhängige Störungsrechnung zum Ausgangspunkt der Goldstoneschen Ableitung machen.

Wir wollen hier die Kenntnis der Arbeit von Goldstone voraussetzen, insbesondere also von seiner Terminologie Gebrauch machen. Den Beweis werden wir jedoch ohne Inanspruchnahme der Feldtheorie und der zeitabhängigen Störungsrechnung durchführen, sondern stattdessen einen elementaren Induktionsschluß bevorzugen. Grundsätzlich kann man auch die graphischen Methoden der einzeitigen Feldtheorie (Tamm-Dancoff-Methode) benutzen.

Unsere Behauptung lautet:

1) Δ enthält alle « unlinked graphs » (abgekürzt: u.g.) in der richtigen Vielfachheit (*). Es enthält keine Produkte von u.g.

2) ψ enthält keine u.g.

Zunächst kann man leicht beweisen, dass diese Behauptungen für die nullte und erste Ordnung richtig sind. Dazu denken wir uns Δ und ψ entwickelt:

$$\Delta = \overset{1}{\Delta} + \overset{2}{\Delta} + \ldots = (\chi_\varrho, \overset{1}{H}\chi_\varrho) + (\chi_\varrho, \overset{1}{H}Q\overset{1}{H}\chi_\varrho) + \ldots , \tag{A.11}$$

$$\psi = \overset{0}{\psi} + \overset{1}{\psi} + \ldots = \chi_\varrho + \sum_{\nu(\neq\varrho)} (\chi_\nu, Q\overset{1}{H}\chi_\varrho)\chi_\nu + \ldots , \tag{A.12}$$

mit der Abkürzung:

$$Q \equiv \frac{P}{\overset{0}{E}_\varrho - \overset{0}{H}} . \tag{A.13}$$

Die nullte Näherung ist trivial. Die Richtigkeit der ersten Näherung ist ebenfalls sofort klar.

Wir zeigen nun, daß die Behauptungen 1 und 2 richtig sind, wenn man deren Richtigkeit für alle Ordnungen (= Potenzen von $\overset{1}{H}$) bis zur n-ten annimmt. Dazu entwickeln wir:

$$\tilde{\Omega} = \overset{0}{\tilde{\Omega}} + \overset{1}{\tilde{\Omega}} + \overset{2}{\tilde{\Omega}} + \ldots , \tag{A.14}$$

mit

$$\overset{n}{\tilde{\Omega}} = Q\overset{1}{H}\overset{n-1}{\tilde{\Omega}} - \overset{1}{\Delta}Q\overset{n-1}{\tilde{\Omega}} - \overset{2}{\Delta}Q\overset{n-2}{\tilde{\Omega}} - \ldots - \overset{n-1}{\Delta}Q\overset{1}{\tilde{\Omega}} . \tag{A.15}$$

Diese Gleichung entnimmt man unmittelbar aus der Integralgleichung für $\tilde{\Omega}$. Ferner:

$$\overset{n}{\Delta} = (\chi_\varrho, \overset{1}{H}\overset{n-1}{\tilde{\Omega}}\chi_\varrho) . \tag{A.16}$$

Unsere Voraussetzungen besagen, daß $\overset{n}{\Delta}$ und $\overset{n}{\psi}$ oder, was dasselbe ist, $\overset{n}{\tilde{\Omega}}$ die Behauptungen 1 und 2 erfüllen. Wir haben zu zeigen, daß dann dasselbe für

$$\overset{n+1}{\tilde{\Omega}} = Q\overset{1}{H}\overset{n}{\tilde{\Omega}} - \overset{1}{\Delta}Q\overset{n}{\tilde{\Omega}} - \overset{2}{\Delta}Q\overset{n-1}{\tilde{\Omega}} - \ldots - \overset{n}{\Delta}Q\overset{1}{\tilde{\Omega}} , \tag{A.17}$$

(*) Wir bezeichnen hier alle « Vakuum-Vakuum-Amplituden » als « unlinked graphs ».

und

$$\overset{n+1}{\Delta} = (\chi_\varrho, \overset{1}{H}\overset{n}{\tilde{\Omega}}\chi_\varrho)\,, \tag{A.18}$$

gilt, wenn es für alle niedrigeren Ordnungen gilt.

Zunächst beweisen wir dies für $\overset{n+1}{\Delta}$. Es ist

$$\overset{n+1}{\Delta} = \sum_{\nu(\neq\varrho)} (\chi_\varrho, \overset{1}{H}\chi_\nu)(\chi_\nu, \overset{n}{\tilde{\Omega}}\chi_\varrho)\,. \tag{A.19}$$

Nun enthält $\overset{n}{\tilde{\Omega}}_{\nu\varrho}$ nach Voraussetzung keine u.g. Durch das « Vorschalten » von $\overset{1}{H}_{\varrho\nu}$ wird der Zwischenzustand χ_ν in den « Vakuumzustand » χ_ϱ übergeführt, wobei ein u.g. n-ter Ordnung entsteht. Das gilt für jeden möglichen der durch $\overset{n}{\tilde{\Omega}}$ repräsentierten Graphen. Es ist klar, daß das erstens ein einfacher u.g. ist (d.h. kein Produkt von solchen) und daß zweitens jeder mögliche u.g. dieser Art auch vorkommt. Denn $\overset{n}{\tilde{\Omega}}$ enthält ja alle Graphen n-ter Ordnung, die sich durch « Schließung » mit $\overset{1}{H}$ zu einem u.g. umformen lassen.

Nun beweisen wir die zweite Behauptung, d.h. daß $\overset{n+1}{\tilde{\Omega}}$ keine u.g. mehr enthält. Wir betrachten alle Glieder außer dem ersten: sie enthalten alle u.g. erster, zweiter usw. Ordnung wegen des Faktors $\overset{1}{\Delta}$, $\overset{2}{\Delta}$, usw. Die Größen $\overset{n}{\tilde{\Omega}}$, $\overset{n-1}{\tilde{\Omega}}$, ... sind dabei alle möglichen (nach Voraussetzung « linked ») Graphen n-ter, $(n-1)$-ter, ... Ordnung. Von dem ersten Glied werden also alle Graphen abgezogen, die u.g. erster, zweiter u.s.w. Ordnung (bis zur n-ten) als Faktor enthalten. Wenn wir nun zeigen, daß das erste Glied

$$\overset{1}{Q}H\overset{n}{\tilde{\Omega}}\,, \tag{A.20}$$

an u.g. nur dieselben enthält, die in den eben untersuchten letzten Gliedern aus (A.17) vorkommen, dann ist alles bewiesen, wenn man sich noch davon überzeugt, daß u.g. $(n+1)$-ter Ordnung in (A.20) sowieso nicht vorkommen.

Die Form der in $\overset{n}{\tilde{\Omega}}$ stehenden Graphen ist in Fig. 1 angedeutet. Es treten also « getrennte graphen » (g.g.) auf, d.h. solche, die nicht mit den anderen zusammenhängen, aber im Gegensatz zu den u.g. äußere Linien enthalten. $\overset{n}{\tilde{\Omega}}$ beschreibt dann die Gesamtheit aller Graphen ohne u.g., jedoch mit allen möglichen g.g. Darunter sind alle möglichen g.g. mit einer einlaufenden und einer auslaufenden äußeren Linie, und zwar natürlich in der richtigen Vielfachheit (Fig. 2). Nun enthält (A.20) noch die « letzte » Wechselwirkung durch $\overset{1}{Q}H$, die man oben an die Graphen auf alle mögliche Weisen anzufügen hat. Dann gibt es aber alle möglichen u.g., wenn man die durch Fig. 2 dargestellten Graphen mit $\overset{1}{Q}H$ « schließt » (Fig. 3). Damit hat man aber *alle* u.g., und diese sind mit allen möglichen « linked graphs » multipliziert. Sie werden, wie gesagt, durch die restlichen Glieder in (A.17) kompensiert. Schließlich

muß man sich noch davon überzeugen, daß in $Q\overset{1}{H}\overset{n}{\tilde{\Omega}}$ nicht etwa u.g. $(n+1)$-ter Ordnung auftreten können. Das ist deshalb nicht der Fall, weil wegen Q der Endzustand nicht $= \chi_0$ (« Vakuum ») sein kann, also einige Teilchen in angeregten Zuständen sein müssen.

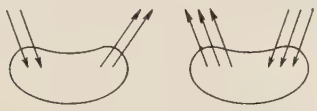

Fig. 1.

Fig. 2.

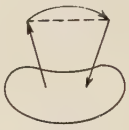

Fig. 3.

Damit ist unsere Behauptung bewiesen. Das gefundene Resultat überträgt sich auf die Ersetzung von v_{ix} durch $v_{ix} - \bar{g}_{ix}$, wie es nach Abschnitt 3 der Brueckner'schen Theorie entspricht (*). Nur hat die Wahl (38) zur Folge, daß noch weitere Terme ausfallen. Das ist im einzelnen von Goldstone diskutiert worden. Das Ergebnis ist jedenfalls, daß man die Entwicklungen für F und ε_i als Summe über « linked » Terme schreiben kann. Damit fallen alle Glieder $\sim A^2, A^3$ usw. heraus.

* * *

Für stetige Förderung und anregende Diskussionen hat der Verf. Herrn Prof. Dr. G. Ludwig sehr zu danken. Herrn Dr. H. Rollnik ist er für die Durchsicht des Manuskripts, für zahlreiche kritische Bemerkungen und Berichtigungen sehr zu Dank verpflichtet.

(*) Dabei läßt man zweckmässigerweise die Vernachlässigungen im 4. Abschnitt weg. Man kommt auf diesem Wege nämlich zu einer (im Prinzip) exakten Theorie.

Anm. bei der Korrektur:

1) Herr Brenig hat inzwischen sein Verfahren [4] weiter so verbessert, daß die oben genannten Nachteile entfallen.

2) Die Herleitung der Brueckner-Theorie läßt sich — jedoch nur für einen *unendlichen* Modellkern — im Anschluß an die von Eden und Francis angegebenen Methoden sehr vereinfachen. Hierüber hatte der Verf. eine anregende Diskussion mit Herrn Beck (München).

RIASSUNTO (*)

Si può ottenere la teoria di Brueckner per mezzo di un appropriato calcolo delle perturbazioni tra un problema applicato a un modello e il problema esatto, se si evita l'insorgenza di cosiddetti termini di linked-cluster. Si riesce a dare esattamente l'interdipendenza tra modello e nocciolo. Trascurando termini dell'ordine di grandezza $1/A$ si arriva poi alle approssimazioni di Brueckner. Si fissa il modello imponendo la condizione che le correlazioni tra due particelle siano tenute presenti per quanto possibile (cioè i tagli siano ridotti al minimo) e che il modello sia accessibile al metodo di Fock-Hartree. Si ottiene la teoria dello scattering di Watson considerando inoltre la trasformazione nel modello del problema dei nucleoni liberi. Una appropriata scelta del modello corrisponde all'approssimazione di Watson.

(*) *Traduzione a cura della Redazione.*

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Probable Nuclear Interaction of a Hydrogen Hypernucleus (*).

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(ricevuto il 20 Marzo 1957)

Summary. — A particle (A) of unit charge, mass (3.6 ± 1.1) proton masses, and velocity $v = (0.26 \pm 0.02)c$, is emitted from a 4.5 GeV π^- star in emulsion. After travelling 2.4 mm, particle A interacts with a nucleus of the emulsion, the only visible product being a helium hypernucleus, identified by direct mass and charge measurements, and by its mesonic decay. The energy of particle A is insufficient to produce directly the observed He_Λ, and the multiple scattering and ionization results on track A are inconsistent with its being a heavy meson or hyperon interacting in flight. The most probable interpretation is to identify particle A as a hydrogen hypernucleus which interacts by charge exchange or proton pickup.

1. – Introduction.

In an emulsion stack exposed to the 4.5 GeV π^- beam of the Berkeley Bevatron, an event has been found which probably can not be explained in terms of any previously observed process. Fig. 1 is a photomicrograph of this event; Table I contains data relevant to its analysis. A grey track (A) (×) of 2.4 mm length leads from a star (O) to a point (O′) from which only a heavily ionizing track (B) emerges. The latter comes to rest after 4.0 mm travel at a point (O″) where it is seen to decay into a proton (C), a negative pion (D), and a short recoil (E). From charge and mass measurements, and from the

(*) Work supported by the Greenewalt Nuclear Physics Fund, the Office of Naval Research and the Atomic Energy Commission.

(+) On leave of absence from the University of Milan.

(×) Symbols A, B, etc., will be used to refer either to tracks of Fig. 1 or to the particles which produced them.

TABLE I.

Track	Polar angle (degrees)	Dip angle (degrees)	Range (or length if it does not stop) (μm)	Energy (MeV)	Identity
A (*)	0 ± 0.2	$+24.2 \pm 1$	2444	—	See text
B	0.6 ± 0.2	-29.9 ± 1	3980 ± 110	126 ± 2 138 ± 2	^{4}He ^{5}He
C	188.4 ± 2	-22.3 ± 3	100 ± 2	$3.6 \pm .04$	p
D	$0 \pm .3$	-17.2 ± 1	15200 ± 500	29.7 ± 0.8	π^- (σ_0)
E	—	—	~ 1	~ 0.7	^{3}He or ^{4}He (from charge balance)

(*) Track A is emitted at 65° with respect to the direction of the primary of star O. Analysis of primary star yielded no associated events.

observed decay, particle B is identified as a helium hypernucleus. Since-star O ($9+0\pi$) is initiated by a beam track, it is assumed to be the parent star of particle A, which therefore traveled from O to O′ and originated the hypernucleus B. With this assumption the following alternative interpretations of this event suggest themselves:

1) Highly inelastic interaction of an energetic He_Λ at point O′, in which case A and B are the same particle;

2) Direct production of a He_Λ by particle A;

3) Interaction of a known heavy meson or hyperon in flight, leading to the production of a He_Λ;

4) Nuclear interaction (proton pickup or charge exchange) of a H_Λ;

5) Production of a He_Λ by an unknown heavy strange particle.

Choice among these interpretations rests on the information contained in the 2.4 mm of track A. Despite this limited length of track, it will be shown in the following that the estimates of M_Λ, Z_Λ, and β_Λ as determined from ionization and scattering measurements favor process 4), best satisfying energy and momentum conservation in the reaction. Conversely, each of the other interpretations, except (5) which cannot be excluded, appears to be incon-

1912

Fig. 1. - Photomicrograph of event EFINS 31.

sistent with at least one of the measured quantities. Details of the various measurement techniques and properties of the emulsions employed are described in the Appendix.

2. – Experimental.

2·1. *Identification of particle* B. – Track B is contained in two pellicles, and has a dip angle of 30° with respect to the plane of the emulsion. Its charge was determined on the basis of a plot of δ-ray density (dN_δ/dR) vs. range (R). This plot is shown in Fig. 2, where the same quantities for calibration tracks

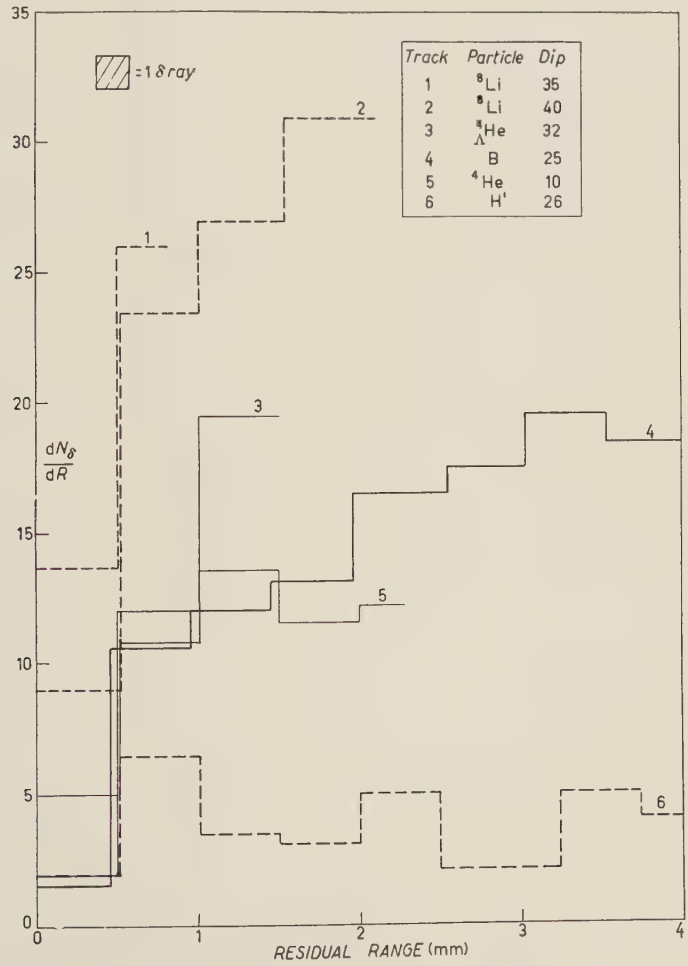

Fig. 2. – Plot of δ-ray density (dN_δ/dR) vs. range (R) for track B and calibration tracks.

of charge 1 (^{1}H), 2 (^{4}He and ^{4}He$_\Lambda$) and 3 (^{8}Li) are also given. It is clear from this figure that the various charges can be readily distinguished with the available length of track, despite the somewhat different dips of the calibration tracks. The histogram for track B falls in the group for doubly charged particles, and also indicates, together with the observed variation of multiple scattering, that particle B is brought to rest at point O″. The mass M_B, from multiple scattering vs. range with $Z_B = 2$, was found to be $(4.5^{+1.6}_{-1.2})$ proton masses. The constant sagitta cell scheme of DILWORTH *et al.* (1) and the mean sagitta for protons given by DI CORATO *et al.* (2) were employed. From these determinations of Z_B and M_B. we conclude that track B is due to a helium isotope.

Furthermore, the emission of a π^- from the secondary star at O″, and the total energy released (about 34 MeV), strongly suggest that track B is due to a helium hypernucleus which decays mesonically at O″. Data relevant to the analysis of the decay are contained in Table I, from which it is apparent that the range of the recoil (track E) is too short to allow positive identification of the hypernucleus as a specific He$_\Lambda$ isotope from momentum balance. We are therefore led to assign the secondary event to one of the known decay schemes:

(1) $$^4\mathrm{He}_\Lambda \rightarrow \pi^- + {}^1\mathrm{H} + {}^3\mathrm{He}\,,$$

(2) $$^5\mathrm{He}_\Lambda \rightarrow \pi^- + {}^1\mathrm{H} + {}^4\mathrm{He}\,,$$

B_Λ can be calculated for either scheme, assuming that the recoil, (E), balances the momentum of the other two decay products. The results, together with the deviations (S) from the mean B_Λ in units of the standard deviations for other He hypernuclei in this stack (3) are shown in Table II. There is a preference for ^{5}He$_\Lambda$, but ^{4}He$_\Lambda$ cannot be excluded. Other decay schemes of ^{4}He$_\Lambda$ or ^{5}H$_\Lambda$ lead for this event to negative binding energies.

TABLE II.

Hypernucleus	Q (MeV)	B_Λ (MeV)	S (Std. devs.)
^{4}He$_\Lambda$	34.1 ± 0.8	2.8 ± 1	2.2
^{5}He$_\Lambda$	33.9 ± 0.8	3.0 ± 1	1.0

(1) C. DILWORTH, S. GOLDSACK and L. HIRSCHBERG: *Nuovo Cimento*, **11**, 113 (1954).

(2) M. DI CORATO, D. HIRSCHBERG and B. LOCATELLI: *Proceedings of the Pisa Conference*, June 1955, *Suppl. Nuovo Nuovo Cimento*, **4**, 275 (1956).

(3) W. SLATER, E. SILVERSTEIN, R. LEVI-SETTI and V. L. TELEGDI: *Bull. Am. Phys. Soc.*, **1**, 319 (1956).

1914

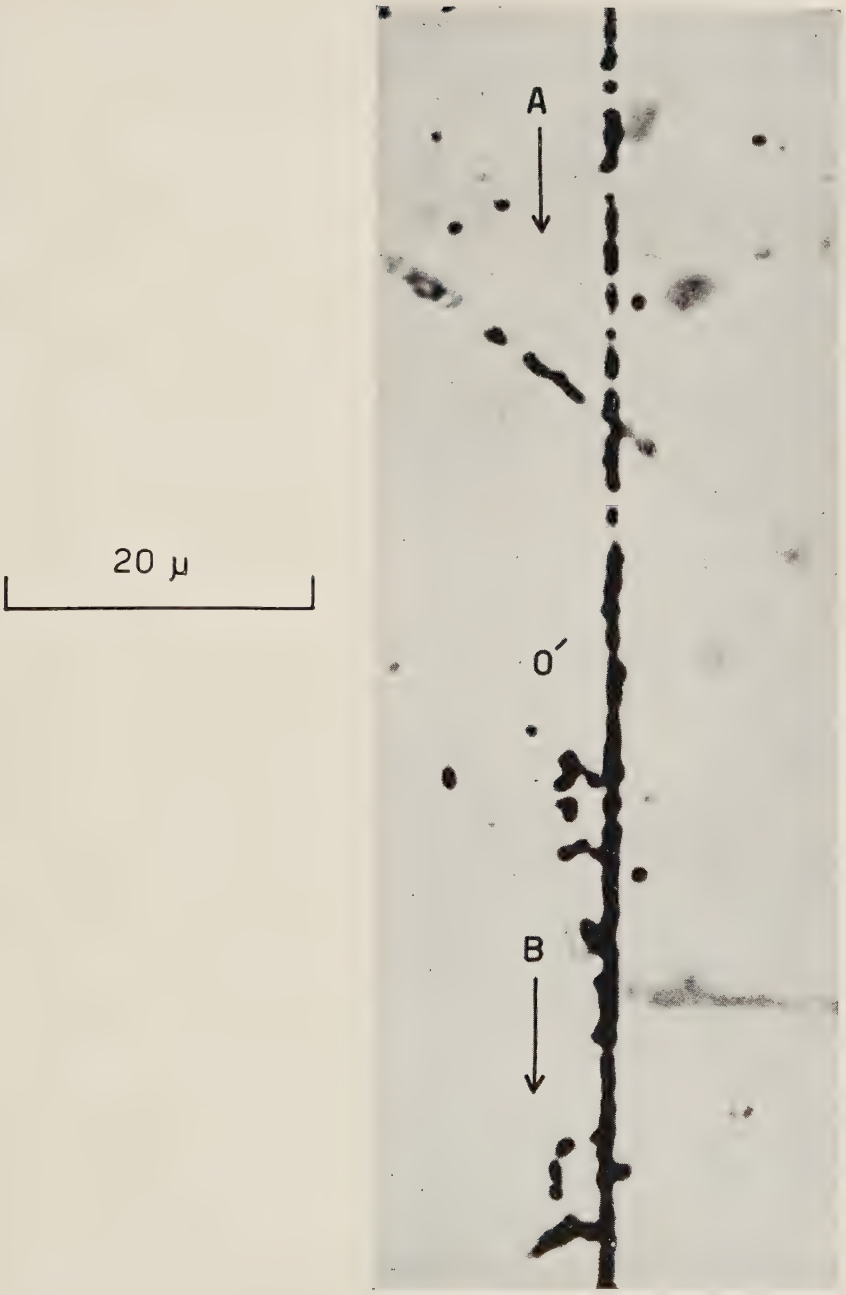

Fig. 3. – Enlarged view of the interaction point O′.

2·2. *Identification of track* A. – Track A is distinguished from track B by an abrupt change at point O′ in ionization, accompanied by an angular deviation of 0.1 radian. This discontinuity is exhibited in the photograph of Fig. 3 and in Fig. 4 where various measures of ionization, viz. δ-ray density dN_δ/dR, mean gap length ($\overline{G}$), and integral gap length per unit length (G), are plotted for track A-B as a function of distance from the primary star O. The possible explanations of this change are: (i) $Z_\Lambda \geqslant Z_B = 2$, implying that $\beta_\Lambda \gg \beta_B$, or (ii) $Z_\Lambda = 1$.

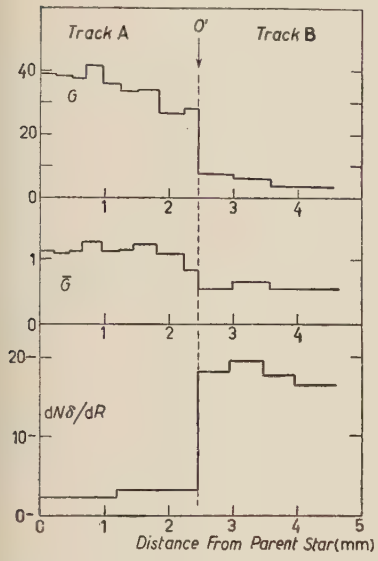

Fig. 4. – Discontinuity in ionization at point O′, shown using integral gap length per unit length (G), mean gap length ($\overline{G}$), and δ-ray density (dN_δ/dR).

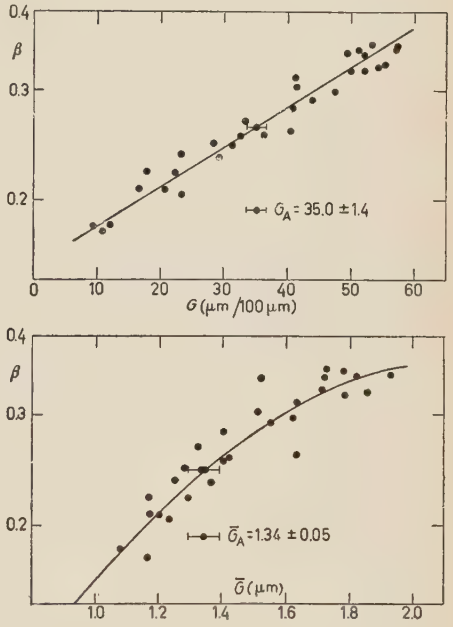

Fig. 5. – Calibration curves relating β to G and $\overline{G}$, obtained from proton tracks.

The information required to state a preference for Z_Λ is drawn from the following measurements: the velocity of track A, assuming Z_Λ is 1 or 2, was determined from a comparison of G_Λ and $\overline{G}_\Lambda$ with the same quantities for protons whose velocities are known from their ranges. Proton tracks having within 1° the same dip as track A were chosen in the same pellicle within a radius of 2 cm from track A. Fig. 5 shows calibration curves, fitted by least squares, of G and $\overline{G}$ vs. log β. Each point was determined from about 120 gaps. Since G_Λ and $\overline{G}_\Lambda$ vary only slightly in the available length of track, the average values from 640 gaps in a single pellicle were employed in the estimate of β_Λ. The values obtained are $G_\Lambda = (35.0 \pm 1.4)$ μm/100 μm, and $\overline{G}_\Lambda = (1.34 \pm 0.05)$ μm

which for $Z_A = 1$ correspond to

$$\beta_A = 0.262^{+.017}_{-.013} \quad \text{and} \quad \beta_A = 0.245^{+.025}_{-.020}, \text{ respectively, using Fig. 5.}$$

With the weighted average β, $0.257^{+.017}_{-.013}$, one can construct the following table (Table III) of kinematic quantities for various possible choices of the identity of track A, which could account for the observation of the hypernucleus B. The same quantities are listed assuming $Z_A = 2$, and for tracks B.

TABLE III.

Track	Choice of identity	β	Kinetic Energy (MeV)	Momentum (MeV/c)	Minimum missing momentum $\lvert p_A - p_B \rvert$ (MeV/c)
A ($Z_A=1$)	K	$0.257^{+.017}_{-0.13}$	$17.3^{+2.5}_{-1.9}$	$133^{+9.5}_{-7.4}$	840 ± 14
	P		$32.7^{+4.6}_{-3.5}$	251^{+18}_{-14}	753 ± 18
	Σ		$41.5^{+5.8}_{-4.5}$	318^{+23}_{-18}	686 ± 22
	Ξ		$46.0^{+6.4}_{-5.0}$	352^{+25}_{-20}	653 ± 23
	$^3H_\Lambda$		104^{+15}_{-12}	797^{+57}_{-44}	224 ± 60
	$^4H_\Lambda$		137^{+19}_{-15}	1045^{+75}_{-58}	111 ± 48
A ($Z_A=2$)	$^4He_\Lambda$	$0.59^{+.04}_{-.03}$	930^{+220}_{-170}	3730^{+390}_{-290}	2570 ± 340
	$^5He_\Lambda$		1160^{+270}_{-210}	4590^{+480}_{-360}	3430 ± 420
B	$^4He_\Lambda$	0.255 ± 0.002	126 ± 2	1002 ± 8	—
	$^5He_\Lambda$	0.240 ± 0.002	138 ± 2	1163 ± 9	—

Further information necessary to assign the charge, and therefore the velocity, and the mass of particle A is given by its mean angle of scattering $\bar{\alpha}_A$. Full details of the method employed in the estimate of $\bar{\alpha}_A$, as well as the results of a scattering calibration on protons, are given in the appendix, Sect. **4**. The measured value of $\bar{\alpha}_A$, deduced from second differences is $\bar{\alpha}''_A = (0.122 \pm 0.032)°/100\ \mu m$; and $\bar{\alpha}'''_A = (0.125 \pm 0.045)°/100\ \mu m$, from third differences. A direct correction for distortion (73 covans) reduces $\bar{\alpha}''_A$ to $(0.113 \pm 0.032)°/100\ \mu m$ which is the value used in the following. The standard deviation on $\bar{\alpha}_A$ is inferred from the results of the scattering calibration. With this result and the information of Table III we can infer Z_A. If A is assumed to be $^4He_\Lambda$ or $^5He_\Lambda$, the interaction at O′ would be an inelastic scattering against a nucleus of the emulsion, involving a very large transfer of energy and momentum (to neutral particles only), accompanied by no evaporation tracks. The minimum energy and momentum transfers are 800 MeV and 2600 MeV/c, respectively, which would require the emission of several energetic neutrons.

1916

Furthermore, on this assumption, track A would show, under the conditions of our measurements, $\bar{\alpha}_A \leqslant 0.02°/100\ \mu m$, incompatible with the measured values above. These arguments hold, *a fortiori*, for $Z > 2$. On the other hand, the energy per nucleon assuming $Z_A = 2$, but A being an ordinary nucleus, is insufficient to produce the observed hypernucleus. We therefore, conclude that $Z_A = 1$.

We can now determine the mass of A, M_A, by comparing $\bar{\alpha}_A$ with the same quantity for protons of velocity β_A, viz. $\bar{\alpha}_p = (0.404 \pm .050)°/100\ \mu m$. The ratio, $\bar{\alpha}''_p/\bar{\alpha}''_A$ gives $M_A = (3.6^{+1.2}_{-0.9})\, M_p$.

3. – Discussion.

Hypothesis (1) that track A be a helium hypernucleus, suffering an inelastic scattering at point O″, has been shown in the preceding section to be untenable. Similarly, the excitation of a stable He to a He_Λ state has been excluded, being forbidden by lack of energy; this argument holds also for the production of particle B by a stable particle of unit charge, since its energy, from $\beta_A = 0.257$, would be insufficient to produce a He_Λ (hypothesis 2).

We are left, therefore, with the consequence that this event represents the interaction of a strange particle of unit charge, either « elementary » (heavy meson or hyperon), or « compound » (hypernucleus), resulting in the emission of a He_Λ. The following reactions may be considered:

Interaction of a K^-. K^- captures at rest are known to lead to the production of hypernuclei ([4]). One supposes that this process can also occur as a result of interactions in flight. However, the probability that $\bar{\alpha}_K = \bar{\alpha}_A$ ($\bar{\alpha}_K$ from $\beta_A = 0.257$ should be $0.77°/100\ \mu m$ whereas $\bar{\alpha}_A = 0.113°/100\ \mu m$) is about 10^{-3}.

Interaction of a $\Sigma^\pm$. It seems reasonable to expect the production of hypernuclei by $\Sigma^\pm$ in flight, since the captures of Σ^- at rest are known to produce ([5]) these particles. In our case, the probability of obtaining the observed $\bar{\alpha}_A$ from the track of a $\Sigma^\pm$ is 2%. The chance that A is a $\Sigma^\pm$ is further reduced by lack of energy conservation in the overall reaction. As-

([4]) J. E. Naugle, E. P. Ney, P. S. Freier and W. B. Cheston: *Phys. Rev.*, **96**, 1383 (1954).

([5]) M. Ceccarelli, N. Dallaporta, M. Grilli, M. Ladu, M. Merlin, G. Salandin and B. Sechi: *Proc. Pisa Conference*, June 1955, *Suppl. Nuovo Cimento*, **9**, 497 (1956); and *Nuovo Cimento*, **2**, 542 (1955).

suming the elementary reaction:

$$\Sigma^{\pm} + \text{nucleon} \rightarrow \Lambda^0 + \text{nucleon} + Q_1 , \qquad Q_1 = (82 \pm 2) \text{ MeV}$$

(intermediate Σ^0 states do not increase the energy released) the maximum energy available to the reaction forming a hypernucleus is

$$T_\Sigma + Q_1 = (124 \pm 7) \text{ MeV} ,$$

where $T_\Sigma = (41.5 \pm 5.8)$ MeV is the kinetic energy of particle A, if Σ, deduced from ionization. The energy required to produce the final state, if track B is $^4\text{He}_\Lambda$, includes: the kinetic energy of track B, $T_B = (126 \pm 2)$ MeV; the energy necessary to remove a ^{3}He from a nucleus of the emulsion, $E_{ext} \cong 17$ MeV; the minimum energy necessary to absorb the excess momentum of ~ 700 MeV/c, if this is taken by a nucleus of $M = 100$, $T_{rec} = 3$ MeV; the energy supplied to the reaction by the binding of the Λ^0 in $^4\text{He}_\Lambda$, $B_\Lambda \cong 2$ MeV. The final state requires, therefore, a minimum energy of:

$$T_B + E_{ext} + T_{rec} - B_\Lambda \geqslant 144 \text{ MeV} .$$

The minimum energy required if B is a $^5\text{He}_\Lambda$ is 146 MeV. There is a deficit in energy, therefore, of (20 ± 7) MeV. The composite probability that energy, is, in fact, conserved, and $\bar{\alpha}_\Sigma = \bar{\alpha}_\Lambda$ is $5 \cdot 10^{-5}$.

Interaction of a Ξ^-. No hypernuclei have been reported so far to be produced by the cascade particle. Such an occurrence is, however, compatible with the conservation of strangeness in fast reactions ([6]), if one of the Λ^0's from

$$\Xi^- + \text{p} \rightarrow 2\Lambda^0 + Q_2 , \qquad Q_2 \cong 27 \text{ MeV}$$

is bound to a nuclear fragment. The energy available to the reaction in this case would be only 73 MeV ($T_\Xi + Q_2$), far less than the minimum final energy required.

From this analysis, it appears that the event can hardly be explained by the interaction of a known elementary strange particle (hypothesis 3).

The interpretation which is supported by the experimental evidence in-

([6]) M. GELL-MANN and A. PAIS: *Proc. of the Glasgow Conference* (1954), London, p. 342.

volves one of the following reactions (hypothesis 4)

$$a) \qquad {}^3\mathrm{H}_\Lambda + (Z, A) \to {}^4\mathrm{He}_\Lambda + (Z-1, A-1) + Q_a, \qquad Q_a \cong 2 \text{ MeV},$$

$$b) \qquad {}^4\mathrm{H}_\Lambda + (Z, A) \to {}^4\mathrm{He}_\Lambda + (Z-1, A) \qquad + Q_b, \qquad Q_b \cong 0 \text{ MeV},$$

$$c) \qquad {}^4\mathrm{H}_\Lambda + (Z, A) \to {}^5\mathrm{He}_\Lambda + (Z-1, A-1) + Q_c, \qquad Q_c \cong 13 \text{ MeV}.$$

These values of Q are computed for an « average » nucleus (Z, A) in which the mean binding energy per nucleon is 8 MeV. Emulsion nuclei deviate at most by 1 MeV from this.

The mass estimate, $M_\Lambda = (3.6^{+1.2}_{-0.9})\, M_p$ does not allow a choice between ${}^3\mathrm{H}_\Lambda$ and ${}^4\mathrm{H}_\Lambda$. Choice among reactions (*a*), (*b*) and (*c*) can be attempted on the basis of balancing energy and momentum. The energy unbalance for each reaction is defined to be $T_B - (T_\Lambda + Q)$, where T_B and T_Λ are the kinetic energies of the visible outgoing and incoming particles respectively (see Table I), and Q is given above for the reactions (*a*), (*b*) and (*c*). Reaction (*a*) is unbalanced by at least (23 ± 15) MeV. The nucleus (Z, A) could not have been a free proton, resulting in a positive Q_a of 5.5 MeV, for in that case there is no particle in the final state to carry off the excess momentum of (224 ± 60) MeV/c.

Reactions (*b*) and (*c*) seem more probable. The energy T_B in both reactions is (137^{+19}_{-15}) MeV, resulting in an energy unbalance for reaction (*b*) of (-11 ± 20) MeV, and an excess momentum of (111 ± 48) MeV/c. For reaction (*c*), the energy unbalance is (-12 ± 20) MeV); the excess momentum is (160 ± 40) MeV/c. The excess momentum in both (*b*) and (*c*), as well as in (*a*) can be absorbed by a recoiling heavy nucleus whose range would be too short to produce a visible track or blob. In reactions (*b*) and (*c*), however, energy can be conserved within the limits of error even if the excess momentum is carried off by a single neutron. On the basis of energy-momentum conservation, therefore, there is no preference between reactions (*b*) and (*c*). Some preference for (*c*) is due to the better agreement of the B_Λ of particle B with that of ${}^5\mathrm{He}_\Lambda$, as was noted in Sect. **2**.

Pickup reactions (7), involving ordinary tritons and deuterons, have been observed, but cross-sections are known only at low energies ($2 \div 3$ MeV). The occurrence of a nuclear interaction of a hypernucleus is not unlikely, considering the total length of hyperfragment tracks (~ 8 cm) seen so far in the world in comparison with, for example, the nuclear mean free path in emulsion (~ 25 cm).

Finally, one might consider the possibility that the event represents the interaction in flight of an unknown heavy strange particle leading to the pro-

(7) R. Sher and J. J. Floyd: *Phys. Rev.*, **102**, 242 (1956); J. B. Marion and G. Weber: *Phys. Rev.*, **102**. 1355 (1956).

duction of a hypernucleus (hypothesis 5). This interpretation, however, is not supported by any direct evidence, while the simpler explanation in terms of reaction (*b*) or (*c*) satisfies the observations.

* * *

It is a pleasure to thank Dr. E. J. LOFGREN and the Bevatron staff for their kind and able assistance in obtaining and carrying out the exposure. Thanks are also due to Professor V. L. TELEGDI for his continued encouragement and many useful discussions.

APPENDIX

1. – The emulsion stack.

The stack was composed of 50 1 mm, and 35 0.6 mm pellicles, each 4 in. × 6 in. in area. These were processed unmounted; alignment of the stack and location of events were achieved by means of a grid printed on each pellicle ([8]). Lateral shrinkage was determined by comparing a distance measured between points on the grid of the pellicle with that between the same points of a master grid. Vertical shrinkage is known from the original average thickness of each pellicle determined both from density and direct measurement. Stopping power was measured by means of μ-meson ranges from pion decays. The density deduced from the muon range is (3.79 ± 0.03) g/cm^3; the measured density is (3.78 ± 0.01) g/cm^3. Energies were determined from the R-E tables of BARKAS ([9]).

2. – δ-ray calibration.

The δ-ray density, $\mathrm{d}N_\delta/\mathrm{d}R$ was determined by counting δ's selected with the aid of a micrometer eyepiece incorporating a movable double hairline. The equivalent distance between these lines, with the objective lens employed, was 0.75 μm. Only δ's having a projection more than this length in a direction perpendicular to the track under measurement were counted ([10]).

([8]) E. SILVERSTEIN and W. SLATER: *Journ. Sci. Instr.*, **33**, 381 (1956).
([9]) W. H. BARKAS: *UCRL* 3384 (1956).
([10]) J. CRUSSARD: *Thesis* (University of Paris), June 1952.

1956

3. – Measurement of ionization.

The choice of G and $\overline{G}$ to measure ionization rather than simple blob count N was dictated by the following properties of this stack.

a) The rather flat maximum in the curve of N vs. R/m, ($N_{max} = 3.5 N_{plateau}$) occurs at about $R/m = 4$ $\mu m/m_e$. Since the R/m of track A is about 2.5 $\mu m/m_e$, the parameter N gives a very unreliable estimate of the velocity. The displacement of the position of our maximum N from the more usual position in the region 10-20 $\mu m/m_e$ is partly due to the underdevelopment of the pellicles ($N = 13.5$ blobs/100 μm).

b) G and $\overline{G}$ are particularly favorable in the region of R/m of track A. In fact, the slopes $dG/d(R/m)$ and $d\overline{G}/d(R/m)$ are 0.72 $m_e/\mu m$ and 0.21 m_e respectively. These may be compared with the corresponding values of 0.50 and 0.18 given by the Rome group ([11]) for a plateau value of $N_{plateau} = 25$ blobs/100 μm.

Gap length was measured with a filar micrometer eyepiece. Gaps of length less than 0.5 μm have been discarded. The values of G and $\overline{G}$ quoted in Fig. 3 are corrected for dip by a factor of $\sec\theta$. Correction for shadowing ([12]) has an effect of less than 1% on the velocity of track A.

A check of the fluctuations of G and $\overline{G}$ has been made on the calibration points. Assuming the dispersion $\sigma_{\overline{G}}/G = 1/\sqrt{N}$ where N is the number of gaps per point, one would expect a mean spread of about 9%. From the 30 calibration points, we find a dispersion of about 6% around the least squares curve. There is a corresponding spread of 10% in G. The spreads $\sigma_{\overline{G}}/G$ and $\sigma_{\overline{G}}/\overline{G}$ for track A, based on 640 gaps, were estimated assuming $N^{-\frac{1}{2}}$ dependence for both quantities.

4. – Multiple scattering.

The mean angle of scattering $\overline{\alpha}_A$ for track A was measured using the procedure described by DI CORATO *et al.* ([2]). Track A contained 62 basic cells of length $t = 33$ μm. Actual cells of length $3t$ and $4t$ were used for the computation of $\overline{\alpha}_A$ from second and third differences, respectively. In the absence of an adequate calibration in unmounted pellicles, no correction was applied to the relation $\overline{\alpha}'' = \overline{\alpha}'''\sqrt{\frac{3}{2}}$. No angle was larger than $4\overline{\alpha}$. To check the reliability of this scattering measurement, analogous measurements were performed on 12 tracks identified as protons from ionization vs. range, having the same dip as A within 5°, and therefore having about the same number of basic cells as track A. These calibration tracks were further chosen so that the point at $R/m = 2.5$ $\mu m/m_e$ was contained in the pellicle of track A.

For each calibration track, an expected $\overline{\alpha}_p$ over the measured length was

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([11]) C. CASTAGNOLI, G. CORTINI and A. MANFREDINI: *Nuovo Cimento*, **2**, 301 (1955).

([12]) C. O'CEALLAIGH: *CERN Bulletin*, BS11.

estimated using the scattering constant of PICKUP and VOYVODIC ([1]), and the actual value of $\bar{\alpha}_{sc}$ was determined. The energy loss in the measured interval was taken into account by calculating $\bar{\alpha}_p$ corresponding to an « effective » range ([14]). From the ratios $\bar{\alpha}_p/\bar{\alpha}_{sc}$ one derives an estimate both of the proton mass, and of the dispersion in $\bar{\alpha}_{sc}$ for our statistics. We find:

	$(\bar{\alpha}_p/\bar{\alpha}_{sc})$	$(\bar{\alpha}_p/\bar{\alpha}_{sc})^{2.38} = M/M_p$
2nd diff.	1.027 ± 0.077	1.06 ± 0.29
3rd diff.	1.09 ± 0.12	1.22 ± 0.35

From the error curves of di Corato *et al.* ([2]) we expect average dispersions of 22% and 26% for 2nd and 3rd difference determinations of $\bar{\alpha}_{sc}$ respectively. We find 26% and 36%, respectively, from the calibration tracks. These measurements indicate the absence of large systematic errors; in view of the smaller dispersion, however, preference has been given, in the choice of $\bar{\alpha}_\Lambda$, to the value given by second differences, directly corrected for distortion, rather than that given by third differences.

([13]) L. VOYVODIC and E. PICKUP: *Phys. Rev.*, **85**, 91 (1952).
([14]) M. G. K. MENON and O. ROCHAT: *Phil. Mag.*, **42**, 1232 (1951).

RIASSUNTO

Una particella (A) di carica unitaria, massa (3.6 ± 1.1) masse protoniche, e velocità $v = (0.26 \pm 0.02)c$, è emessa da una stella provocata da un π^- di 4.5 GeV in emulsione nucleare. Dopo un percorso di 2.4 mm, la particella A interagisce con un nucleo dell'emulsione, originando come unico prodotto visibile un ipernucleo di elio, identificato da misure dirette di massa e carica e dal suo decadimento mesonico. L'energia della particella A è insufficiente a produrre direttamente l'He_Λ osservato, e i risultati delle misure di scattering e ionizzazione sulla traccia A sono inconsistenti con l'ipotesi che quest'ultima sia dovuta a un mesone pesante o iperone interagente in volo. L'interpretazione più probabile consiste nell'identificare la particella A con un ipernucleo di idrogeno, che interagisce per scambio di carica o per cattura di un protone.

Angular Correlation in the π-μ-e Decay of Cosmic Ray Mesons (*).

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(ricevuto il 23 Marzo 1957)

Summary. — A measurement has been made of the angular correlation in the decay for mesons produced at high altitude by cosmic rays incident on a 22.4 liter emulsion stack. Within our statistics the angular distribution of the electrons is isotropic. The value of « a » in the expression $dN/d\Omega = (1+a\cos\theta)$ is $-.03 \pm .04$. This result is in disagreement with the experiments using accelerator produced mesons which show a backwards asymmetry with « a » $= -.15 \pm .02$. Possible explanations for the difference are discussed.

Experiments at Columbia (1) and Chicago (2) have shown an angular correlation in the π-μ-e decay with accelerator produced π^+-mesons. We have studied the same angular distribution for cosmic ray produced π^+-mesons. The π-μ-e decays which we have measured were produced in a stack of 600 μm emulsions having a total volume of 22.4 liters. The exposure was made in a balloon flight in Minnesota at a geomagnetic latitude of 56° on September 18, 1956. The flight had a duration of 8 hours at an altitude of 116 000 feet. Because of the possible depolarization effect of external magnetic fields, we have investigated the magnetic field in the gondola and find that the magnetic field did not exceed the earth's field of 0.5 gauss throughout the emulsion stack.

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(+) On leave of absence from the University of Bristol.

(×) On leave of absence from the University of Brasil and the Centro Brasileiro de Pesquísas Físicas.

(1) R. L. Garwin, L. M. Lederman and M. Weinrich: *Phys. Rev.* (in press).

(2) J. I. Friedman and V. L. Telegdi: *Phys. Rev* (in press).

A portion of the stack was scanned under 10×15 power with the requirement that the π-μ decay be complete in one emulsion. Measurements were made on those events in which the end of the μ-meson was at least 30 μm from either emulsion surface in the processed emulsion. The second requirement was imposed in order to insure reliable observation of the decay electron regardless of its direction of motion. If such a criterion is not imposed, some electrons will be missed; in addition because of the geometry, the probability of missing will be greater for electrons moving forward with respect to the direction of te μ-meson before it comes to rest. Since this latter direction is highly correlated with the initial direction of the μ-meson originating from the π-μ decay, a loss of these electrons could give rise to an artificial backwards asymmetry. We have measured the space angle θ between the direction of the emission of the μ and that of its decay electron for 2117 events satisfying the prescribed conditions. In only 7 cases we were unable to find the electron track. The resultant angular distribution is shown in Fig. 1.

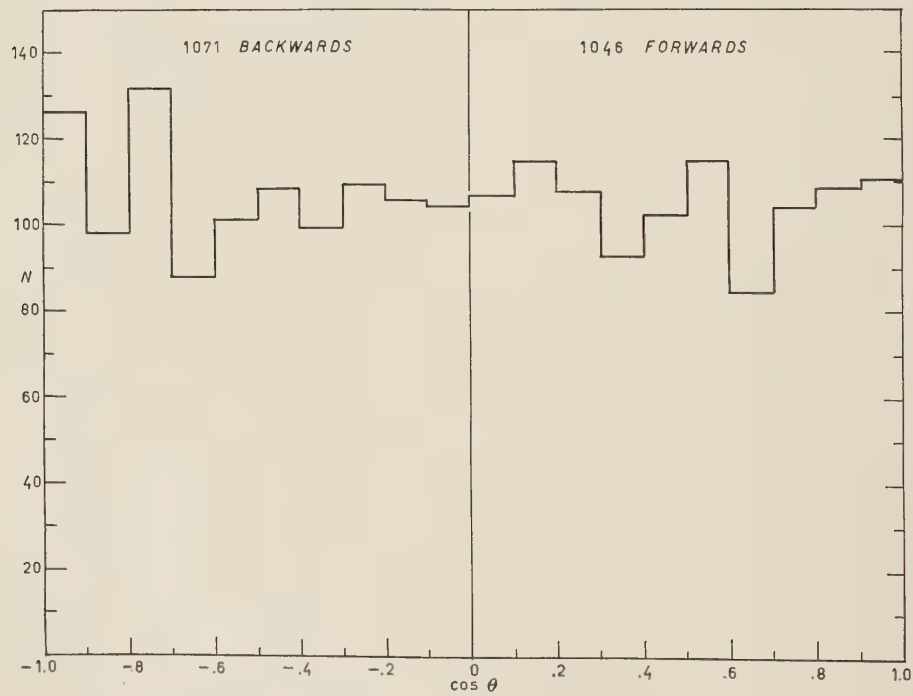

Fig. 1. – Distribution in space angle, θ, between the emission direction of the meson and the electron in π-μ-e decays.

The mean value of $\cos\theta$ was -0.011 ± 0.013. Other investigators have expressed their results in terms of the value of « a » in the expression

$dN/d\Omega = (1 + a \cos\theta)$, where $dN/d\Omega$ is the number of events per unit solid angle as a function of θ. In this expression the constant « a » is 3 times the average value of the $\cos\theta$. Our results, therefore, give « a » $= -0.03 \pm 0.04$, in marked contrast to the values which have been measured for machine-made π^+-mesons. These are tabulated in Table I.

TABLE I. – *Values of « a » determined for machine-made π^+ mesons.*

	« a »	Accelerator in which the mesons were produced	
Columbia [1]	-0.16 ± 0.03	Columbia	$\sim$ 400 MeV
Chicago [2]	-0.17 ± 0.04	Chicago	$\sim$ 450 MeV
Göttingen [3]	-0.09 ± 0.04	Chicago	$\sim$ 450 MeV
Rochester [4]	-0.19 ± 0.055	Cosmotron	$\sim$ 2000 MeV

The great discrepancy between our result and the average machine result suggests some systematic difference between the experiments. The possibilities which we have considered are the following:

1) Magnetic field conditions.

2) Depolarization due to chemical effects such as anomalous muonium formation.

3) Energy of collisions in which the π^+ are produced.

4) Possible difference between cosmic ray primaries and laboratory protons.

5) The presence in our experiment of a large quantity of local matter in which the mesons are produced.

1. – Magnetic field conditions.

The magnetic field in our experiment was ~ 0.5 gauss mainly in the plane of the emulsion. It was shown by the Columbia experiment that the mesons that show the asymmetry precess at a rate of $1.35 \cdot 10^4$ rad s^{-1} G^{-1}. This rate of precession produces a negligible depolarization in our magnetic field, unless anomalous muonium formation takes place.

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(3) N. N. BISWAS, M. CECCARELLI and J. CRUSSARD: Circulated preprint.
(4) M. F. KAPLON: Private communication.

2. Depolarization due to chemical effects such as anomalous muonium formation.

It is known that the asymmetry is sensitive to the material in which the μ^- is brought to rest ([1,2]). Small changes in composition of the emulsion or water content could perhaps catalyze muonium formation which would make the asymmetry decrease. This possibility cannot be ruled out, but it should be pointed out that *all* accelerator experiments with G-5 emulsion exposed at approximately 70 °F have shown the asymmetry. These were the conditions of exposure in our experiment. Furthermore, the value of the mean range of the μ-mesons in our emulsion (602 ± 4 μm) indicates that the density during exposure is typical of that normally encountered.

3. – Energy of collisions in which the π^+ are produced.

We investigated the origin of 50 low energy $\pi^\pm$ which were created and brought to rest in the same emulsion sheet. The $\pi^\pm$-mesons that are brought to rest in the stack are mainly those with energies less than 100 MeV (range $= 10$ cm). It is known that the energy spectrum of $\pi^\pm$ varies only slowly with the number of shower particles in the star. It is, therefore, plausible to assume that the 50 stars investigated are fairly representative of the origin of the π^+-mesons measured in our experiment. The mean number of charged shower particles N_s for the stars was five, and $65\,\%$ and three or more fast charged shower particles. The slow pion was not counted as a shower particle. The number of $\pi^\pm$ produced therefore was approximately $(\bar{N}_s - 1)$. This situation is completely different from Columbia and Chicago whose production is single, and Brookhaven where multiple production of three or more mesons is small.

4. – Possible difference between cosmic ray primaries and laboratory protons.

The difference between the accelerator result and the cosmic ray result may be due to some inherent difference in the nature of the pion producing particle. As YANG and LEE ([5]) have suggested, particles may exist which differ in « handedness » from laboratory particles. Even if there is no such fundamental difference between cosmic ray protons and laboratory protons, the null result for the cosmic ray asymmetry may arise because of the fact

([5]) T. D. LEE and C. N. YANG: *Phys. Rev.*, **104**, 256 (1956).

that in cosmic ray events neutrons and pions as well as protons can give rise to meson production. In accelerator experiments the pion producing particles have been protons. Of the primary cosmic ray nucleons possessing energy in excess of 1 GeV, $\sim 14\%$ are neutrons, which occur as constituents of α-particles and heavier nuclei. Further, as the cosmic ray beam progresses down through the atmosphere (or emulsion), the percentage of neutron primaries for meson producing events increases. Neutrons can make π^+ in multiple production in collision with neutrons as well as with protons in the target nucleus.

5. The presence in our experiment of a large quantity of local matter in which the mesons are produced.

The 22.4 liter stack in which our mesons were produced weighed 300 pounds. The large amount of local matter leads to the followig effects:

a) Many of the pion producing particles are no longer primary cosmic rays as discussed in Sect. **4**.

b) Pions emitted over a range of angles in the center of mass system can be brought to rest in our stack. In a small emulsion stack exposed to cosmic rays only pions emitted in the backwards direction in the C system will be stopped.

c) Because of the large average path of pions in our stack (approximately 10 cm), many of these pions will have suffered nuclear scattering before coming to rest.

6. – Conclusion.

If we assume that all π^+-mesons are identical, it appears that one must accept our result as a consequence either of a large statistical fluctuation or a peculiar chemical effect which was present in our case but absent in all of the accelerator experiments. On the other hand if more than one kind of π^+-mesons is presumed to exist, a different mixture may have been selected in our experiment than that so far observed in accelerator investigations. Such a selection might result if some basic property of the π^+-meson was influenced by the nature of the pion producing particles, by the angle of emission of the meson in the center of mass system of the colliding nucleons, or by processes such as nucler scattering which occurs before the pion is brought to rest.

RIASSUNTO (*)

È stata eseguita una misura della correlazione angolare nel decadimento dei mesoni prodotti ad altitudine elevata dai raggi cosmici incidenti su un pacco di emulsioni di 22.4 litri. Nell'ambito della nostra statistica la distribuzione angolare degli elettroni è isotropa. Il valore di « a » nell'espressione $\mathrm{d}N/\mathrm{d}\Omega = (1+a\cos\theta)$ è $-.03 \pm .04$. Questo risultato è in disaccordo con gli esperimenti che impiegano mesoni prodotti da acceleratori che mostrano una asimmetria indietro con « a » $= -.15 \pm .02$. Si discutono alcune possibili spiegazioni della differenza.

(*) *Traduzione a cura della Redazione.*

On the Theory of Hyperons and K-Mesons.

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(ricevuto il 29 Marzo 1957)

Summary. — An analysis is made of the consequences of the well established law of conservation of « strangeness » on the schemes of quantum numbers attributed to Hyperons and K-mesons. It is shown that the usually accepted Gell-Mann-Nishijima scheme is not unique and that another scheme in which isotopic spin 0 is attributed to K-mesons and $\frac{1}{2}$ to (Λ, Σ) particles is possible. A theory is developed based on general symmetry principles, with this new scheme, and it is shown that, at the stage when only interactions of K-mesons with hyperons are introduced, it is equivalent to Schwinger's 4-dimensional (in isotopic spin space) theory. Some features of interactions of π-mesons with hyperons are considered, as well of weak interactions. One of these features is that for the interactions of the π-mesons with (Λ, Σ) particles we may lose the conservation of the isotopic spin (with the selection rule $\Delta I_3 = 0, \pm 1$) with the new definition of isotopic spin.

Recent experimental results have firmly established the law of conservation of « strangeness » [1] for strong interactions. The concept of strangeness has been already most useful for the analysis and predictions of many reactions and selection rules. This property, or the equivalent ones such as the « attribute » and the « dionic number » [2] which was introduced semi-empirically

(*) Submitted for publication to *Il Nuovo Cimento*. This work was done under the auspices of the Conselho Nacional de Pesquisas.

[1] M. Gell-Mann and A. Pais: *Proc. Glasgow Conf. on Nuclear and Meson Phys.* (1954), p. 324.

[2] R. G. Sachs: *Phys. Rev.*, **99**, 1576 (1955); M. Goldhaber: *Phys. Rev.*, **101**, 433 (1956).

1929

results as a natural consequence from the scheme for attribution of isotopic spin to the hyperons and K-mesons due to NISHIJIMA ([3]) and GELL-MANN ([4]). For this reason it has been frequently assumed that the succes of the application of the conservation of the strangeness principle and of the Nishijima-Gell-Mann scheme implies the solid establishement of the validity of the referred scheme. In the present paper we wish to show that the N-G scheme is not unique and to propose a different scheme which is also in excellent agreement with the known experimental results.

1. – The Gell-Mann-Nishijima scheme and the strangeness.

In this scheme isotopic spin $\frac{1}{2}$ is attributed to the nucleons, cascade particles and K-mesons (in the present paper we shall not be concerned with the difference between θ and τ forms of K-mesons and with parity properties, which we think are to be explained in the lines of such ideas as non conservation of parity in weak interactions ([5])). Thus the neutral and charged forms of these particles are grouped in two component wave operators, the upper and lower ones corresponding to the eigenvalues $+\frac{1}{2}$ and $-\frac{1}{2}$ of the third component I_3 of the isotopic spin operator, respectively

$$(1) \qquad N = \begin{pmatrix} p \\ n \end{pmatrix}; \quad \Xi = \begin{pmatrix} \Xi_0 \\ \Xi_- \end{pmatrix}; \quad K = \begin{pmatrix} K \\ K \end{pmatrix}; \quad \bar{K} = \mathcal{C}\,K^{\dagger} = \begin{pmatrix} -K_0^{\dagger} \\ K_- \end{pmatrix}.$$

On the other hand the Λ particle is assumed to have isotopic spin 0 and the Σ particles are grouped in a triplet corresponding to isotopic spin 1:

$$(2) \qquad \Sigma = \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{pmatrix}; \quad \Sigma_- = \frac{1}{\sqrt{2}}(\Sigma_1 - i\Sigma_2); \quad \Sigma_+ = \frac{1}{\sqrt{2}}(\Sigma_1 + i\Sigma_2); \quad \Sigma_0 = \Sigma_3 .$$

Besides these, except the K particles which have ordinary integer spin, there are the antiparticles corresponding to all the other ones which have ordinary half integer spin. We shall assume that all half integer spin particles (hyperons) have spin $\frac{1}{2}$ and that the K-particle has spin zero.

The strong interactions between these particles is to be described by a term in the Hamiltonian which is invariant under rotations in the isotopic

([3]) T. NAKANO and K. NISHIJIMA: *Prog. Theor. Phys.* (*Japan*), **10**, 581 (1953); K. NISHIJIMA: *Prog. Theor. Phys.* (*Japan*), **12**, 107 (1954).

([4]) M. GELL-MANN: *Phys. Rev.*, **92**, 833 (1953).

([5]) T. D. LEE and C. N. YANG: *Phys. Rev.*, **104**, 254 (1956).

spin space (also in the ordinary 4-dimensional space, of course). This term should lead to conservation of strangeness and of the number of particles.

The concept of strangeness has been incorporated to the theory by d'Espagnat and Prentki (6) who have made use of the complete group of orthogonal transformations in the isotopic spin space, for which there are two kinds of spinors transforming oppositely regarding to reflexions, and introduced an operator U, the isonumber which has eigenvalues $+1$ and -1 corresponding to spinors of the first and second kind, respectively; finally, assuming that N and K are first kind spinors and that Ξ, K′ are of the second kind they were able to show that the « strangeness » is given by

$$S = U - \mathcal{N}\,, \tag{3}$$

where $\mathcal{N}$ is the « number of particles » operator, with eigenvalue $+1$ for spinor particles, -1 for antiparticles and 0 for bosons. For integer isotopic spin U comes out to be zero. It should be mentioned that the isonumber operator has also been introduced independently, by SCHWINGER (7) who called it the hypercharge (Y).

The established values of S as well as the values of U (or Y) are given in Table I.

TABLE I. – *Strangeness and isonumber (of hypercharge) for hyperons,* K- *and* Π-*mesons.*

Particle	S	$U \equiv Y$
N	0	1
Ξ	-2	-1
Σ	-1	0
Λ	-1	0
K	1	1
K′	-1	-1
π	0	0

The expression for the charge (in e units) comes out to be, in the N-G theory:

$$Q = \frac{U}{2} + I_3\,. \tag{4}$$

Now, if the interaction Hamiltonian is invariant under the full 3-dimensional orthogonal group in isotopic spin space the isonumber is conserved; if $\mathcal{N}$ is also conserved then the conservation of S is assured.

(6) B. D'ESPAGNAT and J. PRENTKI: *Nucl. Phys.*, **1**, 33 (1956).
(7) J. SCHWINGER: *Phys. Rev.*, **104**, 1164 (1956).

1931

We should mention at this stage that the only features of the G-N theory which are responsible for its succes in the interpretation of the experimental results are the following ones:

a) The particles N, Ξ, $\mathbf{\Sigma}$ and Λ have half integer ordinary spin, but K, K′ and π have integer spin, conservation of ordinary spin being assumed as usual.

b) The number of spinor particles (hyperons) is conserved.

c) The isotopic spins of N and π are respectively $\frac{1}{2}$ and 1; the sum of the isotopic spins of Λ or $\mathbf{\Sigma}$ and of K or K′ is half integer; isotopic spin is conserved in strong interactions but not necessarily in weak ones (actually only « $\Delta I_3 \neq$ half-integer » has been proved).

d) Strangeness as given by Table I is conserved (or the quantity $U = \mathcal{Q} + \mathcal{S}$ is conserved).

e) The charge operator Q has only the eigenvalues 0, ± 1.

Thus we see that it is only when we wish to keep the definition of the charge as given by (4) that we are led to the N-G attribution of integer isotopic spin for $\mathbf{\Sigma}$, Λ and half integer for K, K′ (in view of the fact that Q has to be 0, ± 1, as U is always integer). Therefore there is place, in principle, for different attribution to these isospins.

2. – Generalized 4-dimensional isotopic spin space.

In a recent paper (7) SCHWINGER has proposed a further extension of the isotopic spin space to a 4-dimensional one. In this way he unifies N and Ξ in one 4-isospinor, Λ and $\mathbf{\Sigma}$ in a 4-isovector and K, K′ in a 4-isospinor. In this way, by imposing invariance of strong interactions with respect to rotations in a 4-dimensional isotopic spin space he unifies the 4 possible terms of the interactions between these particles which are invariant under rotations in the 3-dimensional isotopic spin space.

Thus he generalizes the group of rotations in isotopic spin space in such a way that besides the scalar and 3-vectors quantities there are also 4-vectors and 4-spinors (*). So, if ψ, φ_K (4-spinors) and Σ_μ are given

$$\psi = \begin{pmatrix} \mathrm{p} \\ \mathrm{n} \\ \Xi_0 \\ \Xi_- \end{pmatrix} ; \quad \varphi_K = \begin{pmatrix} \mathrm{K}_+ \\ \mathrm{K}_0 \\ -\mathrm{K}_0^\dagger \\ \mathrm{K}_- \end{pmatrix} ; \quad \Sigma_\mu = (\mathbf{\Sigma}, \Lambda) ; \tag{5}$$

(*) The formulation of the 4-dimensional theory here attributed to SCHWINGER is a reconstitution of what we could infer from his paper and from indirect informations, so it may not coincide with Schwinger's formulation.

a 4-vector B_μ exists, besides the scalars $\bar{\psi}\psi$, and $\varphi_K^\dagger\varphi_K$ and the 3-vectors $\bar{\psi}\boldsymbol{\tau}\psi$ and $\varphi_K^\dagger\boldsymbol{\tau}\varphi_K$, which is given by

$$B_r = i\bar{\psi}\tau_r\varphi_K\,, \qquad B_4 = \bar{\psi}\varphi_K\,, \qquad (r = 1, 2, 3). \tag{6}$$

The scalar quantity which can be formed with ψ, φ_K and Σ_μ is then $B_\mu\Sigma^\mu$ so the interaction Hamiltonian density is taken as:

$$\mathcal{H}_{\text{int}} = gB_\mu\Sigma^\mu + \text{h.c.} = g\bar{\psi}(\Lambda + i\boldsymbol{\Sigma}\cdot\boldsymbol{\tau})\varphi_K + \text{h.c.} \tag{7}$$

The theory is also invariant at this stage with respect to the transformation

$$N \rightleftarrows \Xi\,; \qquad K \rightleftarrows K'\,, \tag{8}$$

which assures the complete symmetry between N and Ξ.

It should be mentioned that according to Schwinger philosophy the masses of *all hyperons* are identical at this stage: not only those of N and Ξ, whose identity is assured by transformation (8), but also those of Λ and $\boldsymbol{\Sigma}$, although they are essentially different because they have isotopic spin different from those of N and Ξ. The introduction of the further interaction of hyperons and K-mesons with π-mesons would break the existing symmetry and result in the splitting of the mass multiplet in the same way as the mass difference between proton and neutron results from the asymmetrical interaction of these particles with the electromagnetic field. We shall maintain this assumption and disregard the mass differences among the hyperons, unless they are explicitely mentioned.

3. – Alternative scheme of quantum-numbers for hyperons and K-mesons.

If we wish to adhere to Schwinger's idea that Λ and $\boldsymbol{\Sigma}$ belong to the same mass multiplet as N and Ξ and that their mass differences result from asymmetric interactions with π-mesons then it seems that they should have similar properties or, in a sense, that they should be different forms of the same particle. Otherwise it would be difficult to explain the coincidence of the masses. This mass degeneracy could even result from invariance of the theory under some transformations. So we should assume, for instance, that they have the same isotopic spin ($\frac{1}{2}$) as well as the same ordinary spin (it is well established, at least, that they are all half-integers).

In this section we shall try to formulate a scheme in which these conditions are fulfilled and, consequently, in which K- and K′-mesons are assumed to

have *integer* isotopic spin. Thus without violating condition (*c*) of the list given at the end of Sect. **1**, all the other conditions should also be satisfied.

For this we first group the wave functions of the hyperons in the following *isospinors*, instead of as in (1), (2):

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi_0 \\ \Xi_- \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} \Sigma_+ \\ \Sigma_n \end{pmatrix}, \quad \Sigma_b = \begin{pmatrix} \Sigma_{n'} \\ \Sigma_- \end{pmatrix}. \tag{9}$$

Here $\Sigma_\pm$ represent the positive or negative Σ and Σ_n, $\Sigma_{n'}$ are linear combinations of the wave operators for the observed Σ_0 and Λ (please, forget their mass difference). It will be shown in Sect. **5** that we should have indeed:

$$\Sigma_n = \frac{1}{\sqrt{2}}(\Lambda - i\Sigma_0); \qquad \Sigma_{n'} = \frac{1}{\sqrt{2}}(\Lambda + i\Sigma_0). \tag{10}$$

In (9) the upper components correspond to isotopic spin $\frac{1}{2}$ and the lower ones to $-\frac{1}{2}$.

It is clear from what was said before that the quantities U in (3) and in (4) cannot be anymore the same because U should now be integer in (3) and half-integer in (4) for Σ and Λ particles. We chose to keep (3) and to modify the definition (4) of Q. In order to avoid confusion with the previous formulae we shall change notation and write, instead of (3)

$$S = Y - \mathcal{Q}, \tag{3a}$$

the hypercharge Y being given in Table I.

For Q we write:

$$Q = J_3 + I_3, \tag{11}$$

where J_3 is a new quantum number which has to be integer or half-integer according to which I_3 is integer or half-integer. It is clear that for hyperons, which have $I_3 = \pm\frac{1}{2}$, the possible values of J_3, are also $\pm\frac{1}{2}$ as Q can have only the values 0, ± 1; for K-mesons J_3 is necessari integer (as I_3).

Now as Y and J_3 are not anymore identical we should find the connection between them. So we introduce a new quantum number J_3' such as:

$$Y = J_3 + J_3', \tag{12}$$

J_3' is integer (of half-integer) if J_3 is integer (or half-integer) because Y is allways integer (see Table I). One finds indeed that, in the same way as I_3 and J_3, J_3' can have also only the values $\pm\frac{1}{2}$ for the hyperons and is integer for K-mesons.

Let us now determine the values of the new quantum numbers J_3 and J_3' for π- and K-mesons.

For π-mesons we take as well established the value one for the isotopic spin ($I_3 = 0, \pm 1$). Thus we find, in view of: $Q = I_3$; $Y = 0$, that:

$$J_3 = J_3' = 0 \quad \text{(for } \pi\text{-mesons)} . \tag{13}$$

For K-mesons we have, at this stage, a large arbitrarity in assigning values to I_3, J_3 and J_3'. Only if we assume isotopic spin zero:

$$I_3 = 0 \quad \text{(for K-mesons)} \tag{14}$$

we have a unique possibility for J_3 and J_3' because now we have:

$$Q = J_3 ; \qquad Y = Q + J_3' .$$

In Table II the values of I_3, J_3 and J_3', as well as of Q and Y, are given for all these particles. In the following section we shall find that (14) is indeed the correct choice if a physical meaning is given to J_3 and J_3' which assures conservation of the hypercharge as a consequence of invariant properties of the interaction Hamiltonian under rotations in a generalized isotopic spin space.

In the following sections we shall develop a theory in which we have, in opposition to the G-N scheme, the scheme of quantum numbers given in

TABLE II. – *Quantum numbers* I_3, J_3 and J_3' *for hyperon,* K- *and* π-*mesons.*

Particle	I_3	J_3	J_3'	$Q = I_3 + J_3$	$Y = J_3 + J_3'$
p	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	1	1
n	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	1
Ξ_0	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1
Ξ_-	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
Σ_+	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	1	0
Σ_n	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
$\Sigma_{n'}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
Σ_-	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	-1	0
π_+	1	0	0	1	0
π_0	0	0	0	0	0
π_-	-1	0	0	-1	0
K_+	0	1	0	1	1
K_0	0	0	1	0	1
K_0'	0	0	-1	0	-1
K_-	0	-1	0	-1	-1

1955

Table II and which satisfies all conditions enumerated in Sect. **1**, necessary for a good agreement with the experimental results, except the conservation laws which still have to be worked out.

4. – Isotopic spin space and hypercharge space.

We wish to define the operators J_3 and J_3' with the eigenvalues considered in the previous section and to build the theory in such a way that the conservation of charge, isotopic spin and hypercharge in fast reactions would result from invariance properties of the interaction Hamiltonian. Alternatively, we should assure the conservation of I_3, J_3 and J_3'.

For the interaction of π-mesons with nucleons this can be assured as usual, by imposing that the Hamiltonian is invariant under rotations in the isotopic spin space. The fact that iI_3 is the infinitesimal operator for rotation in the (1, 2) plane assures the conservation of I_3. In this case the conservation of J_3 and J_3' results from the fact that $J_3 = J_3' = \mathcal{H}$ both for nucleons and π-mesons and that (*)

$$\Delta\mathcal{H} = 0 \,. \tag{15}$$

In general this invariance will not be enough and we have to impose invariance under a more general rotation group. Now as we have for the hyperons 3 dichotomic variables (I_3, J_3 and J_3') which are independent the corresponding operators should commute with each other and their irreducible representation should be given by 8×8 matrices. Thus the wave function should have (at least) 8 components (except those for the ordinary spin) which is exactly the number of the already discovered hyperons: N, Ξ, Σ_a and Σ_b. Therefore we could impose invariance under rotations in a 7-dimensional space (+), which is the widest rotation group with an 8×8 spinor representation.

This 7-dimensional space can be built as the direct sum of the 3-dimensional *isotopic spin space* and a 4-dimensional *hypercharge space*. I_3 being one of the three infinitesimal operators for rotations in the isotopic spin space we should take for iJ_3 and iJ_3' two (commuting) of the six infinitesimal operators $M_{\mu\nu}$

(*) The conservation of N is assured, as it is well known, by imposing invariance of the Hamiltonian under the transformation $\psi \to e^{i\varphi}\psi$, where φ is a real phase, the same for all hyperon particles, having opposite sign for the antiparticles and vanishing for bosons. We shall impose this invariance for our theory, in agreement with condition *b* of Sect. **1**.

(+) A similar theory which starts directly from 7-dimensional space has been worked by Dr. Maurice Neuman from the University of California. We are thankful to him for some information on his work.

of the hypercharge space, so that they all commute:

$$J_3 = \frac{1}{i} M_{12}; \qquad J_3' = \frac{1}{i} M_{34}. \tag{16}$$

In the spinor representation (8×8) each of the operators I_3, J_3 and J_3' have four eigenvalues $+\frac{1}{2}$ and four $-\frac{1}{2}$ as it is necessary for our purpose (this is really true only if the space is cartesian and so we assume this nature of the space). We shall call the hypervector $\boldsymbol{J} = (M_{23}, M_{31}, M_{12})$ the *hyperspin*.

We should mention that from now on we shall not assume invariance of the theory under the whole group of rotations in 7-dimensional space because we would have to introduce three more particles, partners of the K, K′-mesons, which have not been observed—if such particles will be found they can be included in the theory by a natural extension (these particles should have, however, hypercharge zero). We shall indeed assume only invariance under the restricted group of independent rotations in the 3-dimensional isotopic spin space and in the 4-dimensional hypercharge space. So a general tensor quantity will be of the form:

$$B^{r_1 \dots r_m}_{\lambda_1 \dots \lambda_n} \qquad (r_i = 1, 2, 3;\ \lambda_j = 1, 2, 3, 4), \tag{17}$$

being a tensor of rank m in isotopic spin space and n in hypercharge space (we have omitted the ordinary tensor indices).

Now we shall see that there is a very strong restriction on such tensors that can be used to describe bosons if the charge and hypercharge can assume only the value $0, \pm 1$. Indeed it is well known from the theory of representation of the rotation group that the highest eigenvalue of I_3 is m and the highest eigenvalue of J_3 (and J_3') given by (16) is n. Thus the state with $I_3 = m$, $J_3 = n$ has the charge $m+n$ and from the condition that the highest value of the charge be 1 we have

$$m + n = 1.$$

Thus we see that we are left with only two possibilities, viz.

a) $m = 1;\ n = 0$ (isospin 1);

b) $m = 0;\ n = 1$ (hyperspin 1).

The first case corresponds to the π-mesons and is characterized by the fact that there are three states of the particles.

The second case corresponds to a particle with four states and corresponds to the K-mesons. Indeed, it is known that if the K-mesons are described by

the hermitian hypervector

$$\mathrm{K}_{\mu} = \begin{pmatrix} \mathrm{K}_1 \\ \mathrm{K}_2 \\ \mathrm{K}_3 \\ \mathrm{K}_4 \end{pmatrix} \tag{18}$$

then the operators M_{12} and M_{34} have the form:

$$iM_{12} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad iM_{34} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}. \tag{19}$$

So if we make the transformation

$$K_{\mu} \to \widetilde{K}_{\mu} = \begin{pmatrix} K_+ \\ -iK_- \\ K_0 \\ -iK_0^{\dagger} \end{pmatrix} \tag{20}$$

with

$$K_+ = \frac{1}{\sqrt{2}}(K_1 - iK_2); \quad K_0 = \frac{1}{\sqrt{2}}(K_3 - iK_4); \quad K_- = K_+^{\dagger}. \tag{21}$$

we find

$$iM_{12} \to \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}; \quad iM_{34} \to \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

so that Q and Y are diagonalised:

$$Q = iM_{12} \to \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix};$$

$$Y = iM_{12} + iM_{34} \to \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}.$$

Thus we see that we can identify the particles described by the linear combinations (21) with the observed K-particles, as their quantum numbers coincide with those given in Table II.

5. - Hamiltonian for the interaction of K-mesons with hyperons.

As the K-mesons are represented by the hermitian 4-vector (K_μ) in the hypercharge space which is a scalar in the isotopic spin space we should form the invariant interaction Hamiltonian density $\mathcal{H}_{\text{int}}$ by contracting K_μ with a 4-hypervector (scalar in isotopic spin space) formed with hermitian products of the components of the hyperon wave operator ψ. Now if the Γ_μ with $\mu = 1, 2, 3, 4$ are four operators acting on the hypercharge components of ψ (and so commuting with $\boldsymbol{I} = \frac{1}{2}\boldsymbol{\tau}$) and having the anticommutation relations

$$\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\,\delta_{\mu\nu} \cdot 1\,, \tag{22}$$

then $\bar{\psi}\Gamma_\mu\psi$ is a 4-vector in the hypercharge space and a scalar in the isotopic spin space (here $\bar{\psi} = \psi^\dagger\beta$) if the infinitesimal operators for rotation in the first space are taken as

$$M_{\mu\nu} = \tfrac{1}{4}(\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu) \tag{23}$$

and the invariant interaction Hamiltonian (density) will be given uniquely (if K_μ is scalar in ordinary space) by

$$\mathcal{H}_{\text{int}} = gK^\mu\bar{\psi}\Gamma_\mu\psi\,, \tag{24}$$

g being a real coupling constant.

We should notice here that we would not gain any new possibility by considering the alternative expression for $\mathcal{H}_{\text{int}}$ where Γ_μ is substituted by the « pseudo-vector » quantity

$$\Gamma_\mu \to \Gamma'_\mu = i\Gamma_\mu\Gamma_1\Gamma_2\Gamma_3\Gamma_4\,, \tag{25}$$

because this new expression of $\mathcal{H}$ differs from (24) only by a unitary transformation which do not alter the Lagrangian density for free hyperons because no Γ matrices appear in it:

$$\mathcal{L}_0 = \frac{1}{2}\left(\bar{\psi}\gamma^\mu\frac{\partial\psi}{\partial x^\mu} + M\bar{\psi}\psi\right) + \text{h. c.}\,. \tag{26}$$

Also we should not mix these two types of interaction terms because we wish to have all hyperons interacting with K-mesons with the same strength in order to assure that the identity of their masses in (26) will be mantained at this stage of the theory. Such mixture is indeed excluded if we impose also invariance of the theory in relation to inversions in hypercharge space. Such

additional invariance requirement will be analized in the next section. Finally, it is superfluous to mention that if K_μ is pseudo-scalar (in ordinary space) an $i\gamma_5$ matrix should be inserted between $\bar{\psi}$ and ψ in (24).

Now we wish to choose a definite representation of the Γ_μ matrices in order to find how the Hamiltonian (24) compares with previous ones. It is convenient to take the representation:

$$\mathbf{\Gamma} = \begin{pmatrix} 0 & \boldsymbol{\eta} \\ \boldsymbol{\eta} & 0 \end{pmatrix}, \quad \Gamma_4 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{27}$$

so that

$$K_\mu \Gamma^\mu = \begin{pmatrix} 0 & \boldsymbol{K}\cdot\boldsymbol{\eta} - iK_4 \\ \boldsymbol{K}\cdot\boldsymbol{\eta} + iK_4 & 0 \end{pmatrix},$$

where $\boldsymbol{\eta}$ are 2×2 matrices identical in form to the Pauli matrices $\boldsymbol{\tau}$. Thus, if $\psi_1 = \begin{pmatrix} \mathrm{N} \\ \Xi \end{pmatrix}$, $\psi_2 = \begin{pmatrix} \Sigma_a \\ \Sigma_b \end{pmatrix}$ expression (24) for $\mathcal{H}_{\text{int}}$ becomes:

$$\mathcal{H}_{\text{int}} = g\bar{\psi}_1(\boldsymbol{K}\cdot\boldsymbol{\eta} - iK_4)\psi_2 + \text{h.c.} \tag{28}$$

We have also, in view of the relations (21):

$$\boldsymbol{K}\cdot\boldsymbol{\eta} - iK_4 = \sqrt{2}\begin{pmatrix} K_0 & K_+ \\ K_- & -K_0^\dagger \end{pmatrix}$$

so that

$$\mathcal{H}_{\text{int}} = g\sqrt{2}[\bar{N}(K_0\Sigma_a + K_+\Sigma_b) + \bar{\Xi}(K_-\Sigma_a - K_0^\dagger\Sigma_b)] + \text{h.c.} \tag{29}$$

If we introduce the isospinors which have the components:

$$K = \begin{pmatrix} K_+ \\ K_0 \end{pmatrix}; \quad K' = \begin{pmatrix} -K_0^\dagger \\ K_- \end{pmatrix} \tag{30}$$

and the matrix

$$\Sigma = \begin{pmatrix} \Sigma_{n'} & \Sigma_+ \\ \Sigma_- & \Sigma_n \end{pmatrix} \tag{31}$$

then (29) takes the form

$$\mathcal{H}_{\text{int}} = g\sqrt{2}(\bar{N}\Sigma K + \bar{\Xi}\Sigma K') + \text{h.c.} \tag{32}$$

and finally, as we can always write

$$\sqrt{2}\Sigma' = A + i\boldsymbol{\Sigma}\cdot\boldsymbol{\tau}$$

our Hamiltonian takes the form:

$$\mathcal{H}_{\text{int}} = g[\bar{N}(\Lambda + i\boldsymbol{\Sigma}\cdot\boldsymbol{\tau})K + \bar{\Xi}(\Lambda + i\boldsymbol{\Sigma}\cdot\boldsymbol{\tau})K'] + \text{h.c.} \tag{33}$$

where

$$\begin{cases} \Lambda = \dfrac{1}{\sqrt{2}}(\Sigma_n + \Sigma_{n'}), & \Sigma_3 = \dfrac{1}{i\sqrt{2}}(\Sigma_{n'} - \Sigma_n), \\ \Sigma_1 = \dfrac{1}{i\sqrt{2}}(\Sigma_+ + \Sigma_-), & \Sigma_2 = \dfrac{1}{\sqrt{2}}(\Sigma_- - \Sigma_+). \end{cases} \tag{34}$$

We see that $\mathcal{H}_{\text{int}}$ has not only taken the usual form (6), (8) but also has become, quite unexpectedly, *formally identical* to the interaction Hamiltonian (7) of Schwinger's theory. It should be mentioned here that we could modify (34) so that the factor i would not appear (*) in the term $i\boldsymbol{\Sigma}\cdot\boldsymbol{\tau}$, as in reference ([8]). It should be mentioned that one of the reasons of this identity results from the fact that the group of transformations which leaves invariant the Hamiltonian is, in both cases, much wider than the group which was used in the establishment of the theory. It is indeed easy to formulate in the present theory the transformation relations used in Schwinger's theory and to prove the invariance of our Hamiltonian, and therefore, the existence of quantities which correspond to the isotopic spin of Schwinger's (and N-G) theory, with eigenvalues $\pm\frac{1}{2}$ for the quadruplet (N, Ξ), 0, ± 1 for $\boldsymbol{\Sigma}$ and 0 for Λ.

6. – Interactions of the hyperons with π mesons and the splitting of the hyperon mass multiplet.

Our theory is completely symmetrical between all hyperons at this stage (before electromagnetic and π-mesons interactions are introduced). The complete degeneracy of the mass multiplet is assured by general principles, and this is true not only for the bare mass M but also for the self-mass ΔM resulting from the interaction (24). Thus the identity of the masses of the particles of isospin $\frac{1}{2}$ and isospin $-\frac{1}{2}$ in the doublets (9) is assured by the invariance under rotations in the isotopic spin space. The identity of the mass of N and Ξ as well as that of Σ_a and Σ_b is assured by the invariance under rotations in the hypercharge space—here Schwinger's transformation (8) and the corresponding one for Σ_a and Σ_b are nothing more than rotation by

(*) SALAM's result ([9]) that his coefficients g_5 to g_8 are all real, on grounds of invariance under charge conjugation, is based on the assumption that Λ and $\boldsymbol{\Sigma}$ transform in the same way under charge conjugation. If they transform with different signs g_5 could be real and g_6 pure imaginary, for instance, as in (33).

([8]) A. SALAM: *Nucl. Phys.*, **2**, 173 (1956).

an angle π in the (2, 3) plane of the hypercharge space times a fase (i):

$$\psi \to \eta, \psi \; ; \quad K_1 \to K_1 \; ; \quad K_2 \to -K_2 \; ; \quad K_3 \to -K_3 \; ; \quad K_4 \to K_4 \; . \tag{35}$$

Finally the identity of the masses of the (N, Ξ) to that of the (Σ_a, Σ_b) quadruplet (*) is assured by the invariance of $\mathcal{H}_{int}$ under « space » reflexions in the hypercharge space:

$$\psi \to \Gamma_4 \psi \; ; \quad \boldsymbol{K} \to -\boldsymbol{K} \; ; \quad K_4 \to K_4 \; . \tag{36}$$

The introduction of the electromagnetic interaction breaks the symmetry in the isotopic spin space and so leads to the splitting of the charge multiplets. The introduction of the interaction with π-mesons in a way which breaks the symmetries in the hypercharge space will lead to a further splitting. In a forthcoming paper we shall analize the ways in which such interactions may be introduced such that a splitting of the multiplet N, Ξ, Λ and Σ results.

We should anticipate that if this splitting is to be attributed to interactions with π-mesons then a lack of symmetry which leads to non conservation of isotopic spin in some of these interactions will be necessary in order to reproduce simultaneously the different structures of the (N, Ξ) and (Σ, Λ) doublets.

Indeed, in order to reproduce this structure the self-mass ΔM should have the form:

$$\Delta M = A + (B + C\eta_3)(1 + \Gamma_5) + (D + E\boldsymbol{\tau}\cdot\boldsymbol{\eta})(1 - \Gamma_5) \; . \tag{37}$$

here $\Gamma_5 = -\Gamma_1\Gamma_2\Gamma_3\Gamma_4$ has the eigenvalues $+1$ or -1 for the quadruplets (N, Ξ) and (Σ, Λ), respectively.

It is clear that ΔM being not invariant under rotations in the isotopic spin (3-dimensional) space some of the interactions of the π-mesons (with Σ, Λ, for instance) should not have this invariance.

This fact might be taken as an argument against the present formulation in view of the already accepted philosophy about the connection of the strength of the interaction with the lack of symmetry. On the other hand as the interaction with the electromagnetic field is not invariant under such transformations we may expect that it is only the non-conservation of parity (and of the hypercharge) that characterises the weak interactions. In this connection it should also be mentioned that the Σ decay (and Ξ decay) interaction which

(*) We should point out that Schwinger's inference about a possible separation of these quadruplets as a consequence of the interaction with K-mesons (*Seattle conference*, 1956) is not correct in view of the equivalence of his theory with the present one at this stage.

1042

is an *isospinor* in the ordinary formulation (9) takes here the form

$$a\bar{\psi}\Gamma_4\boldsymbol{\tau}\psi\cdot\boldsymbol{\pi} + b\bar{\psi}\Gamma_4\boldsymbol{\eta}\psi\cdot\boldsymbol{\pi}, \tag{38}$$

The fact that it is a weak interaction should result from the insertion in (38) of an operator which determines non-conservation of parity.

7. – Concluding remarks.

In the preceding sections we have succeeded in formulating a theory of the interactions of hyperons with K-mesons which differs from the N-G theory not only in the attribution of isotopic spin to the particles but also in the transformation properties of the fields and so in the form of the interaction. One should then expect that although both theories coincide on the general properties of these particles which have been up to now experimentally determined they would differ in detailed previsions of reactions which could be verified experimentally; thus one might expect that improved experiments could lead to a decision in favour of one of them. We have shown, however, that this is not the case and that the present theory is formally equivalent to the N-G theory in the generalized form of Sect. 2. Thus it will be impossible to decide between the two theories at least solely on the basis of reactions involving only hyperons and K-mesons. It is possible that even when the interactions with π-mesons and the weak interactions are introduced in the present theory this equivalence will be mantained. Even so the present formulation may be still convenient for the understanding of some symmetry properties and for computational purposes as it treats the hyperons more symmetrically.

(9) R. Gatto: *Nuovo Cimento*, **3**, 318 (1956); C. Iso and M. Kawaguchi: *Prog. Theor. Phys.*, **16**, 177 (1956).

RIASSUNTO (*)

Si analizzano le conseguenze della legge ormai affermatasi della conservazione della stranezza sugli schemi dei numeri quantici attribuiti agli iperoni e ai mesoni K. Si dimostra che lo schema di Gell-Mann-Nishijima universalmente accettato non è unico e che è possibile un altro schema in cui al mesone K si attribuisce spin isotopico 0 e alle particelle (Λ, Σ) spin isotopico $\frac{1}{2}$. Si sviluppa con questo schema una teoria basata su principi generali di simmetria e si dimostra che nello stadio in cui si introducono solo interazioni di mesoni K con iperoni detta teoria è equivalente alla teoria quadridimensionale di Schwinger (nello spazio dello spin isotopico). Si considerano alcuni aspetti delle interazioni deboli. Uno di tali aspetti è che con la nuova definizione dello spin isotopico nelle interazioni dei mesoni π con le particelle (Λ, Σ) può andar perduta la conservazione dello spin isotopico (con la regola di selezione $\Delta I_3 = 0, \pm 1$).

(*) *Traduzione a cura della Redazione.*

An Analysis of 419 τ-Meson Decays.

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Summary. — A homogeneous sample of 419 τ-decays found in nuclear emulsions exposed to the Berkeley Bevatron has been used in the present analysis. Special attention has been given to reducing the effects of the experimental biases both for the scanning and for the analysis. The results show that the τ-decay is nearly isotropic, corresponding in the Dalitz treatment to zero spin and negative parity.

1. – Introduction.

The angular and energy distributions of the three pions emitted in the decay of the τ-meson have been investigated by several authors [1] with the object of obtaining information about the spin and parity of the primary

[1] E. Amaldi: *Suppl. Nuovo Cimento*, **4**, 179 (1956); B. T. Feld, A. C. Odian, D. M. Ritson and A. Wattenberg: *Phys. Rev.*, **100**, 1539 (1955); B. Bhowmick, D. Evans, I. J. van Heerden and D. J. Prowse: *Nuovo Cimento*, **3**, 574 (1956); N. N. Biswas, L. Ceccarelli-Fabbrichesi, M. Ceccarelli, K. Gottstein, N. C. Varshneya and P. Waloschek: *Nuovo Cimento*, **3**, 825 (1956); N. Brene, K. H. Hansen, J. E. Hooper and M. Scharff: *Nuovo Cimento*, **4**, 1059 (1956); Y. Eisenberg, E. Lomon and S. Rosendorff: *Nuovo Cimento*, **4**, 610 (1956); R. P. Haddock: *Nuovo Cimento*, **4**, 240 (1956); H. H. Heckman, F. M. Smith and W. H. Barkas: *Nuovo Cimento*, **4**, 51 (1956); J. Orear, G. Harris and S. Taylor: *Phys. Rev.*, **102**, 1676 (1956); H. Winzeler, M. Teucher and E. Lohrmann: *Helv. Phys. Acta*, **29**, 75 (1956).

particle according to the scheme proposed by DALITZ (2), and developed by FABRI (3) and by COSTA and TAFFARA (4), who have considered only the five simple cases 0 —, 1+, 1 —, 2+ and 3 —. However, none of the homogeneous samples which have been analysed so far contains a number of events which substantially exceed one hundred, though in order to reduce the statistical errors inherent in such small numbers, some authors have combined the results of various laboratories. It is not clear how far this procedure is justified since it is difficult to allow for observational and measurement biases which may vary from laboratory to laboratory and which could affect the detection or analysis of certain forms or aspects of the decays.

In order to avoid this difficulty, a homogeneous sample has been obtained which was sufficiently large to be statistically significant, and where the events had been found under conditions such that they were virtually free from both experimental and geometrical biases. The analysis of the 419 events which make up this sample (*) forms the subject of the present work.

2. – Experimental procedure.

2·1. *Method of scanning and selection criteria.* – All 419 events have come from two stacks of Ilford G5 emulsions exposed to the K^+ beam from the Berkeley Bevatron. One stack, the K_1, consisted of 120 pellicles each 27 cm × 39 cm × 600 μm in which the incident K-mesons in our part of the stack had a mean range of 6 cm.

The other stack, the K_2, yielded the majority of the events. It was made up of 300 pellicles each 20 cm × 25 cm × 600 μm in which the ranges of the incident particles were about 13 cm. For both stacks the mean time of flight from the target to the decay point was about $1.4 \cdot 10^{-8}$ s.

Most of the events have been found by area scanning, in which a search was made for K-mesons ending in the emulsion and giving rise to three charged

(2) R. H. DALITZ: *Proc. Phys. Soc.*, A **66**, 710 (1953); *Phil. Mag.*, **44**, 1068 (1953); *Phys. Rev.*, **94**, 1046 (1954); **99**, 915 (1955); *Proc. Phys. Soc.*, A **69**, 527 (1956).

(3) E. FABRI: *Nuovo Cimento*, **11**, 479 (1954).

(4) G. COSTA and L. TAFFARA: *Nuovo Cimento*, **3**, 169 (1956).

(*) An analysis of 187 of these events was communicated by the Padua Group at the Sixth Annual Rochester Conference, 1956 (see *Proceedings*, page V-18); the preliminary results from the complete sample (including 35 additional events from the Padua Group, 66 from the Brussels Group and 121 from the Milan Group) were communicated at the Turin Conference of the Società Italiana di Fisica, September 1956. The conclusions reached in the present work, which contains the whole statistics and a more complete analysis, confirm and strengthen the results of the earlier communications.

secondaries. In a given plate the area selected for examination was that in which the K-mesons of the appropriate momentum were expected to stop, and within that zone the ends of all tracks were carefully scrutinized: 360 events have been found in this way. In view of the characteristic aspect of a τ-decay and the fact that all the primary particles came from approximately the same direction, this visual examination has proved sufficient to discriminate between the decay at rest of τ-mesons and the four-prong stars found in emulsions. On the other hand, the visual analysis adopted can lead to the loss of a certain number of « good » events due to one or more of the following factors:

a) the closeness to the glass or air surfaces of the emulsion;

b) the grain density of the tracks of the secondary particles (which is also connected with the degree and uniformity of the development of the plates);

c) the geometric form of the decay, and in particular the orientation of the decay plane with respect to the emulsion plane, and the orientation of the three secondaries within the decay plane.

In order to ascertain the extent of the losses due to these factors and their selection effects on certain types of event, a partial re-exploration was carried out. About 200 cm² have been carefully re-examined and as a result the losses were found to be less than 5% and there was no evidence of selectivity.

Track scanning was used to find 59 events, and it should be noted that also this type of exploration may be subject to the effects of factors *a*), *b*) and *c*) which might lead to the failure to detect one or more of the secondaries and the consequent mis-classification of the event as representing some other decay mode. However, a systematic re-examination of all the track endings of the K-mesons found by track scanning in the plates from which our τ-mesons came has not led to the finding of a single case of a τ-meson which had been mis-classified as due to a different decay mode.

The distribution of the depth of the decays in the emulsion has been studied, and as was expected in view of the fact that both stacks were slightly underdeveloped and had a development gradient, the number of events in the layer nearest to the glass surface was found to be sensibly below the average for the whole of the emulsion. In view

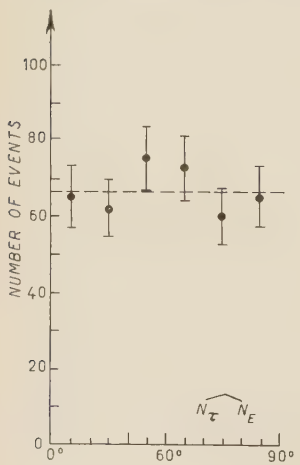

Fig. 1. – Space angle between the normal to the decay plane and the normal to the emulsion, in equal solid angles.

of this effect, all those events lying in the upper 10 μm or in the lowest 40 μm, a total of 15, have been rejected.

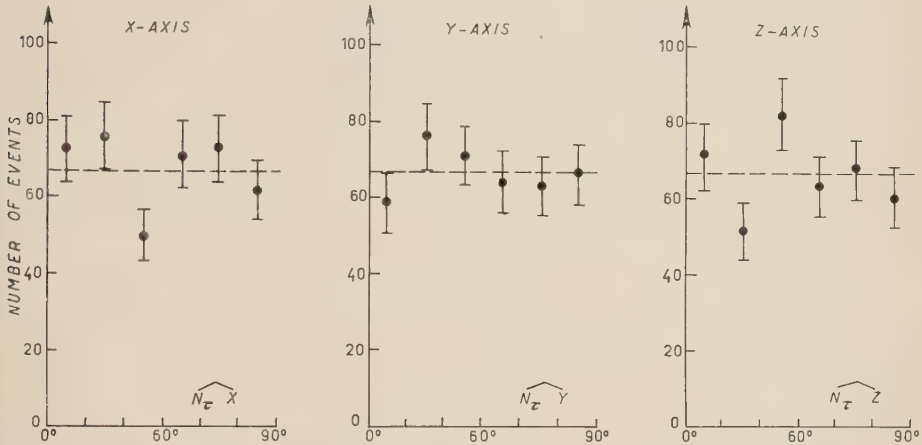

Fig. 2. – Space angles between the normal to the decay plane and the axes defining the production plane (see Appendix I for a description of the co-ordinate system used), in equal solid angles.

The distribution of the angle between the normal to the decay plane and the normal to the plane of the emulsion in given in Fig. 1. It appears to be isotropic, which suggests that if the decay has not introduced any preferential orientations, then the method of scanning has not done so either. This is confirmed by the distribution given in Fig. 2 for the eulerian angles of the normal to the decay plane with respect to the production plane (see appendix I for description of the co-ordinate system).

2·2. *Method of analysis and acceptance criteria.* – For each τ-meson the angles between the secondaries, their energies and the sign of their charges have been determined. The space angles have been derived from the projection and dip angles by means of a stereographic projection (Wulff's disc). The tolerances were chosen to be such that no space angle had an error of more than $\pm 2°$ even in the most unfavourable cases. At the same time the decay was checked for coplanarity: an event was considered coplanar when it was possible to select a meridian on Wulff's disc which was compatible with the errors in the measured projection and dip angles.

The energies and signs of charge have been obtained by following at least two secondaries to their ends: the energy of the third has been calculated by completing the momentum triangle, and its sign of charge determined by assuming that the primary was positive and decayed into one negative and two positive secondaries. When peculiarities of an individual decay made it

advisable, the third secondary was also followed. For those secondaries which were followed the energy could be determined to within $\pm 3\%$, taking into account straggling and errors of measurement, while for those energies which were calculated, the error did not exceed 10% for the most unfavourable cases. The range-energy relation used was that of BARONI *et al.* (5). No special measuments of the stopping power of the emulsion have been made owing to the limits of error which we have accepted.

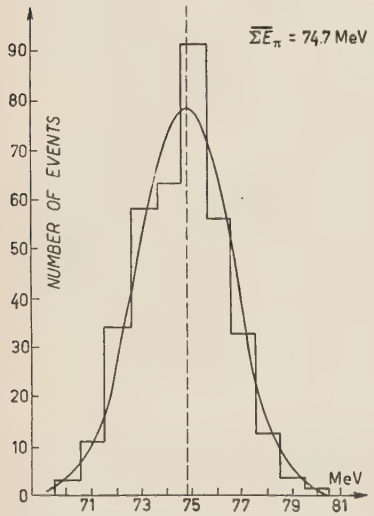

Fig. 3. – Histogram of Q-values. The smooth curve is a gaussian of the same area as the histogram, with dispersion 1.9 MeV.

Almost always the distortion has proved to be small, and can be neglected in view of the tolerance limits accepted for the angular measurements.

400 events have been accepted which have satisfied the following two conditions: *a*) coplanarity of the secondaries (see above), and *b*) the sum of the kinetic energies of the secondaries $\sum E_\pi$ to be compatible with a single Q-value of 75 MeV (6). If an event satisfied condition *a*) it was then required to satisfy condition *b*). The distribution of the Q-value is given in Fig. 3. The smooth curve represents a gaussian of the same area as the histogram; the mean value is 74.7 MeV; the dispersion is 1.9 MeV; in every single case $\sum E_\pi$ lies between 70 and 80 MeV.

Four events have proved to be unmeasurable: two lay in a region of abnormal distortion due to a bubble in the emulsion, while two events each had two secondaries which interacted in flight.

The 400 events satisfying both the acceptance conditions have been considered as the decay at rest of positive τ-mesons. Within the limits of our errors, there is no evidence of decay schemes different from the usual one into three pions.

The fact that no decays in flight were observed is not surprising when one considers that even if the residual momentum of the τ is as much as 30 MeV/c, this only corresponds to a detection probability of 0.2% when area scanning is used: such a residual momentum would have been easily recognized.

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(5) G. BARONI, G. CASTAGNOLI, G. CORTINI, G. FRANZINETTI and M. MANFREDINI: CERN, *Bureau of Stdanards, Bull.* No. 9.

(6) G. L. BACCHELLA, A. BERTHELOT, M. DI CORATO, O. GOUSSU, R. LEVI SETTI, M. RENÉ, D. REVEL, L. SCARSI and G. TOMASINI: *Nuovo Cimento*, **4**, 1529 (1956). Thanks are due to these authors for having made available to us the results on 25 of the 98 τ-mesons found in K_1-stack by the Brussels and Milan Group.

2·3. *Experimental results.* – Fig. 4 gives the energy distributions for the positive and negative secondaries: Fig. 5 gives the corresponding angular distributions. It has already been mentioned in 2·1 that the method of

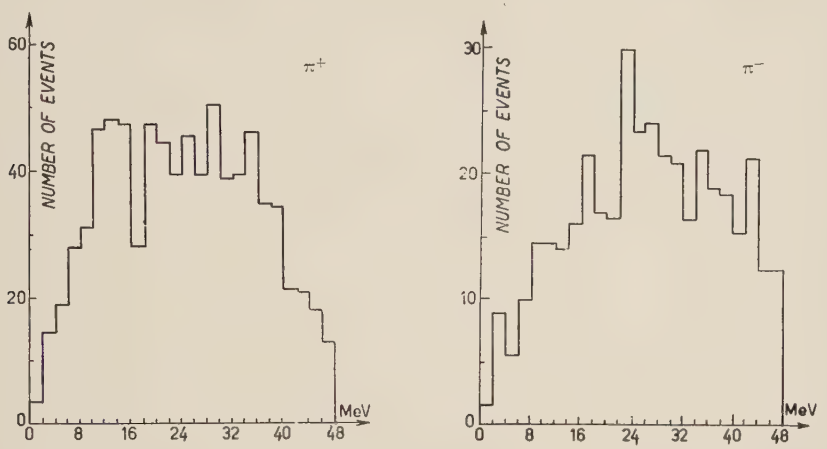

Fig. 4. – Energy distributions for (*a*) the positive pions and (*b*) the negative pions.

scanning may have led to a selection which discriminates against certain types of decay. Such a loss would have especially affected those decays in which at least two of the secondaries were not very visible due to their being relatively energetic: this in turn could have caused the loss of decays in which one of the secondaries (positive or negative being equally affected) was very short, and thus to the impoverishment of the low energy region of the distribution given in Fig. 4. However, this possibility does not seem to be confirmed by the results of the re-exploration.

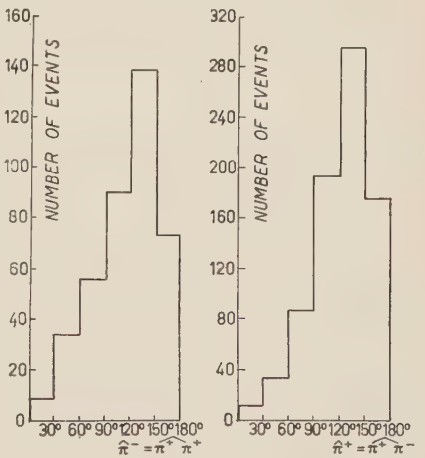

Fig. 5. – Angular distributions for the positive and negative pions.

The effect of the limitations in size of the stack has been repeatedly pointed out (see for example BRENE *et al.* [1]). Both our stacks were well above the critical size of at least 4.5 cm × 4.5 cm × 4.5 cm, and we have only scanned that zone which lay at least 8 mm from any face of the stack. Under these conditions, at least two secondaries should come to rest within the stack, thus allowing the determination of the energy and the sign of charge of the third.

The fact that only 4 events have been rejected owing to experimental dif-

ficulties shows that the selection has not introduced a bias either for or against any particular form of decay. However, one must take into account that while the short range secondaries have generally been followed to their ends, a large part of the long range ones have been calculated: thus the percentage error for the latter is normally larger than for the former. Similarly an even larger error affects the calculated energies of those particles which happen to be opposite a small angle, since small angles are difficult to measure with proportionately small errors. Thus high energy secondaries are subject to a double source of error with respect to low energy secondaries: first because they are calculated instead of being measured, and secondly because the calculation for a high energy secondary is accompanied by a larger error than that for a low energy secondary. This increased error for the higher energies would lead to a spread in the energy distributions: a rough calculation shows that for our events one should expect an upper limit of $1\frac{1}{2}$ cases where the negative pion had an energy greater than 48 MeV (or three cases for the positive pions). In fact one event has been found in which a positive secondary has a calculated energy greater than this value.

3. – Discussion.

DALITZ ([2]) has suggested a scheme for τ-decay which leads to certain curves for the energy and angular distributions as functions of the various possible combinations of spin and parity. In Figs. 6, 7 and 8 the experimental results are compared with the energy distributions for the negative pions, the $\cos\theta$ distribution and those calculated by COSTA and TAFFARA ([4]) respectively. All of these distributions are for the 5 simple cases only, namely $0-$, $1+$, $1-$, $2+$ and $3-$.

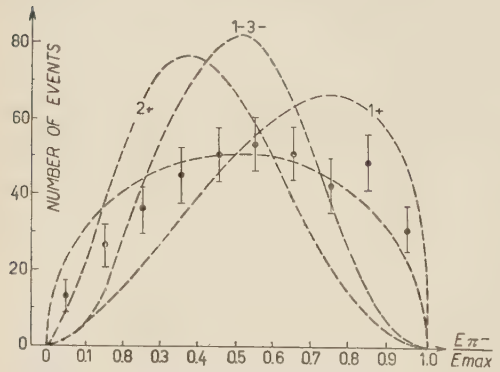

Fig. 6. – The Dalitz curves with the experimental results. The errors indicated are the statistical errors plus the experimental errors.

These theoretical curves are based on various simplifying hypotheses (for example that the radius of interaction is independent of the angular momenta and is roughly equal to the Compton wavelength of the τ-meson; also that the Coulomb effects can be neglected) and therefore the exact forms cannot be obtained. In effect the curves are based on consideration of the momentum space. Energy distribution curves derived in this

way show a maximum sensitivity to the value of the spin in the extreme parts of the spectrum (i.e. near zero or the maximum energy of 48 MeV available to a pion secondary) since in these zones they are less dependent on the simplifying hypotheses used. Similarly the maximum sensitivity for the cos θ distribution is for values near unity. In view of the small number of events in these extreme zones, the most efficient statistical method for the analysis is that of the « maximum likelihood ». The probability function:

$$L = \Pi F_E(E_i) ,$$

has been calculated for each single event between 0 and 10 MeV and between 38 and 48 MeV, while the events with intermediate energies have been grouped in 2 MeV intervals. The analysis has not been made for the cos θ distributions since these are identical for the cases 0 — and 1 +, which are the only reasonable possibilities from amongst the five simple cases.

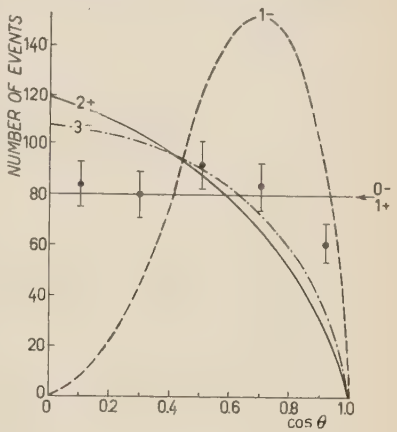

Fig. 7. – The cos θ distributions according to DALITZ.

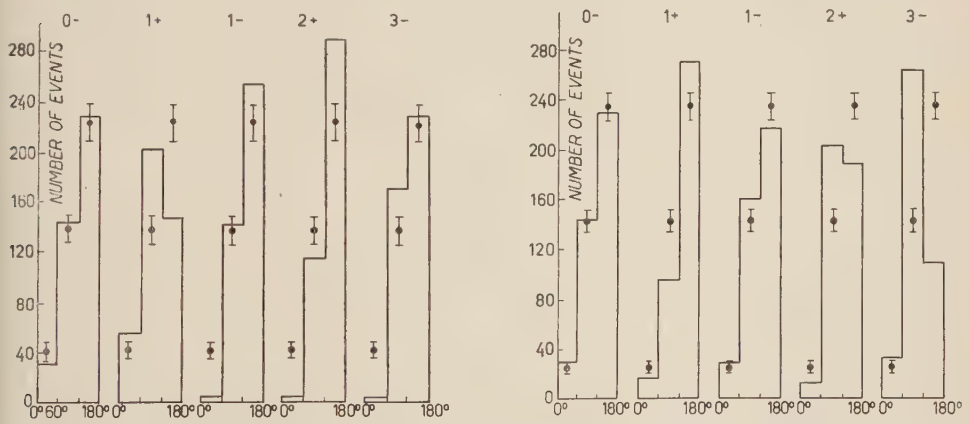

Fig. 8. – Distribution of the angles according to COSTA and TAFFARA (*a*) between the two like mesons, and (*b*) between the unlike mesons.

Table I summarizes the maximum likelihood and χ^2 calculations.

The distributions used by us have been calculated classically. However in the 0 — case an attempt has been made to calculate the relativistic corrections both for the matrix element and for the statistical factor. The variations found lie well within the statistical errors of our sample, further, in

the low energy region (below about 10 MeV) it has been shown [7] that the Coulomb correction neutralizes the relativistic correction. In view of these considerations, we consider that the classical approximation is sufficient for our purposes.

TABLE I.

Spin-parity	Max. likelihood with respect to 0 —	χ^2
0 —	1	7%
1 +	10^{-5}	< 0.1%
1 —, 3 —	10^{-47}	< 0.1%
2 +	10^{-70}	< 0.1%

From the results it would appear that the most likely possibility is that the τ-meson decays in a state of zero spin and negative parity. Since in the Dalitz model this corresponds to a matrix element of 1, one may consider the deviations from the purely statistical factor as being due to the inadequacy of the simplifying hypotheses which form the basis of the model. We have therefore tried to obtain further information about these hypotheses by comparing directly the experimental data with the statistical distribution calculated relativistically. In Figs. 9

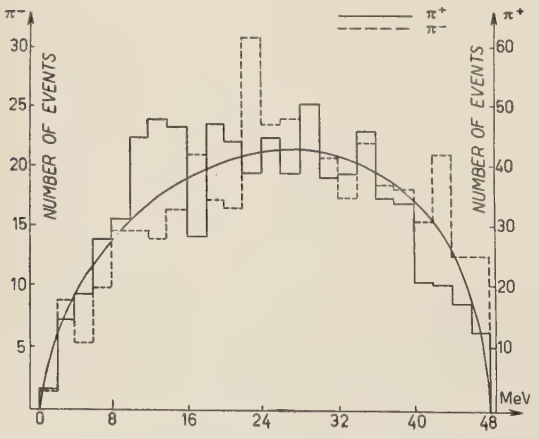

Fig. 9. - Energy distribution using the statistical factor only.

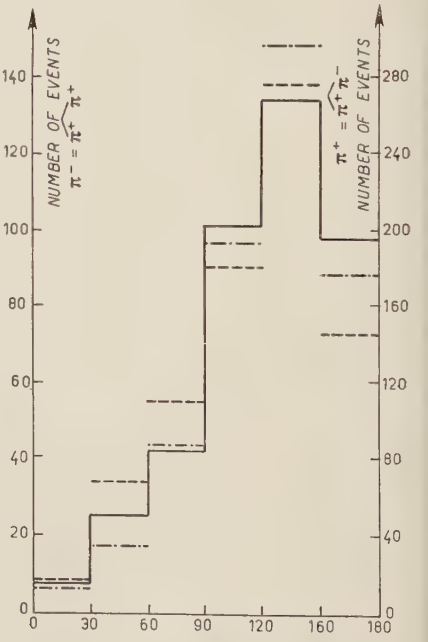

Fig. 10. – Angular distribution using the statistical factor only. — Theoretical curve; ---- π^-; -.-.- π^+

(7) D. M. RITSON: *Massachusetts Institute of Technology, L.N.S. Progress Report* (in preparation).

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and 10 the experimental data and the relativistically correct curves derived from the statistical factor only are compared. In Table II the values of the momenta are given up to the 4th order for the experimental distributions of the π^+ and the π^-, together with those for the theoretical distribution.

TABLE II.

	π^+	π^-	Theoretical
μ_2	132	140	144.34
μ_3	— 255	+ 420	— 118.94
μ_4	35 360	40 600	41 850
γ_1	— 0.167 6	+ 0.253 1	— 0.068 6 ± 0.313 7 (*)
γ_2	— 0.980 0	— 0.933 8	— 0.991 3 ± 0.105 8 (*)

μ_2 = moment of order 2 with respect to the theoretical mean, for the statistical distribution;
μ_3 = moment of order 3 etc.;
μ_4 = moment of order 4 etc.;
γ_1 = coefficient of skewness;
γ_2 = coefficient of excess.

(*) The errors given in column 4 have been calculated on the assumption that the individual pion energies have an error of ± 1 MeV: thus the error given is the differential error, with $\Delta E = 1$ MeV.

The agreement within the experimental errors between the momenta of various orders shows that decay of the τ-meson is substantially isotropic, however it cannot be excluded that the small deviation from the calculated curves may represent a real effect. This can only be decided with a notably larger statistics than that presented here.

4. – Conclusions (*) (+).

A homogeneous sample of 419 τ-decays has been obtained under conditions such that it is virtually free from experimental and geometrical

(*) In a recent paper by SHAPIRO, DOLINSKY and MISHAKOVA [*Nuclear Physics*, **3**, 60 (1957)] which appeared after the present analysis had been prepared for publication, a new criterion is applied to the calculation of the spin-parity curves, namely that « the π-mesons are created in states which are symmetrical with respect to the momenta of all three particles ». The curves obtained in this way are very different from those given by Dalitz, except for the case of 0— which remains the same. If these curves are indeed the correct ones, the conclusions reached in the present analysis not only remain unaltered but are actually strengthened.

(+) After this analysis had been completed, a pre-print was received from J. OREAR (J. OREAR: *Evidence against Spin* 1 *for the* τ-*meson*, to be submitted for publication in the *Phys. Rev.*) in which a similar type of maximum likelihood analysis is made for 835 events gathered from several laboratories. The author concludes that the relative probabilities for spin zero and spin one are in the ratio of 10^4.

biases. A comparison of this data with the curves given by DALITZ shows that the τ-meson decays in a state of zero spin and negative parity. This has suggested a detailed comparison with the curves based on the statistical factor only, and the agreement between these and the experimental data indicates that the effects which were ignored by DALITZ in his model cannot be important. Thus to a first approximation the τ-decay is isotropic, with the Q-energy being statistically divided amongst the three secondaries.

* * *

The authors are very pleased to thank Prof. N. DALLAPORTA for his active interest throughout this work and Prof. G. OCCHIALINI for helpful criticisms and suggestions. They are also indebted to Dr. R. H. DALITZ and Dr. G. MORPURGO for suggestions relating to the analysis of the results. Mr. S. NATALI and Mr. G. VEGNI provided valuable assistance during the later stages of the analysis and the preparation of the data for publication. Thanks are also due to Dr. G. TOMASINI of the Istituto di Fisica di Genova, for having made available to the group a certain number of events found and analyzed by her, and to Dr. G. L. BACCHELLA and Dr. L. SCARSI of the Milan group for help in measurements and useful discussions.

APPENDIX I

If the τ-meson has spin greater than zero and this is polarized, there should be an anisotropy in the direction of emission of the negative pions. Although this type of evidence is less reliable than that already given, in the sense that an isotropic set of results may not indicate the absence of spin but only of polarization (and in addition it has recently been suggested [8] that nuclear emulsions may possess de-polarizing properties), it has nevertheless been thought worthwhile to give the spatial distributions of the negative pions. These distributions are given in Fig. 11 and are relative to a system of axes oriented so that the X-axis corresponds to the direction of the proton beam which produced the K-meson in the target; the Y-axis indicates the direction of emission from the target of those K-mesons selected for the experiment, (at 90° to the proton beam) while the Z-axis has been chosen so as to complete a right-hand system of cartesian co-ordinates. Only the Z-axis lay in the plane of the emulsion.

[8] B. BHOWMIK, D. EVANS and J. D. PROWSE: *Nuovo Cimento*, **5**, 1663 (1957).

The isotropy is very evident about the Y- and Z-axes, and about the X-axis the distribution is not more than 2 standard deviations from isotropy. What is more important, however, is that there is no clear indication of a tendency towards a systematic anisotropy.

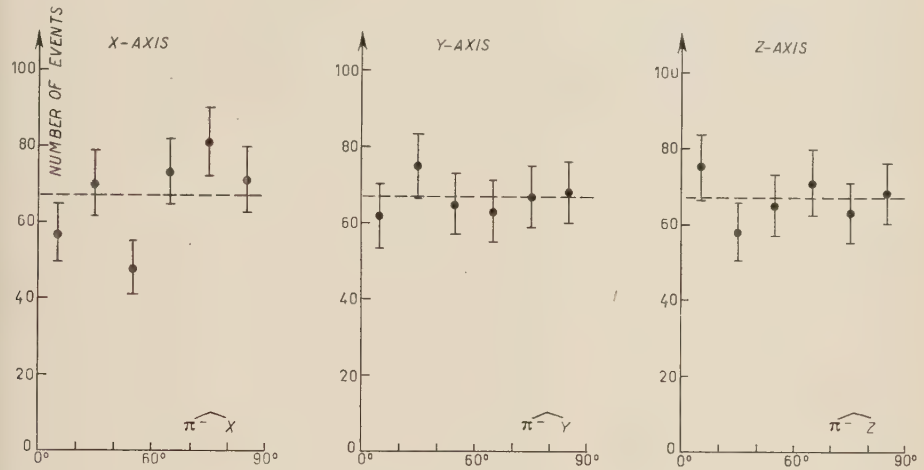

Fig. 11. – The space angles between the negative pions and the axes defining the production plane, in equal solid angle .

Appendix II

The following is a list of the energies of the negative pions from the 400 τ-meson decays used in this analysis. Those which have been calculated are indicated with an asterisk: in the other cases the pion has been followed to its end and the energy obtained from its range.

MeV

1.2	4.0	6.7*	8.4*	10.0	11.3	12.9	14.1
2.0	4.1	6.8	8.5	10.2	11.4	13.0	14.2
2.1	4.15	7.0	8.8	10.2	11.9	13.4	14.2
2.6	5.3	7.1	9.05	10.6	11.9	13.5	14.4
2.75	5.6	7.9	9.2	10.7	12.0	13.5	14.5
3.3	5.8	8.0	9.3	10.8	12.0	13.6	14.5
3.5	6.1	8.0	9.3	10.8	12.3	13.8	14.8
3.5	6.4	8.2	9.5	11.0	12.6	13.9	15.0
3.8	6.4	8.2	9.6*	11.0	12.7	14.0	15.1*
3.9	6.5	8.3	9.7	11.2	12.7*	14.0	15.4

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15.5	19.7*	23.2	26.5*	30.0*	34.1	38.0*	42.4
15.5	19.9	23.2	26.6	30.0*	34.2	38.5	42.5*
15.7	19.9	23.3*	26.6*	30.1*	34.6	38.5*	42.5*
15.8	20.0	23.5	26.7	30.5	34.7	38.5*	42.7*
15.9	20.1	23.5	26.7	30.5	34.8	38.5*	42.8*
16.1	20.2	23.5	26.8*	30.5	35.0	38.6*	43.0*
16.1	20.2*	23.5*	26.9*	30.8	35.0	38.7	43.0*
16.2*	20.5	23.8	27.0	31.0	35.0*	39.0	43.0*
16.5	20.6	23.9	27.0	31.0*	35.0*	39.0	43.2*
16.5	20.8	23.9	27.0	31.1	35.1*	39.0*	43.2*
16.5	21.0*	24.0	27.0*	31.2*	35.2	39.1	43.5*
16.8	21.2	24.0	27.1*	31.3*	35.3	39.2*	43.5*
16.9	21.4	24.1	27.4	31.5	35.3*	39.5*	43.7
17.0	21.5	24.1	27.5	31.5	35.5	39.5	44.0*
17.0	21.6	24.5	27.5	31.5	35.5	39.8	44.0*
17.0	21.6	24.5	27.5	31.5	35.5*	39.8*	44.0*
17.0	21.6*	24.6*	27.5	31.5*	35.5*	39.9*	44.0*
17.1	21.7	24.7	27.5*	31.6*	35.5*	40.0*	44.4
17.1	21.8	24.8	28.0	31.7*	35.5*	40.0*	44.5*
17.2*	22.0	24.8	28.2	31.9	35.5*	40.5*	44.7*
17.5	22.0	24.8	28.2*	32.0	36.0	40.8*	45.0*
17.5	22.1	24.8	28.5	32.0*	36.3	41.0*	45.0*
17.6	22.1*	24.9	28.5*	32.1	36.4*	41.0*	45.0*
17.8	22.2	24.9	28.6*	32.2	36.4*	41.0*	45.0*
17.8	22.3	25.0	28.7*	32.4	36.5*	41.0*	45.0*
18.0	22.3*	25.0	28.8*	32.5	36.5*	41.2*	45.5*
18.0	22.4	25.0	29.0	32.5	36.5*	41.4*	45.8*
18.0*	22.5	25.0	29.0	32.5	37.0	41.5*	46.0
18.2	22.5	25.0	29.0	32.5	37.0*	41.5*	46.6
18.5	22.5*	25.1	29.0*	32.5*	37.0*	41.5*	46.8*
18.5	22.5*	25.3	29.4	33.0	37.2	42.0	47.0*
18.5	22.5*	25.5	29.4*	33.0	37.4	42.0	47.0*
18.6	22.7	25.6	29.5*	33.0*	37.4*	42.0*	47.0*
18.8	22.9	25.7*	29.5*	33.3	37.6	42.0*	47.0*
18.8	23.0	26.0*	29.6	33.4	37.6*	42.0*	47.2*
18.8	23.0	26.2	29.6	33.5	37.7*	42.0*	47.6*
19.0	23.0	26.2	29.8	33.5	37.8	42.0*	48.0*
19.0	23.0	26.3*	29.8*	34.0	37.9*	42.2*	48.0*
19.0	23.0*	26.5	30.0	34.0	38.0	42.3*	48.0*
19.0	23.1	26.5	30.0	34.0*	38.0	42.3*	48.0*

RIASSUNTO

Nel presente lavoro è stato analizzato un gruppo omogeneo di 419 decadimenti di mesoni τ trovati in emulsioni nucleari esposte al Bevatrone di Berkeley. Tanto l'esplorazione quanto l'analisi sono stati eseguite in modo da ridurre al minimo l'errore di bias sperimentale. Dai risultati si ricava che il decadimento del τ è con buona approssimazione isotropo, e corrisponde, secondo DALITZ, a spin zero e parità negativa.

On the Azimuthal Distribution of K Photoelectrons by Polarized Photons.

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(ricevuto il 4 Aprile 1957)

Summary. — The angular distribution of K-photoelectrons has been investigated in Pb using polarized Compton scattered photons of a ^{60}Co source. The selection of the events has been made by associating the photoelectrons to the correspondent fluorescence X-ray. The measurement was performed by determining the behaviour of the asymmetry ratio versus energy: we define the asymmetry ratio as the number of electrons emitted perpendicularly to the Compton scattering plane divided by the number of those emitted in the scattering plane. The results obtained do not seem to agree with Sauter's theory, but rather with Archibald's calculations.

1. – Introduction.

Several authors have calculated, with various approximations the photoelectric cross-section for different values of Z and of the energy of the incident photon. At rather high energies relativistic calculations are necessary and require the application of Dirac's relativistic wave equation.

An analysis of the results of the various theories was made by Kahan (1) and by Davisson and Evans (2), but from the literature available on this subject, it appears that only Sauter (3) has calculated the differential cross-section for the photoelectric effect produced by polarized photons in the relativistic region. On the same problem, Archibald (4) has made not yet pub-

(1) T. Kahan: *Journ. Phys. Rad.*, **10**, 430 (1939).
(2) C. M. Davisson and R. D. Evans: *Rev. Mod. Phys.*, **24**, 79 (1952).
(3) P. Sauter: *Ann. Phys.*, **11**, 454 (1931).
(4) W. J. Archibald: private communication.

1958

lished calculations, which he communicated to us privately. MORETTE's [5] calculations on the angular distribution of photoelectrons refer to the non-relativistic region, and have therefore no value for photons of rather high energy. HULME's [6] calculations refer to any energy and Z but they cannot be converted into equations.

Several authors have carried out experiments to determine the angular distribution of the photoelectrons. In their measurements they have used unpolarized photons; the experimental test can therefore only check the θ distribution of the photoelectrons, the φ distribution being isotropic.

The most recent measurements of this type have been made by HEDGRAM and HULTBERG [7], and HULTBERG [8]; they used a magnetic spectrometer to measure the photoelectron's direction and energy. ROY, GOES and BERGER [9] used a Wilson chamber in a magnetic field. Their results appear in good agreement with Sauter's results concerning the maximum of emission whereas the disagree at $\theta = 0°$ and at large values of θ.

SAUTER [10] himself has revised his formula to reach a better agreement with experiment.

Measurements with polarized photons were carried out by HEREFORD and KEUPER [11] using photons from annihilation of ^{22}Na positive electrons, and by MCMASTER and HEREFORD [12] who polarized the ^{60}Co photons by Compton scattering. In these experiments, made with scintillation counters, Sauter's formula for the φ distribution has been investigated by measuring the ratio between the emission cross-sections of photoelectrons in the plane of the electric vector and in the normal plane of the incident photon.

These author's results seem to be in agreement with the asymmetry ratio expected from Sauter's theory.

The present experiment was designed to repeat the measurements of MCMASTER and HEREFORD with a different experimental arrangement and to compare the results obtained with Sauter's and Archibald's theories.

2. – The experimental arrangement.

A two-Curie ^{60}Co source (Fig. 1), lead screened, provides a collision beam of unpolarized γ-rays.

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(5) C. MORETTE: *Journ. Phys. Rad.*, **7**, 135 (1946).
(6) H. R. HULME: *Proc. Roy. Soc.*, **133** A, 381 (1931).
(7) A. HEDGRAM and S. HULTBERG: *Phys. Rev.*, **94**, 498 (1954).
(8) S. HULTBERG: *Ark. f. Fys.*, **9**, 245 (1955).
(9) R. RAMAN ROY, M. L. GOES and J. BERGER: *Compt. Rend.*, **241**, 1936 (1955).
(10) F. SAUTER and H. O. WUSTER: *Zeits. f. Phys.*, **141**, 83 (1955).
(11) F. L. HEREFORD and J. P. KEUPER: *Phys. Rev.*, **90**, 1043 (1953).
(12) W. H. MCMASTER and F. L. HEREFORD: *Phys. Rev.*, **95**, 723 (1954).

1959

The beam impinges on a small copper cylinder 1 in. $\times$ 1 in.; after a second collimation, with a 8 in. thick lead screen, the source provided a photon beam partially polarized by Compton scattering. Varying the angle θ_1 between the direction defined by this collimation and that of the primary beam, one obtains

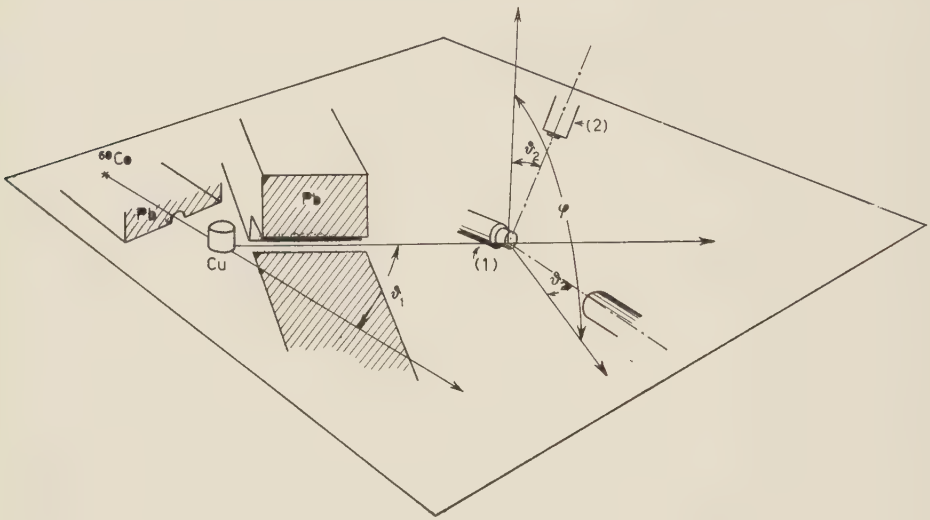

Fig. 1.

the various energies required for the experiment. The target consisted of a circular lead foil 1 in. in diameter and $\sim 0.6/1\,000$ in. thick (~ 16 mg/cm^2). To detect the photoelectric effect, we assumed the criterion to choose those associated events, which could be considered as a pair $e - X$, where X is the photon emitted by the atom after the emission of the electron. This criterion suggested the choice of a Pb target, given the high frequencies of the fluorescence lines.

To this purpose, a cylindrical NaI(Tl) scintillation counter (1) (1 in. in diameter, $\frac{1}{2}$ in. thick, seen by a Dumont 6292 P.M.) detected the X, and an anthracene scintillation counter (2) (also seen by a 6292 P.M.) detected the electrons. For the following reasons a cylindrical anthracene crystal (1 in. diameter, 0.1 in. thick) was selected as electron detector:

a) a screenless receiving surface;

b) good light emission;

c) low efficiency to the γ background;

d) low backscattering percentage.

The solid angle of the X-counter was very near to 2 π sterad, the crystal nearly touching the target. Both scintillation counters were placed in a light-

096

tight parallelepiped box, big enough (35 in. $\times$ 20 in. $\times$ 18 in.) to reduce the γ-background. The box could be rotated around an axis coinciding with the direction of the impinging γ beam.

Coincidences X-e were detected and analyzed by the apparatus whose block diagram is shown in Fig. 2.

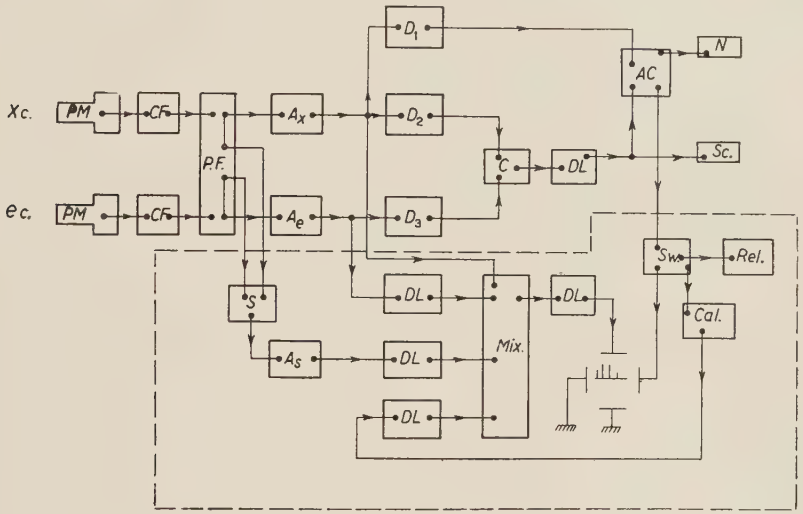

Fig. 2.

The two pulses of both P.M., after suitable amplification and formation, feed on two discriminators D_2 and D_3, setting the bias of the two signals. The pulses overcoming the bias enter a coincidence circuit C, with a resolving time of $\tau \simeq 3 \cdot 10^{-7}$ s.

Then the coincidence pulses feed on an anticoincidence circuit (AC) triggered by pulses higher than a certain bias set by the discriminator D_1 coming from the X-channel. This bias corresponds to the maximum value of the X-rays emitted by the target; higher energy signals are stopped.

Signals coming through are recorded by the numerator N, and trigger a sweep *Sw* of a synchroscope, on which both the X-ray and the corresponding electron can be photographed. *Cal.* represents a calibrator, triggered by the sweep *Sw*, giving a standard pulse which was photographed with the X-ray and the electron pulse.

Another pulse coming from the circuit S, sum of the X and electron pulses, was also photographed. Both the standard and « sum » pulses can control the electronic arrangement and normalize the heights of the two principal pulses, avoiding any possible gain change of the syncroscope.

The last part of the arrangement, concerning the photography of the events, was used only for calibration and preliminary control purposes.

We observe, finally, that the scale *Sc*, counting the events from electron channels, may not appear essential: the normalization of the final results was, on the contrary, based on this counting as is described in a following section.

3. – Efficiency of the experimental arrangement.

The following coincidences are not interesting:

a) chance coincidences;

b) coincidences due to Compton effect in the NaI(Tl) counter which scatters the primary γ in the electron counter;

c) coincidences due to Compton effect in the surrounding materials.

Chance coincidences are made negligible by the resolving time of the coincidence circuit, by a convenient increase of the distance between the diffusing walls of the box and by the bias of discriminators D_2 and D_3.

These biases reduce the effects sub *b*) and *c*) to a minimum. The effect *c*) is also emphasized by the position of the counters with respect to the walls of the box. Therefore in this experiment the walls of the box, where Compton effect due to the γ primaries could happen, were set in a position such that they could see the counters within angles not compatible with the Compton angular distribution.

The residual background was measured by not anticoincided coincidences, coming from an Al target instead of a Pb target, of equal dimensions and thickness in g/cm^2. It follows that the contribution to the coincidences due to photoeffect in Al cannot be revealed, the fluorescent lines being of low energy ($\varepsilon = 0.1$ keV).

The coincidences interesting this experiment can be due to the following events:

a) electron ejected from the *K*-shell coupled to an X of *MK*, *LK*, *ML*, *LK*+*ML* transitions;

b) electron ejected from the *L*-shell coupled to an X of *ML* lines.

To determine the efficiency in detecting these events, the following factors must be considered:

1) the relative intensity of the photo-effect of the *K*- and *L*-shell; the ratio of the two intensities was taken to be 4.9 ([13]);

([13]) G. D. LATISHEV: *Rev. Mod. Phys.*, **19**, 132 (1947).

2) the relative intensity of the decay lines in the case of K-photoelectrons; the decay lines are given by 75% of $ML+LK$ transitions and by 25% of MK transitions ([14]);

3) the fluorescence yields, respectively 89% and 40% for K and L lines ([15]);

4) the solid angle covered by the X-ray counter assumed $\simeq 2\pi$;

5) the self absorption of the Pb target (averaging $\simeq 10$ mg/cm^2) for the various lines. This absorption is valued as follows: line $LK = 3.5\%$; line $MK = 0\%$; line $ML = 8\%$ ([16]);

6) the absorption in the material between the target and the NaI(Tl) counter ($\simeq 0.32$ g/cm^2 Al) valued as follows: line $LK = 10.5\%$; line $MK = 6\%$; line $ML = 88\%$ ([16]);

7) the detection efficiency for the three lines, valued 100% ([16]).

From these data, the available relative intensities of lines LK, MK, ML result to be 100:78:6.7.

Our evaluations allow to state also that 40% of the events corresponding to the lines ML are due to L photo-electrons and 60% to K photo-electrons. Therefore L photo-electrons would represent only $1 \div 2\%$ of the detected events.

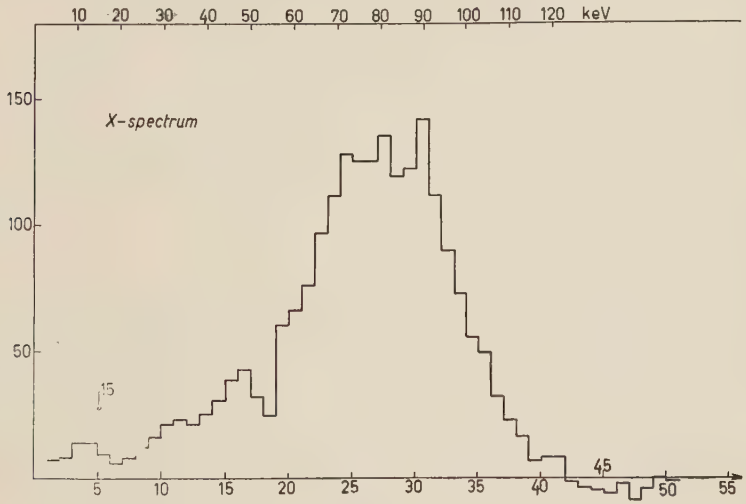

Fig. 3.

([14]) A. H. COMPTON and S. K. ALLISON: *X-Ray in Theory and Experiments* (New York, 1954).

([15]) E. H. S. BURHOP: *Auger effect* (Cambridge Monography on Physics, University Press, 1952), p. 45.

([16]) CLARK: *Applied X-Rays* (New York, 1955), p. 165.

As can be seen, the relative intensities of K and L events are valued on a series of data, which, for their own error, can bring in the final evaluations of the photo-electron intensity, errors of even an order of magnitude. The experimental arrangement in this case, can give an evaluation of the asymmetry ratio even for the L photo-electrons.

An experimental test of these last conclusions was carried out by detecting the differential spectrum of the fluorescence X (naturally corrected for background), shown in Fig. 3. The X of energy below ~ 20 keV, corresponding to the ML line represent only a few percent of the total intensity, as suggested by our estimates. An additional test was made by detecting the differential spectrum of the electrons associated to X-rays of energy below 20 KeV, shown in Fig. 4.

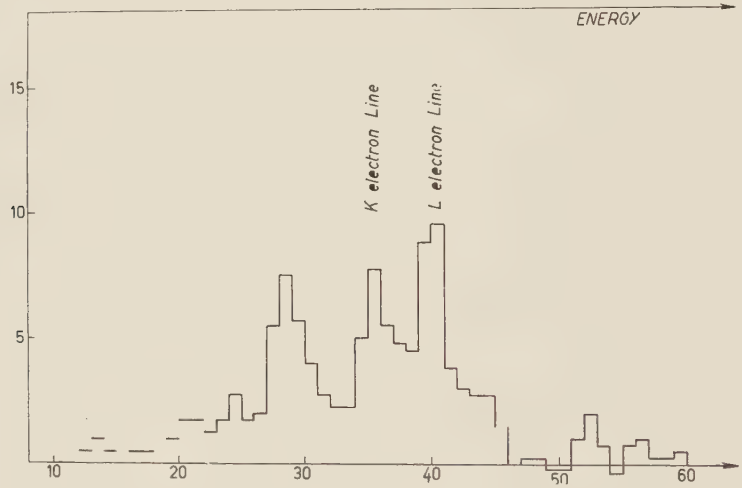

Fig. 4.

The test showed that both intensities, corresponding respectively to K- and L-photoelectrons, are comparable—although we are aware of the lack of precision due to the poor statistics—to the fluctuations in energy loss, to the backscattering (which produces a tail in the range of low energy) to the monocromaticity of the incident γ (which gives the measure a character of internal control).

We can therefore say that the asymmetry ratio measured by our experimental arrangement is actually due only to K-photoelectrons.

4. – The experiment.

To determine the wanted asymmetry ratio, the rate of not anticoincided coincidences (first with Pb target and after with Al target) was measured first

with the system target-counter-electrons in the scattering plane, then in the normal plane.

So the ratio was given by:

$$R = \frac{C_{\perp \mathrm{Pb}} - C_{\perp \mathrm{Al}}}{C_{\parallel \mathrm{Pb}} - C_{\parallel \mathrm{Al}}} = \frac{\Delta C}{\Delta C} \quad (1)$$

The same measurements were made for the scattering angles 65°, 47°, 35°, 20°. In each of these four series of measurements, the bias T_3 of the discriminator D_3 of the electron channel, was fixed so that the ratio r = true coincidences/background should not be too small (we took actually $r = 1 \div 2$).

The value of this bias can be chosen within a certain range of values, without determining any change on the value of R. In fact, the differential photoelectron spectra, respectively in the scattering plane and in that perpendicular to it, are proportional. We have:

$$R = \frac{\Delta C_{\perp}}{\Delta C_{\parallel}} = \frac{k \int_{T_3}^{E_M} f(E)\,\mathrm{d}E}{\int_{T_3}^{E_M} f(E)\,\mathrm{d}E}.$$

Therefore, whichever be the value of T_3, is $R = k$. The most essential point is that T_3 be strictly constant in all the four measurements $C_{\perp \mathrm{Pb}}$, $C_{\perp \mathrm{Al}}$, $C_{\parallel \mathrm{Pb}}$, $C_{\parallel \mathrm{Al}}$, of course remaining constant the total gain in the electron channel and the efficiency to the detection of the X, both depending on the total gain of the X and on the two biases T_1 and T_2.

It is well known that, when measurements with crystals and P.M. are taken for a long time, they are affected by fluctuations, whose principal causes are known.

For this reason we have looked for a criterion which allows us to compare, also in absolute value, measurements made at different times.

We observe that gain variations in every channel can be made to correspond to arranged variations in the bias put in the channels themselves. So we have purposely varied, within certain limits, compatible with the correspondent self variations of the experimental arrangement, each of the three biases T_1, T_2, T_3 in short times with respect to the average time of the self fluctuations. The result was that actually only bias T_3 of the electron channel showed a certain influence on the coincidence counting. Therefore, simultaneously to the coincidences, the single events of the electron channel were counted by the Scale *Sc*.

The correspondent frequencies were assumed as index of the gain of the experimental arrangement.

The frequency behaviour of the coincidences as a function of those corresponding to single events, was empirically found by varying T_3. In counting the singles, even for variations much wider than those obtained in the real

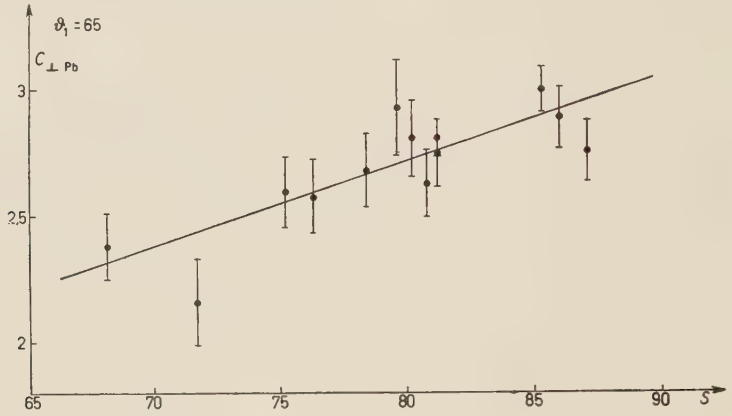

Fig. 5.

measurements, the bond between coincidences C and singles S appear in first approximation linear. Therefore each series of measurements was subdivided in groups (each corresponding to an interval of $3 \div 4$ hours) and in a diagram were reported the coincidences as a function of the singles; an example is given in Fig. 5. By a least squares method, we obtained straight lines characteristic of every series of measurements, namely a straight line for each of the $C_{\perp Pb}$, $C_{\perp Al}$, $C_{\parallel Pb}$, $C_{\parallel Al}$ cases. To find out the four values of C corresponding to the same T_3 (i.e. corresponding to the same gain of the experimental arrangement) it is necessary to determine the values of the respective singles. These values could not be the same, because the background depends on the position of the counter with respect to the γ beam.

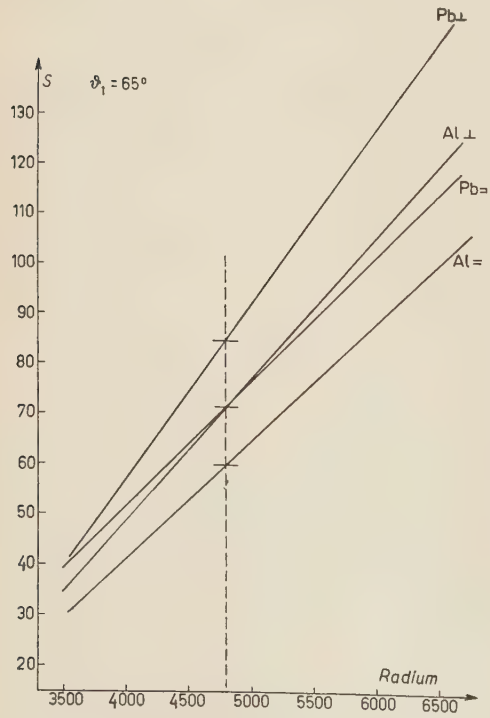

Fig. 6.

To determine these four values of the singles S, we determined the behaviour of the singles versus the frequency (artificially varying the gain

of the electron channel). This frequency was measured by the electron counter, when, taken away the primary γ beam, a radioactive source was placed in a fixed position with respect to the counter itself. Also this behaviour appeared linear, in first approximation. Naturally, to equal frequencies with radium had do correspond equal gains of the experimental arrangement. An example of such behaviours, obtained by the method of least squares, is shown in Fig. 6. The standard frequency was chosen so that the four obtained values of the singles were near to the four correspondent mean values found in the experiment.

In respect of the accuracy of this method, we note that, taking into account the errors on the normalization singles and the arbitrarity of the choice of the standard frequency of radium, the indetermination on the asymmetry ratio of the coincidence due to these causes appeared less than the statistical error.

5. – Theoretical previsions on the value of the measurement.

The differential cross-section of the photoelectric effect for polarized photons gives an angular distribution, which is referred to the direction of the electric field. As in the experiment we used Compton scattering to polarize the photons, and the beam obtained is not totally polarized, the experimental result will depend on the state of polarization of the beam. The probability that a photon, after being scattered to θ_1 angle, has a polarization vector making an angle φ with the normal to the scattering plane, is

$$\pi(\varphi, \theta_1) \propto \left(\frac{\alpha_0}{\alpha} + \frac{\alpha}{\alpha_0} - 2 + 2\cos^2\varphi + 2\sin^2\varphi\cos^2\theta_1\right).$$

The asymmetry ratio computed on the numbers of perpendicularly emitted photo-electrons produced by the beam on the target and of those emitted in a direction parallel to the scattering plane, can be deduced from

$$R = \frac{N_\perp}{N_\parallel} = \frac{\int_0^\pi \pi(\varphi, \theta_1) \left\{ \int_{\Omega_\perp} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\Phi, \theta_2)\, \mathrm{d}\Omega_\perp \right\} \mathrm{d}\varphi}{\int_0^\pi \pi(\varphi, \theta_1) \left\{ \int_{\Omega_\parallel} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\Phi, \theta_2)\, \mathrm{d}\Omega_\parallel \right\} \mathrm{d}\varphi},$$

where $\mathrm{d}\sigma/\mathrm{d}\Omega$ is the differential cross-section of the photoelectric effect.

The integrals were calculated assuming first Sauter's ([3]) cross-section in

the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\varphi, \theta_2)\,\mathrm{d}\Omega = \left(\frac{1}{\varepsilon}\right)\sqrt{1-\beta^2}\,\frac{\beta^2 \sin^2\theta_2}{(\frac{1}{2}-\beta\cos\theta_2)^4}\cdot$$
$$\cdot\left[\frac{\varepsilon^2}{4}(1-\cos\theta_2)+\left\{\sqrt{1-\beta^2}-\frac{\varepsilon}{2}(1-\beta\cos\theta_2)\right\}\cos^2\varphi\right]\mathrm{d}\Omega\,,$$

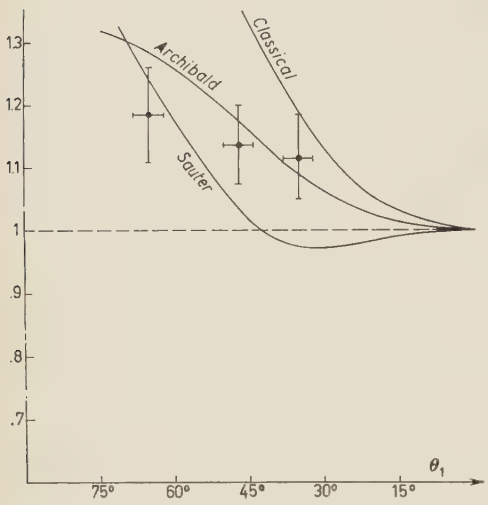

Fig. 7.

and another time assuming Archibald's cross-section. The results are reported in Fig. 7 together with the experimental points. The same figure shows the asymmetry ratio computed by using the classical cross-section.

6. – Experimental results.

We have determined the asymmetry ratio for the four scattering angles, in three various series of measurements, following the criterion of section four. Table I shows the weighted mean of the three values obtained for each scattering angle.

TABLE I.

θ_1	65°	47°	35°	20°
R	0.958 ± 0.061	0.921 ± 0.053	0.903 ± 0.055	0.800 ± 0.100

As it can be seen, all values are below 1. This is due to systematic effect of geometrical asymmetry, due to the collimation system. To have a qualitative indication of such an asymmetry, the photon beam was photographed and by a densimeter were drawn the curves of equal darkness. The result was that the beam had rather well defined edges and the centre of the target did not coincide with the beam axis. This fact does not influence the behaviour of the experimental points.

For the absolute value corresponding to symmetry ($R = 1$), it is to be noted that $R_{20°}$ is, in every theory, within 1 ÷ 3 %, without asymmetry, because for that scattering angle the photons are practically unpolarized. Therefore the true value $R = 1$ should correspond, within few percents, to $R_{20°}$.

Another test of the geometrical asymmetry was made by measuring in the same way, the asymmetry ratio and substituting the scatterer (of course without ^{60}Co source) with a source of 10 m.C. of ^{137}Cs emitting 660 keV photons.

These photons being unpolarized, the asymmetry ratio should be 1, if there were no causes of geometrical asymmetry. Three measurements were taken in different conditions. The weighted mean of three measurements gave the following value $R_{Cs} = 0.819 \pm 0.082$, in agreement with $R_{20°}$. The weighted mean of R_{Cs} and $R_{20°}$ was taken as geometrical asymmetry ratio, and resulted to be $R_g = 0.810 \pm 0.063$.

One reason of deviation in the behaviour could be the fact that the internal walls of the collimator may produce a second scattering. The intensities of these scattered photons in comparison with that due to the real scatterer, was estimated to be of the order of 10%, a value practically independent from the angle of the first scattering. However, the second scattering, because of small values of the probable scattering angles does not affect appreciably neither the polarization nor the energy of the photons coming from the first scattering. Consequently it does not affect the behaviour of the experimental points. Also those primary photons, which, crossing through the scatterer, impinge on an inner wall of the collimation system and are scattered towards the target, have no influence, because their intensity, in the worst conditions, is $1 \div 2\%$ of the direct beam.

Our experimental results can have been influenced by those primary photons which, on the scatterer, are coherently scattered (Rayleigh effect) and produce photoelectric effect on the target. This effect might be felt at large scattering angles; however, its cross-section is, at least, two orders of magnitude below the Compton cross-section.

Therefore, assuming that the state of polarization of elastic scattered photons is the same of those scattered inelastically, we can conclude that their effect on the measured asymmetry ratio is absolutely negligible.

7. – Conclusions.

Our results are not in agreement with McMaster's and Hereford's [12] results. Our seem to agree better with Archibald's [4] theoretic prevision, whereas Hereford's seem to correspond with Sauter's calculations. However, the reasons of these discrepancies are not easy to be explained. Anyhow, considering the experimental arrangement used in their experiments by M.M.H., it seems not possible to neglect an influence of the elastic scattering (particularly on the target); such an effect on the asymmetry ratio works in the opposite way to the photoelectric effect (like the anelastic scattering).

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The authors wish to thank Prof. W. J. Archibald for his private communications, Prof. Puppi for many helpful discussions and Dr. A. Minguzzi for his interest in this experiment.

RIASSUNTO

È stata osservata la distribuzione angolare dei foto-elettroni in Pb da fotoni polarizzati per effetto Compton, usando a questo scopo una sorgente di ^{60}Co collimata. La selezione degli eventi fotoelettrici è stata misurata rivelando i fotoelettroni in coincidenza con i corrispondenti raggi X di fluorescenza. La misura è stata eseguita per determinare l'andamento del rapporto di asimmetria in funzione dell'energia: tale rapporto è stato definito dal numero dei fotoelettroni emessi perpendicolarmente al piano di scattering Compton diviso dal numero di fotoelettroni emessi nel piano di scattering I risu³tati ottenuti non sono in accordo con la teoria di Sauter ma sembrano piuttosto vicini al calcolo di Archibald.

IL NUOVO CIMENTO Vol. VI, N. 1 1° Luglio 1957

On the Classical Fluctuation of a Beam of Light.

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Central Research Institute of Physics, Department of Cosmic Rays - Budapest

(ricevuto il 7 Aprile 1957)

Summary. — The fluctuation of intensity of a light beam emitted by a macroscopically constant source is evaluated. The generating function of the distribution is determined exactly; it is found that the moments of the distribution are in good approximation those of a Gaussian distribution. Recent experiments on photon coincidences are discussed.

1. – In a number of papers E. WOLF [1] investigates the fluctuation in an optical image caused by the interference of components of different frequency and phase, PURCELL [2] has dealt with another aspect of the fluctuation problem of light. It seems to us essential to carry out such investigations using a physical model of the source of light. We shall treat the problem in a purely classical manner, but it is to be expected that the results obtained follow also from a quantum theoretical treatment.

We thus investigate the fluctuations of light in a point of observation O; the light is supposed to be emitted by a small source S at a large distance R from O. We suppose schematically that the source contains a large number of emitting atoms. These atoms are supposed to become excited at random times and to emit a damped oscillation each after the excitation has taken place. We shall consider the accumulative effect of the trains of damped wave bands reaching the point of observation and shall work out the distribution function of the intensity in O. The fluctuations arise from (1) the random times

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[1] E. WOLF: *Nuovo Cimento*, **12**, 884 (1954); *Proc. Roy. Soc.*, A **225**, 96 (1954); **230**, 246 (1955); *Phil. Mag.* (in the press).

[2] E. M. PURCELL: *Nature*, **178**, 1449 (1956).

1971

of arrivals of the wave fronts, (2) the random phases of these fronts, (3) the random directions of polarizations. We give the detailed calculation in the following sections.

2. – A light source, although macroscopically of constant intensity, is expected to show fluctuations due to the fact that the emission is composed of the emissions of individual atoms.

From the purely qualitative point of view we may remark the following: Suppose a source to contain N atoms which emit light at random phases. In a distant point O the *expected* value of the amplitude of the oscillation is zero, since the phases of the oscillations are distributed at random and the various light impulses extinguish each other by interference. To see this in more detail, let us select at random a phase, e.g. $\varphi = 0$; denote the oscillations with phases $-\pi/2 < \varphi < +\pi/2$ as oscillations *in phase* with the originally selected one, and let us denote the other oscillations as *out of phase* with the original one. The expected number of oscillations *in phase* is $N_+ \approx N/2$, and the expected number of those *out of phase* is $N_- \approx N/2$, thus the oscillations *in phase* and *out of phase* are about equal in number, and they largely compensate each other. However, according to the laws of random fluctuation the number of oscillations in the two groups will differ by about $\sqrt{N}$, thus it is very likely that in a given case N_+ and N_- differ so that

$$|N_+ - N_-| \sim \sqrt{N} .$$

Thus owing to statistical fluctuation, there is to be expected a number of about $\sqrt{N}$ of uncompensated oscillations. As the light intensity is proportional to the square of the total amplitude, we expect a remaining intensity proportional to

$$(\sqrt{N})^2 = N .$$

Thus the uncompensated intensity is roughly proportional to the number N of signals arriving; the latter result is caused by the random fluctuation in the distribution of phases.

We note, however, that the number of phases not compensated in a given case is only roughly equal $\sqrt{N}$, comparatively large fluctuations around this value are to be expected. Thus the intensity must be expected to show large fluctuations around its average value, which itself is proportional to N (*).

(*) This consideration was put forward by LORD RAYLEIGH: *Theory of Sound* (1894), see e.g. (New York, 1945), p. 35.

1972

3. – We treat presently the fluctuation described above qualitatively in a more quantitative manner.

Suppose the light source to contain atoms which emit light through damped oscillations of frequency ω and with a damping constant γ. Thus the electric vector of the light in the point of observation, caused by a single emission process at a time t is given by

$$\boldsymbol{E}(t) = \boldsymbol{E}_0 \exp[-\gamma(t+t')] \cos(\omega t + \varphi), \qquad t + t' > 0, \tag{1}$$

$$= 0, \qquad t + t' < 0,$$

where t' is the instant in which the atom starts emitting. We are not so much interested in the instantaneous value of the electric vector as in the amplitude of the oscillation; the square of the amplitude can be obtained if we add to the square of the instantaneous value of the field strength the square of the time derivative with a suitable factor. We suppose the following orders of magnitude:

$$\gamma \sim 10^8 \text{ s}^{-1}, \qquad \omega \sim 10^{14} \text{ s}^{-1},$$

thus γ is always small in comparison with ω; furthermore we can put in good approximation for the amplitude of the oscillation

$$\boldsymbol{E}^2(t) + \frac{1}{\omega^2}\dot{\boldsymbol{E}}^2(t) = \boldsymbol{E}_0^2 \exp[-2\gamma(t+t')].$$

We suppose that the direction of propagation of the light is parallel to the x-axis, thus we may put

$$E_{0x} = 0, \qquad E_{0y} = E_0 \cos\varepsilon, \qquad E_{0z} = E_0 \sin\varepsilon, \qquad 0 \leqslant \varepsilon < \pi,$$

where

$$E_0 = |E_0|,$$

and ε is the angle of polarization. The electric vector at the time $t = 0$ in the point of observation can be built as the superposition of a number of vectors of the form (1). We may thus put for the values of the components of the electric vector at the time $t = 0$

$$\begin{cases} E_y = E_0 \sum_k \exp[-\gamma t_k] \cos\varphi_k \cos\varepsilon_k, \\ E_z = E_0 \sum_k \exp[-\gamma t_k] \cos\varphi_k \sin\varepsilon_k, \end{cases} \tag{2a}$$

further we put

$$(2b)\qquad \begin{cases} F_y = E_0 \sum\limits_k \exp\left[-\gamma t_k\right] \sin \varphi_k \cos \varepsilon_k \,, \\ F_z = E_0 \sum\limits_k \exp\left[-\gamma t_k\right] \sin \varphi_k \sin \varepsilon_k \,, \end{cases}$$

where $-t_k$ are the times at which the fronts of emissions from the various atoms arrive, φ_k their respective phases and ε_k ($k = 1, 2, \ldots$) their angles of polarization. The total intensity at any instant of time is given by

$$(3)\qquad J^2 = E_y^2 + E_z^2 + F_y^2 + F_z^2 \,.$$

As all three quantities $t_k \geqslant 0$, φ_k and ε_k are subject to random fluctuations, the field components (2) can be regarded as stochastic variables. We shall derive below the simultaneous distribution

$$P(E_y, E_z, F_y, F_z) \,,$$

giving the probability that at a given moment the four components take certain given values; with help of the above distribution the fluctuation properties of the beam can be determined.

The distance between the source of light and the point of observation may be denoted R, thus the time of travel of an emitted front is

$$T = R/c \,.$$

The fronts arriving at times $-t_1$, $-t_2$, ... have started at times $-T-t_1$, $-T-t_2$, ...

Suppose that the probability of an emission to start in a time interval dt to be $A\,dt$ where A is a constant, characteristic for the intensity of the source. Each emission which takes place at a time

$$t < -T$$

affects the electric field in the point of observation. An emission which took place at the time $t = -t' - T$ with a phase $\varphi' - \omega T$ and an angle of polarization ε' makes in the point of observation the following contributions to the field strengths:

$$(4)\qquad \begin{cases} x_1 = E_0 \exp\left[-\gamma t'\right] \cos \varphi' \cos \varepsilon' \quad \text{to} \quad E_x \,, \\ x_2 = E_0 \exp\left[-\gamma t'\right] \cos \varphi' \sin \varepsilon' \quad \text{to} \quad E_y \,, \\ x_3 = E_0 \exp\left[-\gamma t'\right] \sin \varphi' \cos \varepsilon' \quad \text{to} \quad F_y \,, \\ x_4 = E_0 \exp\left[-\gamma t'\right] \sin \varphi' \sin \varepsilon' \quad \text{to} \quad F_z \,. \end{cases}$$

1974

We introduce a three-dimensional space with the co-ordinates t', φ', ε' with the following restrictions

$$t' > 0\,, \qquad 0 \leqslant \varphi' < 2\pi\,, \qquad 0 \leqslant \varepsilon' < \pi\,.$$

We can subdivide this space into elements $\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'$, each emission can be taken to come out of one of these elements. Most of the elements do not contain the beginning of an emission, the probability that an emission should after all begin in a given element is

$$\frac{A}{2\pi^2}\,\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'\,.$$

If an element contains an emission, then this emission contributes to the field strength the amounts E_x, E_y, F_x, F_y given in (4). Denoting these contributions e_y, e_z, f_y, f_z, we can put for the probability that out of the element $\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'$ the field strength in O is contributed to by components e_y, e_z, f_y, f_z,

$$(5) \qquad P_{t'\varphi'\varepsilon'}(e_y, e_z, f_y, f_z) = \left(1 - \frac{A}{2\pi^2}\,\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'\right)\delta(e_y)\,\delta(e_z)\,\delta(f_y)\,\delta(f_z) +$$

$$+ \frac{A}{2\pi^2}\,\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'\,\delta(x_1 - e_y)\,\delta(x_2 - e_z)\,\delta(x_3 - f_y)\,\delta(x_4 - f_z)\,.$$

The first term on the right hand side of (5) expresses that there is a probability $1 - (A/2\pi^2)\,\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'$ to have no emission in the element $\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'$; if there is no emission, we are sure that

$$e_y = e_z = f_y = f_z = 0\,.$$

The second term on the other hand takes care of the case, where there is an emission in the element $\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'$. If there is such emission then its contribution to the field strength in the point O amounts to

$$e_y = x_1\,, \qquad e_x = x_2\,, \qquad f_y = x_3\,, \qquad f_z = x_4\,.$$

So as to get the distribution (3) of the field strength, we have to determine the probability that the sums of all contributions e_y arising from different elements give together E_y, the sum of all contributions e_z give exactly E_z, etc. Thus the distribution $P(E_y, E_z, F_y, F_z)$ is obtained as the *folding* of all distributions $P_{t'\varphi'\varepsilon'}(e_y, e_z, f_y, f_z)$, each of which arises from the contribution out of a possible element $\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'$.

So as to carry out the folding process, we build the generating function.

We have by definition of the generating function

$$(6)\qquad G(P_{t'\varphi'\varepsilon'};\,\lambda_1,\,\lambda_2,\,\lambda_3,\,\lambda_4) =$$

$$= \int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\exp\left[\lambda_1 e_y + \lambda_2 e_z + \lambda_3 f_y + \lambda_4 f_z\right] P_{t'\varphi'\varepsilon'}(e_y, e_z, f_y, f_z)\,\mathrm{d}e_y\,\mathrm{d}e_z\,\mathrm{d}f_y\,\mathrm{d}f_z\,.$$

Introducing (5) into (6) we find

$$(7)\qquad G(P_{t'\varphi'\varepsilon'};\,\lambda_1,\,\lambda_2,\,\lambda_3,\,\lambda_4) =$$

$$= 1 + \frac{A}{2\pi^2}\,\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'\left(\exp\left[\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4\right] - 1\right),$$

where the values for the x_i, $i = 1, 2, 3, 4$ have to be inserted from (4). The generating function of the distribution (5) is obtained as the product of the generating functions (7) taken for all elements $\mathrm{d}t'\,\mathrm{d}\varphi'\,\mathrm{d}\varepsilon'$. So as to evaluate this product, it is convenient to introduce logarithmic generating functions, one finds thus with help of (4)

$$(8)\qquad \ln G(P, \lambda_1, \lambda_2, \lambda_3, \lambda) = \sum \ln G(P_{t'\varphi'\varepsilon'};\,\lambda_1,\,\lambda_2,\,\lambda_3,\,\lambda) =$$

$$= \frac{A}{2\pi^2}\int_0^{\infty}\int_0^{2\pi}\int_0^{\pi}\left(\exp\left[E_0 e^{-\gamma t'}(\lambda_1 \cos\varphi' \cos\varepsilon' + \lambda_2 \cos\varphi' \sin\varepsilon' + \right.\right.$$

$$\left.\left. + \lambda_3 \sin\varphi' \cos\varepsilon' + \lambda_4 \sin\varphi' \sin\varepsilon')\right] - 1\right)\mathrm{d}\varepsilon'\,\mathrm{d}\varphi'\,\mathrm{d}t'\,.$$

4. – With help of the generating function given in (8) we can determine the various moments of the distribution of the field strength. One finds differentiating (8) into one of the λ's

$$(9)\qquad \left(\frac{\partial \ln G}{\partial \lambda_i}\right)_{(0)} = 0\,, \qquad i = 1, 2, 3, 4,$$

where we have written for short G instead of $G(P;\,\lambda_1,\,\lambda_2,\,\lambda_3,\,\lambda_4)$. The lower index (0) signifies that after differentiation the λ's have to be put equal zero. From (9) it follows that the expected values of all four field components are equal zero, i.e.

$$\langle E_y\rangle = \langle E_z\rangle = \langle F_y\rangle = \langle F_z\rangle = 0\,.$$

Differentiating twice into λ_i we have further

$$(10)\qquad \left(\frac{\partial^2 \ln G}{\partial \lambda_i^2}\right)_{(0)} = \frac{A E_0^2}{8\gamma}\,, \qquad i = 1, 2, 3, 4,$$

thus the expectation value of the total intensity J is

$$\langle J \rangle = \langle E_y^2 \rangle + \langle E_z^2 \rangle + \langle F_y^2 \rangle + \langle F_z^2 \rangle = \frac{A E_0^2}{2\gamma} . \tag{11}$$

$E_0^2/2\gamma = i_0$ is a quantity proportional to the energy of one light pulse emitted, and we may write

$$\langle J \rangle = A i_0 .$$

Differentiating (8) into λ_i and λ_k, $i \neq k$, we find

$$\left(\frac{\partial^2 \ln G}{\partial \lambda_i \partial \lambda_k} \right)_0 = 0 , \qquad \text{for} \qquad i \neq k ,$$

thus the correlations between the field components are all equal zero, we have exactly

$$\langle E_y E_z \rangle = \langle E_y E_z \rangle = \ldots = 0 .$$

Determining various other moments with the help of (8), we find that each moment can be expressed as a polynomial containing powers of A/γ.

E.g. the mean square fluctuation of the square of the field strength is

$$\langle (\delta E_y^2)^2 \rangle = \langle E_y^4 \rangle - \langle E_y^2 \rangle^2 ,$$

and

$$\langle (\delta E_y^2)^2 \rangle = \left(\frac{\partial^4 \ln G}{\partial \lambda_1^4} \right)_{(0)} + 2 \left(\frac{\partial^2 \ln G}{\partial \lambda_1^2} \right)^2 .$$

Computing the differentiations, we find

$$\left(\frac{\partial^4 \ln G}{\partial \lambda_1^4} \right)_0 = \frac{A E_0^4}{2\pi^2} \int_0^\infty \exp\left[-4\gamma t' \right] \mathrm{d}t' \int_0^{2\pi} \cos^2 \varphi' \, \mathrm{d}\varphi' \int_0^\pi \cos^4 \varepsilon' \, \mathrm{d}\varepsilon' = \frac{9 A E_0^4}{256 \gamma} .$$

Thus with help of (10)

$$\langle (\delta E_y^2)^2 \rangle = E_0^4 \left(\frac{1}{32} \frac{A^2}{\gamma^2} + \frac{9}{256} \frac{A}{\gamma} \right) .$$

For all practical purposes it can be assumed that

$$A/\gamma \gg 1 ,$$

and it is sufficient to determine the term of the highest power of A/γ in each case. Using this approximation, the moments obtained are those of a Gaussian; we find that the distribution

$$P_1(E_y, E_z, F_y, F_z) = \left(\frac{2}{\pi J_0}\right)^2 \exp\left[-\frac{E_y^2 + E_z^2 + F_y^2 + F_z^2}{J_0/2}\right], \tag{12}$$

is a good approximation of the exact distribution $P(E_y, E_z, F_y, F_z)$ introduced in Sect. 3. Using the definition (3) of the total intensity J we can derive the distribution function of the total intensity. In the approximation (12) we get in the usual way

$$P_1(J) = \frac{4J}{J_0^2} \exp\left[-2J/J_0\right]. \tag{13}$$

The mean square fluctuation as obtained from (13) is found to be

$$\langle J^2\rangle - \langle J\rangle^2 = \tfrac{1}{2}J_0^2,$$

thus

$$\bar{a}^2 = \frac{\langle J^2\rangle - \langle J\rangle^2}{\langle J\rangle^2} = \frac{1}{2} \tag{14}$$

is the relative mean square fluctuation of the beam. Carrying out the calculation with the exact distribution, corresponding to the generating function given in (8), we obtain instead of (14)

$$\bar{a}^2 = \frac{1}{2} + \frac{1}{2}\frac{\gamma}{A}.$$

If we pass the light through a polarizing filter before measuring the intensity, then two of the four components can be put equal zero. Thus the intensity of polarized light is given by

$$J^{(p)} = E_y^2 + F_y^2.$$

The expected value of $J^{(p)}$ is obtained from (8) as

$$J_0^{(p)} = \langle J^{(p)}\rangle = \langle E_y^2\rangle + \langle F_y^2\rangle = \tfrac{1}{2}J_0.$$

The distribution of $J^{(p)}$ using the approximation (12) is thus

$$P_1(J^{(p)}) = \frac{1}{J_0^{(p)}} \exp\left[-J/J_0^{(p)}\right],$$

1978

and therefore

$$\langle J^{(p)2}\rangle - \langle J^{(p)}\rangle^2 = J_0^{(p)2},$$

or

$$\bar{a}^2 = \frac{\langle J^{(p)2}\rangle - \langle J^{(p)}\rangle^2}{\langle J^{(p)}\rangle^2} = 1. \tag{15}$$

The exact expression for the fluctuation of polarized light is found from (8) to be

$$\bar{a}^2 = 1 + \frac{1}{2}\,\frac{\gamma}{A}.$$

The circumstance that polarized light shows greater fluctuations than not polarized light can be understood easily. The correlation coefficient between the components polarized into the y resp. z direction is given by

$$\Gamma = \frac{\langle (E_y^2 + F_y^2)(E_z^2 + F_z^2)\rangle - \langle E_y^2 + F_y^2\rangle\langle E_z^2 + F_z^2\rangle}{(\langle (E_y^2 + F_y^2)^2\rangle\langle (E_z^2 + F_z^2)^2\rangle)^{\frac{1}{2}}}.$$

Evaluating the various terms in the above expression with the help of the generating function given in (8) we find

$$\Gamma = \frac{\gamma}{4A} + \text{terms in } \left(\frac{\gamma}{A}\right)^2.$$

We see therefore that the correlation between the two differently polarized components is only of the order of γ/A and thus the components can be considered to be practically independent. The superposition of the practically independent fluctuations of the two components reduces thus the relative mean square fluctuation of the superposed intensity by about a factor 2.

5. – The values of the intensity J taken at two different times, e.g. at $t = 0$ and $t = \tau$ are correlated to some extent. So as to investigate this correlation, we consider the values of the field strength

$$E_y^{(0)},\quad E_z^{(0)},\quad F_y^{(0)},\quad F_z^{(0)} \qquad \text{and} \qquad E_y^{(\tau)},\quad E_z^{(\tau)},\quad F_y^{(\tau)},\quad F_z^{(\tau)},$$

at the times $t = 0$ and τ. If no new front were to arrive in the interval $0 \to \tau$, then the values of the field strength at the time $t = 0$ could be exactly determined from the values taken up at $t = 0$; in general a number of fronts do arrive in the interval $0 \to \tau$, thus the values of the field components at $t = \tau$

1970

can be split into two parts, e.g.

$$E_y^{(\tau)} = E_{y_1}^{(\tau)} + E_{y_2}^{(\tau)} ,$$

where $E_{y_1}^{(\tau)}$ gives the contribution of the wave fronts which have arrived before the time $t = 0$, while $E_{y_2}^{(\tau)}$ represents the contribution of the fronts which have arrived later. The two contributions are statistically independent since the very acts of emissions which have given rise to the wave fronts are independent of each other and independent of any emission processes which might have taken place before. The intensity at the times 0 and τ are thus given by

$$J(0) = E_y^{(0)2} + E_z^{(0)2} + F_y^{(0)2} + F_z^{(0)2} ,$$

and

$$J(\tau) = J^{(1)}(\tau) + 2J^{(1,2)}(\tau) + J^{(2)}(\tau) , \tag{16}$$

where

$$J^{(1)}(\tau) = E_{y_1}^{(\tau)2} + \ldots ; \qquad J^{(2)}(\tau) = E_{y_2}^{(\tau)2} + \ldots , \tag{17}$$

and

$$J^{(1,2)}(\tau) = E_{y_1}^{(\tau)} E_{y_2}^{(\tau)} + \ldots . \tag{18}$$

The expectation values of the three components of $J(\tau)$ can be worked out easily. Firstly

$$\langle J^{(1,2)}(\tau) \rangle = 0 ,$$

ince $E_{y_1}^{(\tau)}$ and $E_{y_2}^{(\tau)}$, etc. are independent. Secondly, from (2) and (3) it is seen that

$$J^{(1)}(\tau) = \exp[-2\gamma\tau] J(0) , \tag{19}$$

therefore

$$\langle J^{(1)}(\tau) \rangle = \exp[-2\gamma\tau] \langle J(0) \rangle = J_0 \exp[-2\gamma\tau] .$$

Finally, as the expectation value of the intensity is constant in time, we have

$$\langle J_2(\tau) \rangle = J_0(1 - \exp[-2\gamma\tau]) .$$

We work out the expectation value of the product of the intensities taken at the time $t = 0$ and τ. The expectation value of the product of two intensities existing at the instants $t = 0$ and τ can be obtained if the expressions

are resolved into the dependent and independent components. We find from (16)

$$\langle J(0)J(\tau)\rangle = \langle J(0)J^{(1)}(\tau)\rangle + 2\langle J(0)J^{(1,2)}(\tau)\rangle + \langle J(0)J^{(2)}(\tau)\rangle \,. \tag{20}$$

The first term of the right hand side can be evaluated with help of (19); we have

$$\langle J(0)J^{(1)}(\tau)\rangle = \exp[-2\gamma\tau]\langle (J(0))^2\rangle = \exp[-2\gamma\tau]J_0^2 + \langle (\delta J(0))^2\rangle \,,$$

the middle term on the right hand side of (20) vanishes, as can be seen easily if we write down with help of (18) the explicite expression. For the third term we find

$$\langle J(0)J^{(2)}(\tau)\rangle = \langle J(0)\rangle\langle J^{(2)}(\tau)\rangle = J_0^2(1 - \exp[-2\gamma\tau]) \,.$$

Together we find thus

$$\langle J(0)J(\tau)\rangle = J_0^2 + \exp[-2\gamma\tau]\langle (\delta J(0))^2\rangle \,. \tag{21}$$

For the interpretation of experiments it is necessary to evaluate certain time integrals of the intensity. We give a few relevant expressions.

Denote

$$\bar{J}(T) = \frac{1}{T}\int_0^T J(t)\,\mathrm{d}t \,, \tag{22}$$

we have

$$\langle \bar{J}(T)\rangle = \frac{1}{T}\int_0^T \langle J(t)\rangle\,\mathrm{d}t = J_0 \,. \tag{23}$$

Further

$$\langle (\bar{J}(T))^2\rangle = \frac{2}{T^2}\left\langle \int_0^T \mathrm{d}t \int_0^t J(t)J(t')\,\mathrm{d}t\,\mathrm{d}t' \right\rangle \,. \tag{24}$$

For $t \geqslant t'$ we have

$$\langle J(t)J(t')\rangle = J_0^2 + \exp[-2\gamma(t-t')]\langle (\delta J)^2\rangle \,.$$

Thus interchanging the integration signs and the expectation sign, we have instead of (24) when we subtract J_0^2 from both sides

$$\langle (\delta\bar{J}(T))^2\rangle = \frac{2\gamma T - 1 + \exp[-2\gamma T]}{2\gamma^2 T^2}\langle (\delta J^2)\rangle \,. \tag{25}$$

Thus, taking the interesting limiting cases

$$\frac{\langle(\delta\bar{J}(T))^2\rangle}{\langle(\delta J)^2\rangle} \approx \begin{cases} (1-\frac{2}{3}\gamma T) & \text{if} \quad \gamma T \ll 1\,, \\ 1/(\gamma T) & \text{if} \quad \gamma T \gg 1\,. \end{cases}$$

In a similar way we can show that

$$\left\langle \frac{1}{2T}\int_{-T}^{T} J(0)J(t)\,\mathrm{d}t \right\rangle = J_0^2 + \frac{1-\exp[-2\gamma T]}{\gamma T}\langle(\delta J)^2\rangle\,. \tag{26}$$

6·1. – We discuss the relevant experiments shortly. Twiss and Brown [3] have amplified the photocurrents obtained from two photocells illuminated by coherent beams of light; the amplifier had a band width of $3 \div 27$ MHz. According to the theory the amplitude of the fluctuations decreases with decreasing frequency, therefore the fluctuations with 3 MHz can be neglected in first approximation when compared to those with 27 MHz and therefore the signals obtained from each of the two amplifiers can be roughly taken to be equal to

$$\bar{J}(T) = \int_0^T J(t)\,\mathrm{d}t\,, \qquad T \sim 1/27\cdot 10^6 \sim 3\cdot 10^{-8}\ \mathrm{s}\,.$$

The signals from the amplifiers are multiplied and then averaged. Thus the recorded quantity can be taken to be roughly equal to $\langle(\delta\bar{J}(T))^2\rangle$ given by (25). If the photocells are illuminated by two different parts of the optical image of the light source, then the arrangement should register

$$\delta\bar{J}(T)\,\delta\bar{J}'(T)\rangle = 0\,,$$

where $\bar{J}(T)$ and $\bar{J}'(T)$ are the independent intensities in two points of the image. The effect observed by Brown and Twiss may be the classical fluctuation, although this cannot be checked quantitatively from the data given by the authors.

6·2. – Coincidence experiments. Ádám, Jánossy and Varga [4], and Brannen and Ferguson [5] have carried out experiments investigating whether or

[3] R. Hanbury Brown and R. Q. Twiss: *Nature*, **177**, 27 (1956).

[4] A. Ádám, L. Jánossy and P. Varga: *Acta Phys. Hung.*, **4**, 301 (1955); *Ann. d. Phys.*, **16**, 408 (1955).

[5] E. Brannen and H. I. S. Ferguson: *Nature*, **177**, 481 (1956).

1982

not there exist coincidences between the photons in coherent beams of light. The purpose of these experiments was not the investigation of the classical fluctuation, nevertheless this fluctuation might conceivably have affected the experimental results; we give an analysis of the experiments presently.

If we separate a beam of light into two coherent components and register them with two electron multipliers, then there should be no correlations between the times of arrivals of photons on the two cathodes provided the intensity of the beam is constant. However, as we have seen, the intensity of a beam, even if macroscopically of constant intensity, shows microscopic fluctuations and these are bound to give rise to a certain amount of correlation.

Suppose the probability of a photon to arrive in a time interval t, $t + \mathrm{d}t$ to be $J(t)\,\mathrm{d}t$; if a photon arrives in the interval $t - \tau$ to $t + \tau$ on the second cathode, then these two photons are recorded as a coincidence when using an arrangement with resolving time τ. The probability of such a coincidence to happen is thus

$$J(t)\,\mathrm{d}t \int_{-\tau}^{+\tau} J(t + t')\,\mathrm{d}t' .$$

The rate of twofold coincidences to be expected is thus

$$C_2 = \frac{1}{T}\int_0^T J(t)\,\mathrm{d}t \int_{-\tau}^{+\tau} J(t + t')\,\mathrm{d}t' .$$

The expected value of the rate of coincidences can thus be evaluated with help of (26), we find

$$C_2 = c_2\left(1 + \frac{1 - \exp\left[-4\gamma\tau\right]}{2\gamma\tau}\,\frac{\langle(\delta J)^2\rangle}{J_0^2}\right),$$

where

$$c_2 = 2\tau J_0^2$$

is the number of casual coincidences to be expected in the absence of fluctuations. c_2 is also the rate of coincidences to be expected between the photons of two incoherent beams.

In the experiments quoted above no excess of coincidences was found when comparing the rates of coincidences obtained between coherent beams, and those obtained between incoherent beams. The reasons for the absence of extra coincidences in case of coherent beams may be the following.

1983

In the case of Ádám, Jánossy and Varga it was $\tau \sim 2 \cdot 10^{-6}$, thus

$$\gamma\tau \sim 0.01 .$$

The effect, if it existed at all, would have been inside the margin of experimental error. In the experiments of Brannen and Ferguson the effect might have been observed under favourable conditions; presumably, however, the optical arrangement in these experiments was chosen such that the fluctuations of the various parts of the image of the light source on the cathode were not in phase with each other and thus the independently fluctuating parts reduced the effect largely. We hope to carry out more experiments to clear these points.

RIASSUNTO (*)

Si valuta la fluttuazione di intensità di un raggio di luce emesso da una sorgente macroscopicamente costante. Si determina esattamente la funzione generatrice della distribuzione: si trova che i momenti della distribuzione corrispondono con buona approssimazione a quelli di una distribuzione gaussiana. Si discutono esperimenti recenti sulle coincidenze fotoniche.

(*) *Traduzione a cura della Redazione.*

Thin Down and Breakup of a Large Z Cosmic Ray Primary.

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Summary. — A high Z primary cosmic ray particle is observed in G-5 emulsion exposed in Texas at an altitude of 104 000 ft. Thin down measurements indicate a value of Z between 24 and 40. The thin down is accompanied by a partial breakup which permits an interesting determination of the charge on the primary, and suggests that the charge on the primary is between 35 and 40.

1. – Introduction.

In the course of an investigation on the large Z spectrum of cosmic ray primaries, the interesting thin down and breakup shown in Fig. 1 was observed. The photo is a mosaic of the event, while the superposed drawing is a schematic diagram of the event. The event was found in a stack of 600 μm Ilford G-5 pellicles exposed to cosmic rays at an altitude of 104 000 ft, at San Angelo, Texas, 41° N latitude. All the tracks stopped in the emulsion. Employing δ-ray count and multiple-scattering measurement, using the constant sagitta method (1), the charge and mass of the five fragments were determined. The results are tabulated in Table I.

(*) Work supported in part by the National Science Foundation.

(1) R. G. Glasser: *Phys. Rev.*, **98**, 174 (1955).

TABLE I. – *Measurements on the fragment particles.*

Particle	θ	Mass	Charge	Range (μ)	Energy (MeV/nucleons)	Momentum $\left(\frac{\text{MeV}}{c}/\text{nucleons}\right)$	$P\cos\theta$ $\left(\frac{\text{MeV}}{c}/\text{nucleons}\right)$	Identity
1	85°	$1.7^{+3.5}_{-1.0}$	1	240± 7	6,2±0.1	107.6±3	9.4±0.3	p, d, t
2	14°	4.6±0.5	2	2570± 75	24.2±0.5	214 ±2	207.6±2	α
3	−25°	1	1	5400±108	37.0±0.8	266 ±3	241 ±3	p
4	−30°	1	1	3772± 76	30.1±0.5	239 ±2	216 ±2	p
5	−25°	1	1	360± 18	7.9±0.2	122 ±1.5	106 ±1.5	p

2. – Determination of charge on the primary.

The primary particle entered the stack at an angle of 29° with the zenith and traveled 8.2 cm before it interacts and stops. The charge on the primary was obtained by the method of thin down length for heavy nuclei ending in emulsion. All investigators ([2-4]) have described the thinning as chiefly due to variations in maximum energy and frequency of secondary electrons which are too numerous to be individually resolved. The relationships given by the various theories are:

$$a) \qquad Z = \frac{L}{10} \qquad \text{HOANG},$$

$$b) \qquad Z^2 = \frac{L}{0.7} \qquad \text{PERKINS},$$

$$c) \qquad Z^2 = \frac{L}{0.5} \qquad \text{FREIER } et\ al..$$

where L is the thin down length in microns.

Measurements on the primary track relating width of track as a function of residual range are given in Fig. 2 and indicate a thin down length L of (400 ± 50) μm.

([2]) T. F. HOANG: *Journ. Phys. Radium*, **12**, 739 (1951).
([3]) D. H. PERKINS: *Proc. Roy. Soc.*, A **203**, 399 (1950).
([4]) P. FREIER, E. J. LOFGREN, E. P. NEY, F. OPPENHEIMER, H. L. BRADT and B. PETERS: *Phys. Rev.*, **74**, 213 (1948); P. FREIER, E. J. LOFGREN, E. P. NEY and F. OPPENHEIMER: *Phys. Rev.*, **74**, 1818 (1948).

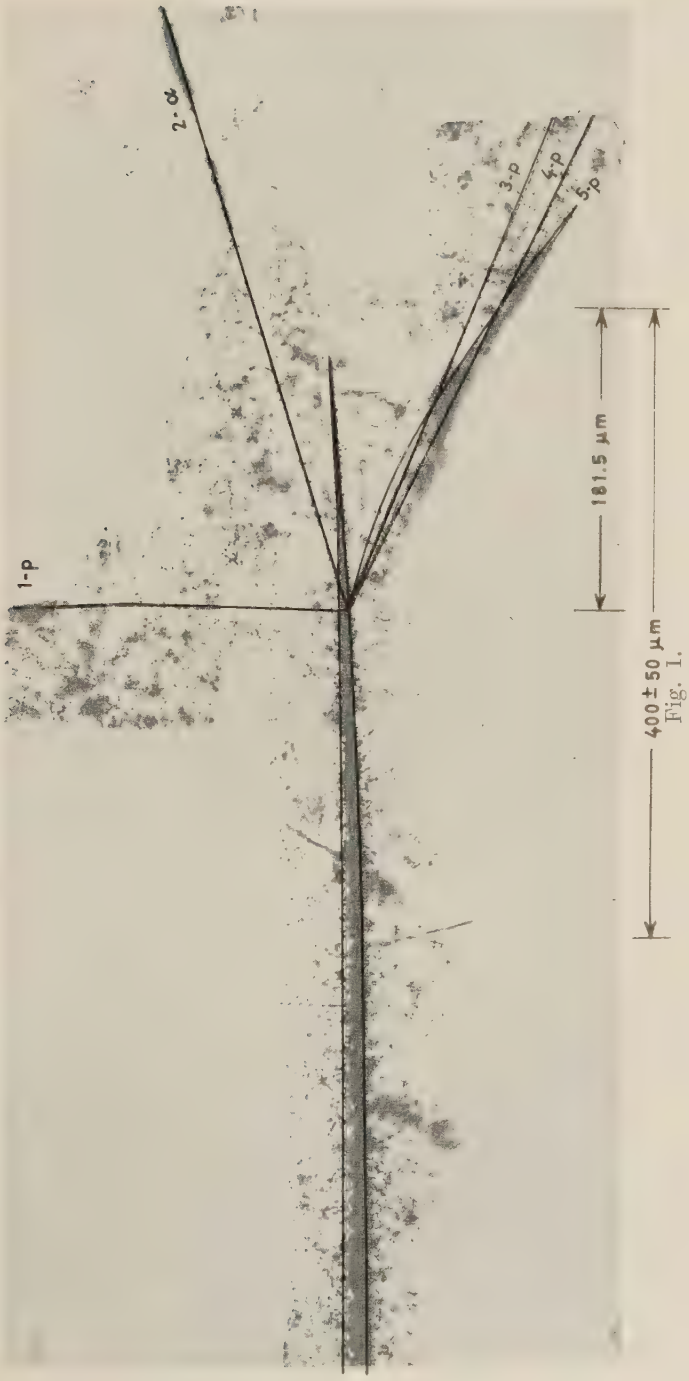

Fig. 1.

If we assume an average value of $L = 400\ \mu m$, we get for the charge of the primary a value of 40 (HOANG), 24 (PERKINS) and 27 (FREIER *et al*). If the primary particle actually undergoes a breakup, a sudden change in the thin down width should appear. This is not indicated by the data given in Fig. 2. The section of track at the point of breakup was enlarged 2000× and the change in thin down width becomes quite apparent (Fig. 3). The breakup appears at a residual range of 181.5 μm.

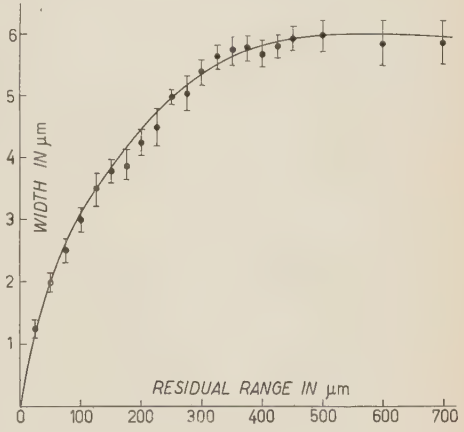

Fig. 2.

In analysing the event the possibility of coincidence was considered. The probability that the event in the thin down was caused by a neutral particle or a charged particle in the same direction as the heavy primary was computed for a residual range of 2000 μm and average cross-section of 4 μm × 4 μm. By considering a residual range of 2000 μm we assume that there would be nothing unique had the breakup occurred some distance beyond the thin down region. Five thin down events were observed in 40000 events. The sensitive volume of the stack was 290 cm³. If we consider the thin down volume per cubic centimeter to be the probability of finding a thin down event in one cubic centimeter of emulsion, and the events per cubic centimeter as the probability of finding an event in one cubic centimeter of emulsion, then the product of the two would represent the probability of a thin down and some other coinciding. The chance of coincidence is less than one in a million.

A further analysis of the event is of interest if we are permitted to assume that the breakup is due to a peripheral proton in the primary particle colliding with a proton in the emulsion. This assumption seems plausible if we consider the slow proton, track No. 1, to be the incident proton after collision, and track No, 5 the recoil proton. The collision also results in the breakup of a small part of the primary particle equivalent to a charge of four. Returning to Table I we see that the horizontal components of particles 2, 3 and 4 are very nearly the same and equal to a rms value of 221 MeV/c. On the assumption that the breakup particles are emitted as two protons and one α having a forward momentum equal to that of the primary, a check on the charge of the primary at the instant of breakup can be made. The range of a proton of momentum 221 MeV/c will be 2840 μm. Knowing that

$$R_Z = \frac{M}{Z^2} R_p , \tag{1}$$

and that the residual range of the heavy nuclei after the interaction is 181.5 μm, we can determine the charge Z. The ratio

$$\frac{R_P}{R_Z} = \frac{Z^2}{M} = \frac{2\,840}{181.5} = 15.6 \,. \tag{2}$$

Tabulating various values of Z^2/M results in the ratio of

$$\frac{37^2}{85.5} = 16.1 \,,$$

or

$$\frac{36^2}{84} = 15.4 \,,$$

$$\frac{35^2}{80} = 15.3 \,,$$

$$\frac{34^2}{79} = 14.6 \,,$$

$$\frac{33^2}{75} = 14.5 \,.$$

The results indicate that the charge of the primary after the interaction is about 36 and that the initial charge could be as high as 41.

If we assume that the charge on the particle is 35, a study of the probability of the particle to survival can be made, At latitude 41° N and zenith angle 29° W, we find the cut-off energy ([5]) to be 2.2 GeV per nucleon or 176 GeV for the particle. Since the particle has a range of 8.2 cm in emulsion, a proton of the same velocity would have a range of 125 cm, equivalent to an energy of 1 GeV. This represents the energy per nucleon of the primary heavy particle at the instant it enters the stack. Between the stack and the top of the atmosphere the particle must lose 1.2 GeV per nucleon of 96 GeV for a particle of mass 80. A 2.2 GeV proton entering the atmosphere loses energy at the rate of 1.45 MeV per $g \cdot cm^{-2}$, hence, a particle of charge 35 will lose energy at the rate of 1.77 GeV per $g \cdot cm^{-2}$. This implies that the particle enters the stack after passing through 54 $g \cdot cm^{-2}$ of air. As the decrease in Z from 35 becomes appreciable, the rate of loss in energy decreases which implies that the particle must travel a distance greater than 54 $g \cdot cm^{-2}$ outside the stack in order to be stopped in 8.2 cm of emulsion. This implies a greater probability of inter-

([5]) E. FERMI: *Nuclear Physics*, p. 230 (Chicago).

Fig. 3.

action, hence, a smaller probability of survival.

* * *

The authors are very grateful to Professor MARCEL SCHEIN of the University of Chicago for his generous assistance in regard to this problem, and to Dr. M. KOSHIBA, also of the University of Chicago, for his very helpful and stimulating discussions.

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RIASSUNTO (*)

In un'emulsione G-5 esposta nel Texas ad un'altezza di 104 000 piedi, è stata osservata una particella cosmica primaria di Z elevato. Le misure di assottigliamento indicano un valore di Z tra 24 e 40. L'assottigliamento è accompagnato da un parziale spezzettamento che consente una determinazione interessante della carica del primario e ne suggerisce il valore compreso tra 35 e 40.

(*) *Traduzione a cura della Redazione.*

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Nuclear Interactions of Neutral K-Mesons of Long Lifetime.

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(ricevuto il 13 Aprile 1957)

Summary. — An emulsion stack was exposed to a neutral beam from the Berkeley Bevatron. The target to stack distance was 9.5 ft, which excluded θ_1-mesons from the stack. The plates were scanned for K^-, $\Sigma^\pm$ and hyperfragments which could have come from the nuclear interactions of the long-lived θ_2-meson. Five Σ^- and three Σ^+ hyperons have been found, all of which originated in small stars in the emulsions. In five cases the Σ hyperon was accompanied by a light meson, presumably a π-meson, each of which had an energy of less than 150 MeV. In two cases the mass of the incoming particle could be measured, and was found to be $(1\,000^{+\,30}_{-\,180})\,m_e$ and $(995^{+\,75}_{-\,270})\,m_e$ respectively. In addition, ten double stars with short connecting tracks have been observed which can be interpreted as hyperfragments. The production of the events in or near the stack by energetic nucleons, pions or K-mesons can be excluded. The events are interpreted as representing the nuclear interactions of long-lived θ_2-mesons in the $\bar{\theta}$ mode by the following reaction:

$$\theta_2 + \mathcal{N} \rightarrow \begin{pmatrix} \Sigma \\ \Lambda \end{pmatrix} + \pi .$$

1. – Introduction.

The possible existence of neutral K-mesons of long life has been predicted from considerations of charge conjugation by Gell-Mann and Pais (1). Ex-

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(+) On leave of absence from St. Paul's University, Tokyo.

(1) M. Gell-Mann and A. Pais: *Phys. Rev.*, **97**, 1387 (1955).

amples of the decay of a long lived neutral K-meson have recently been observed in a cloud chamber ([2]), and it has been suggested that these particles correspond to those predicted by the above authors. Several cases of the decay of a neutral K-meson by a mode other than the two pion mode have been seen ([3]). Recently the anti-theta has been observed in a bubble chamber by ALVAREZ *et al.* ([4]). Possible examples of the nuclear interaction of long lived K-mesons have been seen in emulsions ([5,6]). In all cases the observed properties of these particles are in agreement with those predicted by theory.

In order to study the mode of nuclear interaction of these particles, a stack of nuclear emulsions has been exposed to a neutral beam from the Berkeley Bevatron under conditions such that the time of flight of particles coming from the target was about 10^{-8} s, and virtually excluded the arrival at the stack of all particles of lifetime less than 10^{-9} s.

2. – Experimental method.

Exposure. – Of utmost importance in the exposure was the need to obtain a beam of neutral K-mesons free from all particles capable of creating strange particles in or near the stack of emulsions.

The stack was therefore exposed to a neutral beam coming from a 3 inch hole in the iron yoke of the magnet of the Berkeley Bevatron at 90 degrees to the target. The field of the magnet was used to sweep out the charged particles, while a γ-ray converter consisting of 1 inch of Pb was placed at the inner end of the hole. The total distance of the stack from the target was

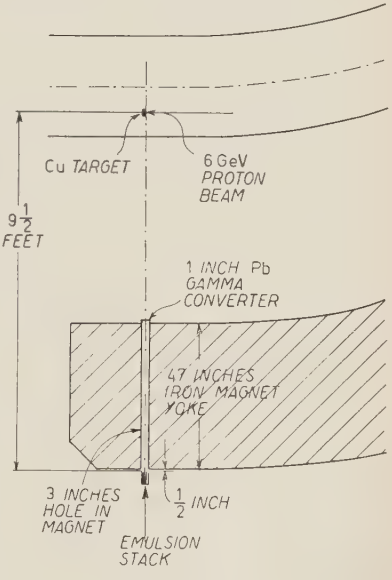

Fig. 1.
Geometry of the Exposure.

([2]) K. LANDE, E. T. BOOTH, J. IMPEDUGLIA, L. M. LEDERMAN and W. CHINOWSKY: *Phys. Rev.*, **103**, 1901 (1956); K. LANDE, L. M. LEDERMAN and W. CHINOWSKY: *Nevis*, 38 (February, 1957).

([3]) R. W. THOMSON: *Progress in Cosmic Ray Physics* (New York, 1954), vol. 3.

([4]) L. W. ALVAREZ, H. BRADNER, P. FALK-VAIRANT, J. D. GOW, A. H. ROSENFELD, F. SOLMITZ and R. D. TRIPP: UCRL-3583 *Nuovo Cimento* **4**, 1026 (1957).

([5]) W. F. FRY, J. SCHNEPS and M. S. SWAMI: *Phys. Rev.*, **103**, 1904 (1956).

([6]) R. G. GLASSER and N. SEEMAN: *Bull. Am. Phys. Soc.*, **1**, 320 (1956).

1961

9.5 ft. and the length of the channel through the iron was 47 inches. The geometry of the exposure is shown in Fig. 1. Of the various channels available, that at 90 degrees was chosen since K-mesons are known to be still relatively abundant, while from kinematic considerations the number of energetic neutrons emitted from the target is expected to be small. Unfortunately the stack was placed only one-half inch from the exit of the channel, and this led us to consider the magnet itself as a supplementary source of contamination in making our background control.

The copper target was one inch thick in the beam direction, but only 60 percent of the target thickness was seen by the channel. The target was one inch long in the direction of the neutral beam. The total flux on the target was $8 \cdot 10^{12}$ protons of 6 GeV. The general backgound of tracks resulting from low energy neutron interactions in the emulsion limited the intensity of the exposure. The proton flux was chosen on the basis of previous test exposures.

Scanning. – On the basis of earlier observations it seemed reasonable to assume that long-lived neutral K-mesons would interact and produce other strange particles, some of which would be charged and therefore detectable in emulsions.

The model of Gell-Mann and Pais indicates that a θ_2 should be capable of interacting either with strangeness $+1$, or with strangeness -1. We could therefore look either for K^+-mesons produced by charge exchange of the θ mode or for K^-, $\Sigma^\pm$ and hyperfragments produced by interactions of the $\overline{\theta}$ mode analogous to the K^- interaction process.

The background of tracks and small stars in our plates was high, with 5 tracks coming to rest in a field of about 800 μm, and 174 stars per cm^2 of emulsion surface. In order to make a first rapid examination of the plates to determine if θ_2 interactions were present, we have made a fast scan under low dry magnification (26×8) looking for the more visible events, i.e. $\Sigma^+ \rightarrow p + \pi^0$ decays at rest, Σ^- and K^- interaction stars, and hyperfragments.

3. – Results.

A total of 218 cm^2 of emulsion were scanned for $\Sigma^\pm$ and for K^- captures at rest. Five Σ^- hyperons were found, three Σ^+ decays and seven K^- captures at rest. All the $\Sigma^\pm$ hyperons when traced back were found to originate in small stars in the emulsion; the seven K^--mesons came from outside the stack from the direction of the beam.

A total of 337 cm^2 was scanned for hyperfragments and 10 events were found in which two stars were linked by a black track. As is generally found

Fig. 2. – Photograph of Event Pd_1 (observer: A. Bernardi).

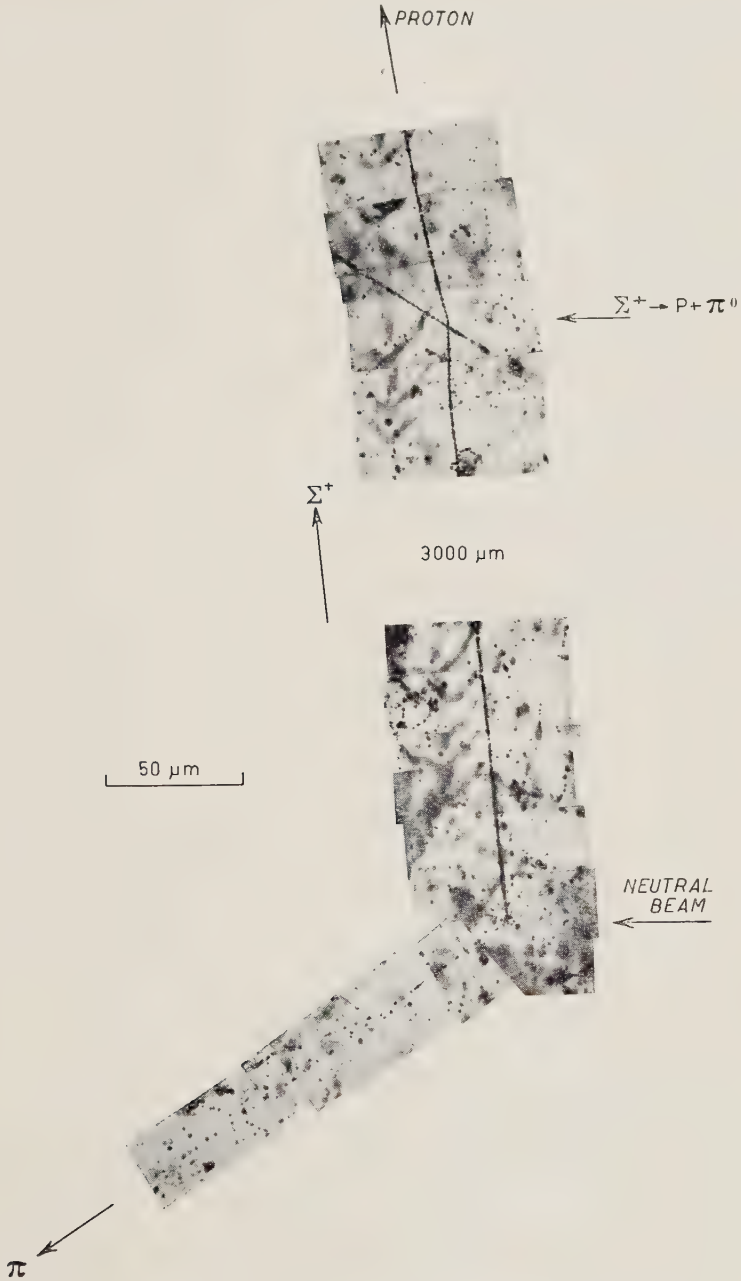

Fig. 3. – Photograph of Event Pd_1 (observer: G. Gesuato).

in the search for hyperfragments, the majority of the double stars are linked by a connecting track so short that their identification as hyperfragments rather than as slow pions or collision processes of nuclear particles, can not be established.

All of the events found are described in Table I. All the negative Σ-hyperons in the events to be described below, have been identified by their capture stars and by multiple scattering (constant sagitta). Fast particles have all been identified by means of ionization and multiple scattering as L-mesons, presumable pions, and their energy determined by multiple scattering. Positive Σ-hyperons have been identified by the characteristic range of their proton secondaries.

Those events which have a charged hyperon will now be described in detail:

Event Pd_1. – This event was recognized by the characteristic capture star of the stopped Σ^- hyperon, which was then followed back to the point of production. A photograph of the event is reproduced in Fig. 2. The star consists of only two tracks; that of the hyperon whose mass was found to be $(1760 \pm 330)\, m_e$, and a pion of 105 MeV. There is no evidence for a nuclear recoil or electrons at the origin of the hyperon and π-meson. The kinetic energy of the hyperon was 32 MeV. The sum of the momenta of the two particles is 119 MeV/c and lies within 68° of the direction of the neutral beam. From conservation of charge, it is most likely that the target nucleon was a neutron and that the π-meson was positively charged. If we assume that the interaction was of the type:

$$\theta + n \rightarrow \Sigma^- + \pi^+ ,$$

then it is possible to calculate the mass of the θ-meson, which is found to be $(1000^{+\,30}_{-180})\, m_e$. The errors quoted arise from assuming that the neutron had a momentum of 200 MeV/c due to its nuclear motion. Since the momentum of the neutron could have been of the same order of magnitude as that of the K-meson it is not surprising that the combined momentum vector of the hyperon and π-meson does not coincide with that of the beam.

Event Pd_4. – This event is very similar to the previous one, except that the Σ hyperon was positive. A photograph of the event is reproduced in Fig. 3.

The point of origin of the Σ^+ hyperon, the decay point and the proton all lie in one plate. The energy of the pion was 140 MeV. There is no evidence of a nuclear recoil. The probable reaction is

$$\theta + n \rightarrow \Sigma^+ + \pi^- .$$

The mass of the θ has been calculated in the same way as for the previous event and was found to be $(995^{+\,75}_{-270})\, m_e$. The direction of the combined mo-

TABLE I (*A*). – *Σ-hyperon events.*

Event number	Strange particles	Probable production interaction	Parent star: Number of prongs	Parent star: E_Σ in MeV	Parent star: E_π in MeV	Total visible energy	Energy of neutral K-meson	Remarks
Pd_1	Σ^-	$\bar{\theta}+n \rightarrow \Sigma^- + \pi^+$	2	32 ± 1	105 ± 20	137	~ 14	Estimated mass of neutral K-meson $1\,000^{+30}_{-180}\ m_e$
Pd_2	Σ^+	$\bar{\theta}+p \rightarrow \Sigma^+ + \pi^0$	2	6 ± 0.5	—	41	—	—
Pd_3	Σ^-	$\bar{\theta}+n \rightarrow \Sigma^- + \pi^+$	7	14.5 ± 1	—	72	—	Probable reabsorption of the π^+
Pd_4	Σ^+	$\bar{\theta}+n \rightarrow \Sigma^- + \pi^-$	2	15 ± 1	140 ± 15	155	~ 37	Estimated mass of neutral K-meson $995^{+75}_{-270}\ m_e$
Pd_5	Σ^-	$\bar{\theta}+n \rightarrow \Sigma^- + \pi^+$	2 + recoil	44 ± 2	67 ± 15	> 110	—	—
Pd_6	Σ^-	$\bar{\theta}+n \rightarrow \Sigma^- + \pi^+$	3 + recoil	17.5 ± 1	50 ± 15	> 185	—	—
Pd_7	Σ^+	$\bar{\theta}+p \rightarrow \Sigma^+ + \pi^0$	1	49 ± 2	—	49	—	—
Mi_5	Σ^-	$\bar{\theta}+n \rightarrow \Sigma^- + \pi^+$	4	6.4 ± 0.5	100 ± 15	~ 140	—	—

TABLE I (*B*). – *Possible Hyperfragment events.*

Event number	Parent star			Secondary star					Remarks
	Type	Visible kinetic energy in MeV	Emission of π^--meson	Length of connecting track	Number of prongs	Visible energy release in MeV	Total visible charge	Type of decay	
Pd_3	8+0n	250	no	2 μm	2+recoil	> 100	⩾3	Non-mesonic	—
Pd_9	5+0n	65	no	2 μm	3	130	⩾3	Non-mesonic	—
Pd_{10}	7+0n	125	yes	4 μm	3	70	3	Non-mesonic	—
Pd_{11}	10+0n	100	no	2 μm	3	15	3	Non-mesonic	—
Mi_1	11+0n	160	no	3 μm	4	> 38	4	Non-mesonic	—
Mi_2	9+0n	126	no	3 μm	3+recoil	29	3	Non-mesonic	—
Mi_3	2+1n	> 6	possible	15 μm	2+recoil	> 10	> 2	Non-mesonic	—
Mi_4	4+0n	> 55	no	5 μm	2+recoil	10	> 2	Non-mesonic	—
Mi_6	1+0n	20	no	60 μm	2	5	2	—	Linking track with $Z > 1$
Mi_7	2+0n+recoil	12	no	18 μm	2	11	2	—	It is not possible to decide which is the parent and which the daughter star.

menta of the Σ hyperon and the π-meson makes an angle of 15° with the direction of the beam.

Event Pd_5. – A drawing of the event is shown in Fig. 4. The interaction star consists of three particles; a π-meson of (67 ± 15) MeV, a short recoil of 12 μm, and a Σ^--hyperon of range 6 200 μm. The mass of the hyperon was found to be (2 800 ± 210) m_e.

The association of a charged π-meson with a negative hyperon suggests that the basic interaction was with a neutron. A mass estimate of the incoming particle is probably not meaningful since it seems likely that nuclear excitation followed by neutron emission accompanied the interaction.

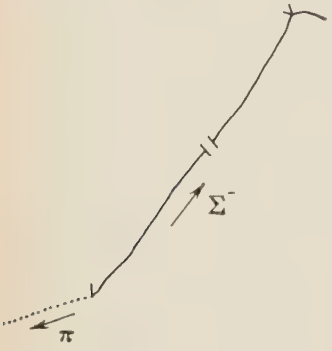

Fig. 4. – Drawing of Event Pd_5 (observer: A. Bernardi).

Event Pd_6. – A drawing of this event is shown in Fig. 5. The event consists of four tracks. Track one is due to a pion of (50 ± 15) MeV. Track two was produced by a proton of 150 MeV, while track three is only 5 μm long and cannot be identified; it could be a nuclear recoil of a light element. Track four is 1255 μm long and has a three pronged star at its end. Scattering measurements along this track give for the mass of the particle (2 300 ± 700) m_e, and since it gave rise to a star at the end of its range we assume that it was a negative hyperon: one of the tracks from this hyperon star ends with another star of three prongs which is interpreted as the non-mesonic decay of a hyperfragment.

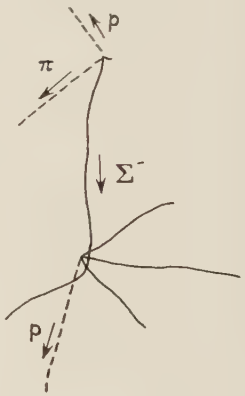

Fig. 5. – Drawing of Event Pd_6.

Event Pd_3. – This event is interpreted as the nuclear interaction of a neutral K-meson with the production of a negative Σ-hyperon and a π^+-meson which was absorbed in the same nucleus. The hyperon was absorbed at rest and produced a hyperfragment.

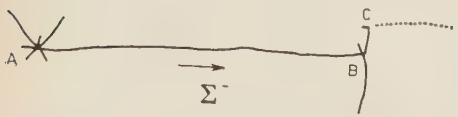

Fig. 6. – Drawing of Event Pd_3 (observer: S. Marchetti).

The event consists of three connected stars as is shown in the drawing Fig. 6. Star A consists of 7 prongs all of which are grey or black. The connecting track between stars A and B is 900 μm long; it is black and quite straight. Although this track is not long, a scattering measurement was made

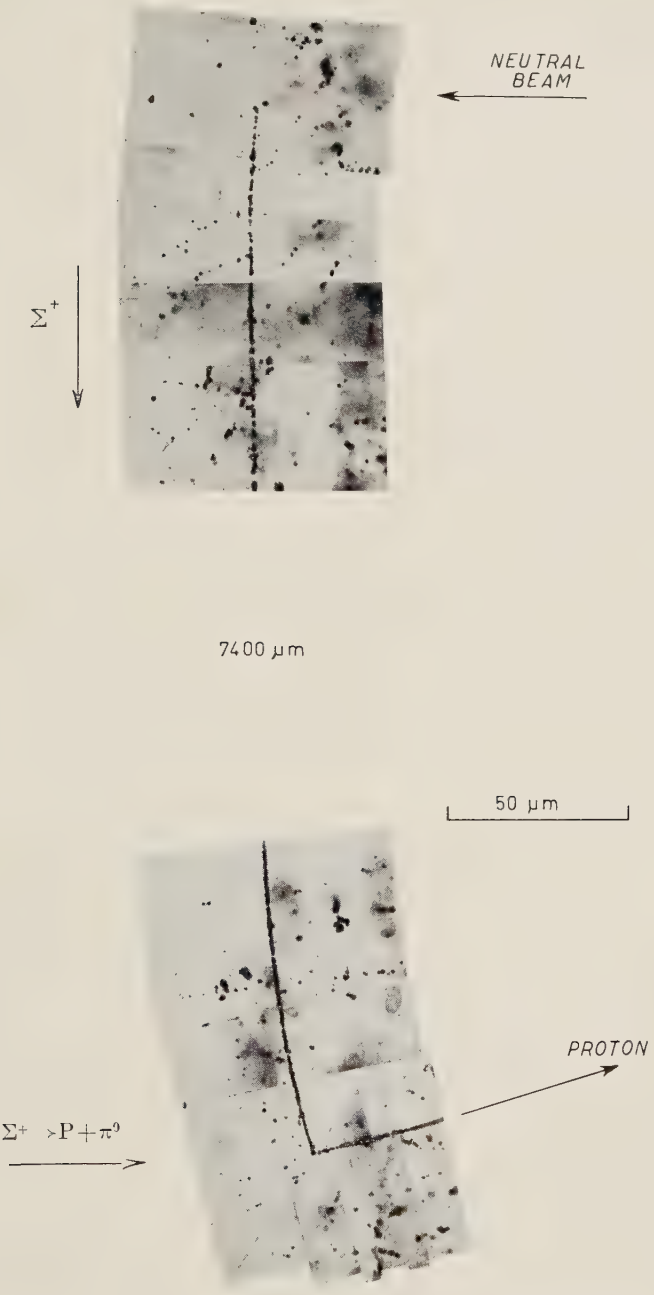

Fig. 8. – Photograph of Event Pd_7 (observer: F. Franceschini).

which gave $(2100 \pm 670)\, m_e$ for the mass of the particle. The fact that it produced a star at the end of its range is strong evidence that it was either a negative hyperon or a negative K-meson. In view of the mass measurement it seems more likely that it was a negative hyperon.

One of the tracks from star B ends with star C. This connecting track is only 17.5 μm long, is straight and is saturated. Star C consists of only two tracks; that of a fast particle and a short recoil of 1.5 μm. It is interpreted as representing the decay of a hyperfragment produced in star B by the absorption of the negative hyperon from star A.

Event Pd_2. – The interaction star in this event also consists of only two outgoing charged particles; one a proton of 35 MeV and a second particle which appeared to stop and give rise to a proton of range 1590 μm (see Fig. 7). The mass of this second particle could not be determined from its track because of its short range, but it is clear that it stopped before it gave rise to the proton. The range of the proton strongly suggests that it came from the decay at rest of a positive Σ-hyperon.

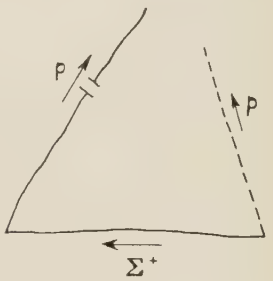

Fig. 7. – Drawing of Event Pd_2 (observer: S. Marchetti).

This event can be interpreted in the same way as the previous events if one assumes that the basic interaction was

$$\theta + p \rightarrow \Sigma^+ + \pi^0 ,$$

accompanied by the emission of a proton of 35 MeV, which may have arisen from the interaction of the π^0-meson within the parent nucleus.

Event Pd_7. – In this event a positive Σ-hyperon is produced in the middle of the emulsion unaccompanied by other charged particles. At the point of origin of the hyperon there is one additional grain which could be interpreted as a recoil but it could also be a background grain. The hyperon came to rest after 7.4 mm and decayed into a proton of range 1650 μm. A photograph of the point of origin and the decay point in shown in Fig. 8. It is believed that the event is an example of the interaction of a θ with a proton leading to the production of a Σ^+-hyperon and a π^0-meson. It is not possible to decide whether the interaction was with a free proton or with a proton in a nucleus, because in many cases of the interaction with a single nucleon in a nucleus, the nuclear excitation is very small and cannot be detected (as is evident from the other events).

Event Mi_5. – A drawing of this event is shown in Fig. 9. The parent star consists of a pion of 100 MeV, two nuclear particles and a fourth particle

which stops after 260 μm and produces a single track due to a doubly charged particle. Delta rays along this connecting track exclude a charge of two and therefore it could not have been a hyperfragment. The characteristics of the connecting track exclude a negative pion, and therefore it must have been a negative strange particle. By analogy with events previously described, taking into account the presence of the charged pion in the parent star, it is reasonable to assume that it was a Σ^--hyperon rather than a K^--meson.

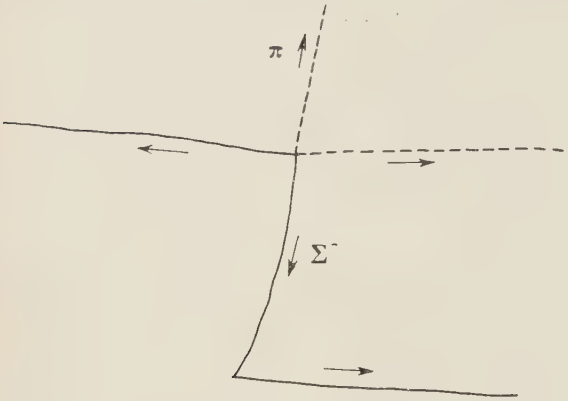

Fig. 9. – Drawing of Event Mi_5 (observer: S. Bonetti).

4. – Interaction mode of the neutral K-mesons.

The events have been found by looking for the typical decays or captures of strange particles and then following them back to their points of origin. In this way the production stars have been found without appreciable bias. One of the most striking features of these stars is that they are very small; in fact in some cases they consist of only a charged hyperon and a light meson of relatively low energy. Of those cases where a charged π-meson is not observed only one represents the production of a negative Σ-hyperon and here the parent star is somewhat larger than the others and consistent with the assumption that a positive π-meson was also produced but was subsequently absorbed in the same nucleus. In two of the three cases of the production of a positive Σ-hyperon, a charged π-meson is not observed. In these cases it is possible that the interaction was with a proton and that it was a neutral π-meson which was emitted. In events Pd_1 and Pd_4, it is possible to calculate the mass of the incoming particle which is found to be that of a K-meson. All of the charged hyperon events are consistent with the reaction:

$$\theta + \mathcal{N} \rightarrow \Sigma + \pi \,,$$

in which the θ-meson has a kinetic energy of less than 200 MeV.

5. – Discussion.

It might be argued that the events of interest were not produced by neutral K-mesons from the target, but by other sources. Among these sources are:

1) the direct production of the events observed by fast neutrons from the target;

2) the production in the stack of neutral K-mesons by high energy neutrons, pions and protons;

3) the charge exchange of negative and positive K-mesons which originate outside the stack, and the subsequent interaction of the neutral K-mesons in the stack;

4) the interaction of hyperons in the stack, giving rise to neutral K-mesons which subsequently interact;

5) production by cosmic rays.

In order to investigate the first two possibilities, a study of all types of stars in an area of 8.5 cm^2 has been made. The average prong number of all stars is about 3: the number of stars with one shower in this area is 51.

A part of the stars with one shower prong may be the interaction of an incoming particle (represented by the minimum track) or the production of a fast meson by a neutral particle. Only 5 stars have been observed which have two shower particles, and none with more than two shower particles. The stars with two shower particles can be either double-production of π-mesons by neutrons, or single-production by nucleons or π-mesons.

FOWLER *et al.* [7] have found that the probability of the production of two shower particles by 1.5 GeV neutrons is about 50 percent. This result can be used to estimate the number of high energy interactions that are present in our emulsion.

In order to determine the true number of shower tracks per star in the plates, it is necessary to estimate the scanning loss of minimum ionizing tracks. Observations on π-μ-e decays were made and showed that the loss of single minimum ionizing tracks in our plates was 20 percent.

We therefore estimate that the number of π-mesons, from interactions comparable to 1.5 GeV, is $(5 \cdot 2)/(0.5 \cdot 0.8) = 25$ in the volume scanned for the background study. This means that in the total volume studied for strange particles (218 cm^2) there would be about $25 \cdot (218/8.5) = 640$ pions produced

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[7] W. B. FOWLER, G. MAENCHEN, W. M. POWELL, G. SAPHIR and R. W. WRIGHT: *Phys. Rev.*, **101**, 911 (1956).

1999

in the high energy reactions. The probability that such interactions can lead to the production of a K-meson can be estimated from the π-meson to K^+-meson ratio of 570 found by HARRIS *et al.* [8] for protons of 3 GeV on a copper target. This means that the number of charged K-mesons produced by high energy neutrons should be about 1. Assuming that charged hyperons and K^+-mesons are produced in equal numbers, then this number of hyperons, namely 1, is to be compared to that of 8 charged hyperons observed, which represent 35 charged hyperons after correcting only for the scanning loss (see below). This source for the production of the events of interest would therefore seem unimportant.

Source 2 for the production of the events by protons or neutrons can be immediately excluded since this must necessarily be less than source 1 owing to the small probability that the meson, when so produced, will in turn interact in the plates (this probability is about 1/12). This also renders unimportant the contribution to K^0-meson production by fast π-mesons.

The production of the events by source 3 can be excluded by the following considerations. In the scanning, 7 examples of the capture of negative K-mesons at rest have been found. All of these negative K-mesons came from the direction of the beam, and had an energy less than 100 MeV. The dimensions of the stack were such that all K-mesons of energy less than about 170 MeV would have stopped in the plates. Therefore we believe that the number of those which did not stop in the stack because of their high energy is negligible. It is known that the probability of charge exchange as opposed to hyperon production is quite small for negative K-mesons (probability less than 10 percent). When one further considers the small probability that the neutral K-meson so produced will interact, one sees that this effect is completely negligible. Only if we assume the Gell-Mann and Pais duality of the θ-particle could the charge exchange of the K^+ lead to hyperon production. For this process to lead to the production of the number of events which we observe, there should have been a flux of the order of 10^4 K^+-particles, of which a high proportion of τ, τ' and $K_{\mu 3}$ should have been detected.

Source 4 can be immediately excluded because the lifetime of the hyperons is short and therefore they can not reach the stack.

The stack was not finally assembled until immediately before the exposure and therefore the events which were not associated with the machine exposure could not be followed from one plate to another. At least one of the tracks from each hyperon event passed into a second plate thereby proving quite conclusively that they were produced after the stack was assembled. The

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[8] G. HARRIS, J. OREAR, S. TAYLOR and P. BAUMEL: *Production of* K^+*-mesons by* 3 GeV *protons on hydrogen* (abstract for the *Washington Meeting of the Am. Phys. Soc.*, 1957).

2000

number of energetic cosmic rays incident on the stack after assembly was negligible, as is clear from the small number of large stars found in the scan for the background study.

From these arguments we conclude that the neutral K-mesons responsible for the events came from outside the stack.

The immediate vicinity of the stack has been monitored by the observations on the stack itself. We have no reason to expect a sharp discontinuity in flux of high energy particles or K-mesons between it and the end of the channel, and therefore whether we assume that the neutral K-mesons came from the walls of the channel or from the target itself, the conclusions are the same; namely that they lived for a time long in comparison with the lifetime of the normal θ.

The K^--mesons which were observed to come from the direction of the channel were probably the result of the charge exchange of the θ_2-mesons in the walls of the channel itself, since the number of θ_2 incident on the front edge of the channel is much larger than that of the incident K^-.

An estimate can be made of the total number of both charged and neutral hyperons which were produced in the volume which was scanned. We have observed 3 positive and 5 negative Σ-hyperons. In our method of scanning we would not expect to observe decays in flight of Σ-hyperons of either sign, and further, a positive hyperon which decays from rest into a charged pion would also be missed because of the difficulty of observing the fast pion.

For the positive hyperons, only the decay at rest into a proton would be expected to be observed. The branching ratio of the charged Σ-hyperons and the ratio of the number of Σ-hyperons to Λ-hyperons can be estimated from previous studies of the capture of negative K-mesons at rest in the emulsion ([9]). One would expect that the $\bar{\theta}$ mode would be basically the same as that of the negative K-meson, but one difference is that the neutral K-mesons interacted in flight while the data on negative K-mesons comes from captures at rest. However, it is probable that the relative importance of the various isotopic spin states will not be greatly changed. The probability that a positive hyperon will decay from rest into a proton and will be observed, can be estimated from known lifetime and branching ratio data. This probability of detecting the Σ^+-hyperon is estimated to be 0.18. Using this detection efficiency, the total number of positive Σ-hyperons becomes 17.

The detection efficiency of negative Σ-hyperons is not so easily estimated. It is known that an appreciable fraction of stopped negative Σ-hyperons do not produce a visible star, and it has been estimated that about 50 percent fall into this class ([9]). The detection efficiency for Σ^--hyperons in this ex-

([9]) W. F. Fry, J. Schneps, G. A. Snow and M. S. Swami: *Phys. Rev.*, **100**. 950 (1955).

2001

periment is probably lower than in the negative K-meson work because of the very high background of stopping particles. Small stars with a single short prong could easily have been mistakely classified as the scatter of a stopping particle. Correcting for decays in flight we estimate the detection efficiency for Σ^--hyperons to be about 0.27. The total number of Σ^--hyperons in the area scanned is then estimated to be 18. In the capture of negative K^--mesons at rest in emulsions ([10]) the fraction of stars from which charged hyperons are emitted is estimated as 0.13. Assuming the analogy between K^- and $\bar{\theta}$ interactions, the total number of neutral K-meson interactions in the area studied is about 270 (excluding those interactions which give rise to neutral and positively charged K-mesons because in our method of scanning, positive K-meson would not have been detected). It is to be expected that a neutral hyperon escapes from the greater part of those interactions in which no charged hyperon is observed.

6. – Conclusions.

An emulsion stack was exposed to a neutral beam from the Berkeley Bevatron at a distance of 3 m from the target.

Five examples of the production of negative Σ-hyperons and three examples of positive Σ-hyperons have been observed, of which five were accompanied by a charged π-meson of energy less than 150 MeV. In two cases the mass of the incoming particle could be calculated and was found to be that of a K-meson within narrow limits.

In addition to these hyperon events, 10 double stars have been observed, with a short linking track, of which some may be hyperfragments produced by neutral mesons (*).

Direct production of these events, or the production in or near the stack of neutral K-mesons which in turn were responsible for our events, has mainly been excluded by a study of the background stars in the emulsion. The charge exchange in or near the stack of incoming K-mesons, and the production by hyperons can also be excluded. The neutral K-mesons responsible for the

([10]) W. M. Fry, J. Schneps, G. A. Snow, M. S. Swami and D. C. Wold: *Charged hyperons from* 1001 K^-*-meson stars* (to be published in the *Phys. Rev.*).

(*) *Note added in proof.* – One τ^+-meson, five K^--mesons and additional $\Sigma^\pm$-hyperons have been found in subsequent scanning; all of these particles originated in small stars without a charged incoming particle. The production of the τ^+ as well as the K -mesons and $\Sigma^\pm$-hyperons indicates that the neutral 0 can produce particles of both strangenesses – a fundamental property predicted by Gell-Mann and Pais.

2002

observed interactions were produced far from the stack, probably in the target, and were therefore of long life.

The general characteristics of these interactions and the long lifetime suggest that the particle responsible should be identified with the θ_2 proposed by GELL-MANN and PAIS. All of the events are compatible with the interaction of the θ_2 by the $\overline{\theta}$ mode with the reaction

$$\theta_2 + \mathcal{N} \rightarrow \begin{pmatrix} \Sigma \\ \Lambda \end{pmatrix} + \pi \, .$$

* * *

The authors are grateful to those at Berkeley who made it possible to use the facilities of the Bevatron accelerator. The exposure was made for us by Dr. R. LEVI SETTI to whom we are greatly indebted. Discussions with Professors N. DALLAPORTA, M. MERLIN and G. OCCHIALINI were very helpful.

One of us (W.F.F.) wishes to thank all of those in the group for making it possible to be one of them in this work. Another of us (H.H.) wishes to thank the Italian Government for a scholarship.

RIASSUNTO

Un pacco di emulsioni nucleari è stato esposto ad un fascio neutro ottenuto dal Bevatrone di Berkeley. La distanza fra il pacco ed il bersaglio era di 3 m, in modo da escludere dal pacco i mesoni θ_1. Si sono ricercati mesoni K negativi, $\Sigma^{\pm}$ ed iperframmenti che potrebbero provenire da interazioni nucleari di mesoni θ_2 di lunga vita media. Sono stati trovati cinque Σ^- e tre Σ^+, tutti provenienti da piccole stelle dell'emulsione. In cinque casi l'iperone Σ era associato ad un mesone leggero, probabilmente un mesone π, di energia minore di 150 MeV. In due casi si è potuto misurare la massa della particella incidente che è risultata $(1\,000^{\pm}_{-}{}^{30}_{180})\,m_e$ e rispettivamente $(995^{+}_{-}{}^{75}_{270})\,m_e$. Inoltre sono state trovate dieci stelle doppie collegate mediante brevi tracce che possono essere interpretate come iperframmenti. Si può escludere la produzione degli eventi da parte di nucleoni veloci, pioni o mesoni K in vicinanza o nel pacco. Gli eventi possono interpretarsi come interazioni nucleari di mesoni θ_2 di lunga vita media, secondo l'interazione $\overline{\theta}$, mediante la seguente reazione:

$$\theta_2 + \mathcal{N} \rightarrow \begin{pmatrix} \Sigma \\ \Lambda \end{pmatrix} + \pi \, .$$

2003

IL NUOVO CIMENTO Vol. VI, N. 1 1° Luglio 1957

The Decay of a Σ^+ Hyperon-Proton Fragment.

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(ricevuto il 13 Aprile 1957)

Summary. — An event, has been found in an emulsion stack which is interpreted as a (Σ^+, proton) compound which decayed from rest into a pion, a proton and a neutron with an energy release of (106±14) MeV.

1. – Introduction.

It has been pointed out by several authors ([1,2]) that only two basic types of compounds of Σ-hyperons and nucleons could be stable for a time comparable to the lifetime of the hyperon, namely a Σ^+-hyperon with one or more protons, or a Σ^--hyperon with one or more neutrons. A charged Σ-hyperon bound in a fragment containing both protons and neutrons should, by a fast process, produce a Λ-hyperon, and therefore such a fragment would not be expected to be observed.

It is not surprising that a Σ hyperfragment has not yet been identified in view of the fact that only a relatively small number of slow charged Σ-hyperons have been studied. In addition there is the difficulty of detecting the (Σ^-, neutron) fragment; further, the possible number of (Σ, nucleon) compounds is strictly limited.

(+) On leave of absence from the University of Wisconsin, Madison, Wisc.. Supported in part by the Atomic Energy Commission (USA) and in part by the Graduate School, University of Wisconsin.

(×) On leave of absence from St. Paul's University, Tokyo.

(1) W. G. Holladay, quoted by R. G. Sachs: *Phys. Rev.*, **99**, 1573 (1955).

(2) R. H. Dalitz: *Nuclear Physics*, **1**, 372 (1956).

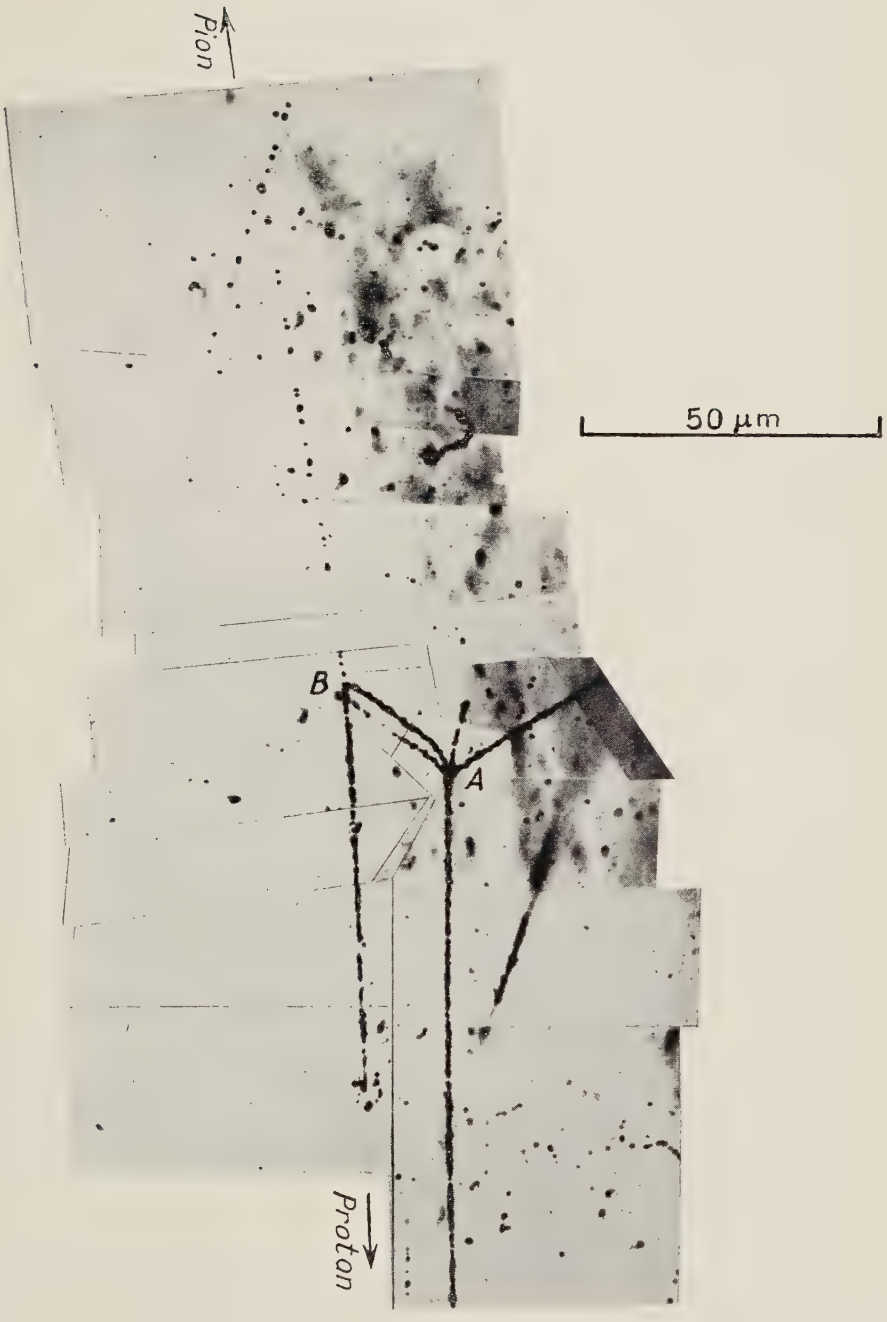

Fig. 1. – Photograph of the Event. Of the seven prongs of star « A » only five are visible in the photograph, since one was nearly vertical in the emulsion and another was only a short recoil. (Observer: A. Zeviani).

As in the case of the Λ hyperfragments, a study of Σ hyperfragments should yield considerable information about the properties of the Σ-hyperon and its interaction with nuclei.

2. – Description of the event.

The event was found in a systematic search for Λ hyperfragments in a stack exposed to 4.5 GeV π^--mesons. A photograph is shown in Fig. 1. Star « A » is of the type (7+0n) and one prong connects with star « B ». Only five of the seven prongs of star « A » are visible in the photograph, since one was nearly vertical in the emulsion and another was only a short recoil. The total visible energy in this star is about 70 MeV: there are no minimum ionizing tracks associated with it. The connecting track between the two stars in only 22 μm long. It is saturated and shows some scattering, which suggests that it was produced by a stopping particle.

The secondary star « B » consists of only two tracks: a lightly ionizing track, and a black track which stops in the plate containing the event. There is no indication of a short recoil track or a slow electron, the presence of which would have suggested the nuclear capture of the connecting particle. The black track is 1 170 μm long, and multiple scattering measurements give for the mass of the particle $(1\,570 \pm 700)\,m_e$: we therefore assume it to be a proton of 15.5 MeV. A grain count along the lightly ionizing track gives for the ionization $(1.27 \pm 0.05) \cdot I_{min}$. The track has been followed for 13 mm after which it disappears in the emulsion without associated tracks: scattering measurements have been made over 11 mm yielding a $p\beta$ of (129 $\perp$ 16) MeV/c. This together with the ionization measurements shows that the particle was a light meson. The fact that the particle charge-exchanges identifies it as a pion, and clearly shows that its direction was away form star « B ». The energy of the pion at production was (85 $\pm$ 11) MeV (taking into account the energy loss over the portion of track used for scattering measurements). A careful search along the pion track has been made for knock-on electrons of high enough energy to determine the direction of the pion. Such an electron was found: it had an energy of 0.16 MeV and made an angle of 71° with the pion. From the angle and the energy of the electron the energy of the pion was calculated and was found to be consistent ($\sim$ 80 MeV) with that found from scattering measurements. It seems very unlikely therefore, that this electron was a coincidence of a background track, since it gave the correct energy for the pion. This is additional evidence that the direction of the pion was away from the star « B ».

Although the connecting track is not long (22 μm), it is possible to obtain

considerable information from measurements of the profile of the track ([3,4]), and a comparison of the width with known stopping particles such as α-particles, protons and pions. The results of the profile measurements are tabulated in Table I.

TABLE I. – *Summary of the profile measurements.*

Type of particle	Total number of tracks measured	Total number of cells	Mean track width in microns
Alpha	8	69	0.65 ± 0.008
Proton	11	110	0.58 ± 0.008
Pion	11	112	0.55 ± 0.007
Connecting track	1	9	0.64 ± 0.02

It has been found that the measured deviations in the widths of tracks produced by one type of particle, for example protons, comes from actual deviations in the track width and not from experimental errors in measuring the width. For this reason it was decided to determine the error in the measurements by employing the deviations from the mean in the distribution of track widths of particles of one type. The error in the mean track width for α-particles, protons and pions is the standard deviation of the mean from the experimental distribution. Since such an estimate cannot be made for one track, the estimated error for the connecting track is taken to be equal to the standard deviation of the distribution for the α-particles. This assumption seems to be justified because the standard deviations (and therefore also the error in the mean as shown in Table I) for α-particles, protons and pions are nearly equal.

The dip angle of the connecting track is 22°. Only tracks with a dip angle less than 30° were used for the calibration of the width. It has been shown([3,4]) that the dip correction is unimportant for tracks which have a dip angle less than 30°.

As will discussed later, the two most likely interpretations of the event are that the connecting track was produced by either a negative K-meson or a (Σ^+, proton) compound. The width of the connecting track is such that if the particle responsible had been of charge one, its mass would have been

([3]) G. ALVIAL, A. BONETTI, C. DILWORTH, M. LADU, J. MORGAN and G. OCCHIALINI: *Suppl. Nuovo Cimento*, **4**, 244 (1956).

([4]) S. NAKAGAWA, E. TAMAI, H. HUZITA and K. OKUDAIRA: *Journ. Phys. Soc. Japan*, **11**, 191 (1956).

at least that of a deuteron; the fact that the width is at least three standard deviations from that of a K-meson or a proton indicates that the charge is probably two. In either of these two cases it is probable that the track was not produced by a negative K-meson. Of course it must be borne in mind that the errors are based on a relatively small number of measurements, namely 8 α-particles, 11 protons and 11 pions, but the large difference in the widths of α-particles and protons makes it relatively easy to distinguish between them.

3. – Interpretation.

The following factors suggest the interpretation of the event as the decay of a (Σ^+, proton) fragment: the excellent agreement of the calculated total energy release of (106$\pm$14) MeV with the expected value of (110 MeV — binding energy), the relatively high energy pion, the complete absence of evidence for the nuclear capture of the connecting particle, and the consistency of the charge of the connecting particle with that of the decay products.

The proton and pion tracks are not collinear and have an angle of 147° between them. If it is assumed that the residual momentum was carried away by a single neutron, its energy is found to be (5.2$-$1.4) MeV, and therefore the total energy release was (106$\pm$14) MeV. The error in the total energy comes principally from the determination of the energy of the pion by the multiple scattering. The expected energy release from the decay of a (Σ^+, proton) compound cannot be accurately predicted because the binding energy is not known due to the absence of information about the strength of the Σ-nucleon force. If one assumes that the Σ-nucleon force is comparable to the nucleon-nucleon force, then the system would be similar to the normal deuteron which is bound with about 2 MeV. If the binding energy of the (Σ^+, proton) fragment is also 2 MeV the energy release from the decay would be 108 MeV. This value is in excellent agreement with the energy release in the event observed (106$\pm$14) MeV.

The parent star « A » of the event does not have a charged primary or a high energy secondary which is charged. The absence of high energy charged particles would suggest that the Σ^+-hyperon was the result of the interaction of a neutral secondary particle such as a Θ-meson or a Λ-hyperon having apreciable kinetic energy. In fact one might expect that the probability of formation of a Σ compound would be higher from a Θ interaction than in an associated production process, because the velocity of the Σ-hyperon would be expected to be slower in the former process. Of course, the production by a neutron of a Σ^+-hyperon in association with a neutral K-meson cannot be excluded.

Several alternative explanations for the event have been considered, namely

2007

(i) that the connecting track was due to a K^--meson which stopped, and which on capture produced star « B », (ii) that star « B » was produced by a neutral particle, and that the connecting track caused star « A », (iii) that a Σ^--hyperon or a Ξ^--hyperon was produced in star « A » and its nuclear capture led to star « B », or (iv) that there was an accidental association of two unrelated stars, one of which happened by chance to give the correct energy release for a Σ hyperfragment.

Of all the above possibilities, the first would seem *a priori* to be the most likely alternative explanation. It might be argued that the connecting track was produced by a slow K^--meson which gave rise to star « B » without any nuclear recoil or electron, and that the capture products were a Λ-hyperon, a fast pion and an evaporation proton; however, the profile measurements on the connecting track are inconsistent with this interpretation.

Interpretation (ii) is rather improbable for the same reason as (i) since in both cases it is necessary to postulate that the connecting track was due to a negative K-meson: the total visible energy in star « A » excludes a Σ^- or a Ξ^--hyperon. Furthermore, the inconsistency of the total charge from star « A » ($\geqslant 6$) with that of the connecting track, excludes the decay of a hyperfragment which originated at « B » and decayed at « A ».

Consideration (iii) is not possible, because a stopped Σ-hyperon should lead to the production of a Λ-hyperon. Even though the Λ-hyperon might become trapped in a nucleus following the capture of the Σ-hyperon, the pion observed is too energetic to have come from the mesonic decay of such a Λ-hyperon. The same argument may be applied to the Ξ^--hyperon.

Of course consideration (iv) cannot be completely excluded, but it seems very improbable because not only must one track from star « B » end at the centre of star « A » (or the converse) but the total energy of the pion, proton and neutron must accidentally coincide with the expected Q-value for the Σ-hyperon decay: further, the connecting track must have been due to an α-particle. That all of these factors should have been a coincidence seems extremely improbable.

4. – Conclusions.

The event has been interpreted as the decay from rest of a (Σ^+, proton) fragment into a pion, a proton and a neutron. The energy release cannot be measured with enough precision to enable the binding energy of the compound to be estimated.

The fact that a two-body system consisting of a Σ-hyperon and a nucleon is bound shows that the Σ-nucleon force is probably greater than the

2008

Λ-nucleon force, since there is good evidence that the two-body system containing a Λ-hyperon is unbound.

* * *

Many interesting discussions with Prof. N. Dallaporta and Prof. M. Merlin have been helpful and stimulating. One of us (W.F.F.) wishes to thank those at Padua for permitting him to work with them in their laboratory on this problem. Another of us (H.H.) wishes to thank the Italian Government for a scholarship.

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RIASSUNTO

È stato trovato un evento in lastre nucleari che è stato interpretato come un composto (Σ^+, protone) che decade a riposo in un pione, un protone e un neutrone con un Q del valore di $(10\,\pm 14)$ MeV.

2009

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On a Macroscopic Measurement of the Spin of Electromagnetic Radiation.

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(ricevuto il 13 Aprile 1957)

Summary. — The spin of electromagnetic radiation can be revealed at microwave frequencies by the torque exerted by a circularly polarized wave upon a screen which can conduct current only parallel to a given direction. In order to make an accurate quantitative measurement, one has to know the angular momentum cross section of the screen. The first few terms of a series expansion of the cross section are evaluated, with the assumption that the screen is a circular disc whose radius is small with respect to the wavelength. In order to assess the accuracy with which the polarization must be circular, also the imaginary part of the cross section is evaluated. Explicit expressions are given for the scattered field over the screen. The feasibility of the experiment is discussed.

1. – Introduction.

It was shown by Beth [1] that the spin of electromagnetic radiation can be revealed by a macroscopic measurement. He succeeded in measuring the torque exerted by a suitably polarized beam of light on a doubly refracting plate and found a good agreement with the theory.

A different experiment was suggested by Carrara [2], who made use of microwaves. He revealed the torque that a circularly polarized wave exerts upon a plane screen (or a system of plane screens) which can conduct the current only parallel to a given direction.

(1) R. A. Beth: *Phys. Rev.*, **50**, 115 (1936).
(2) N. Carrara: *Nuovo Cimento*, **6**, 50 (1949); *Nature*, **164**, 882 (1949).

2010

As is well known, field theory predicts that a circularly polarized wave should carry a spin per unit time per unit surface given by S/ω, where S is the magnitude of Poynting's vector and ω the angular frequency. If the wave carries N photons per unit time per unit surface, we shall have $S = N\hbar\omega$, whence the spin carried by each photon turns out equal to $\hbar$, as it should be.

Apart from microwave applications (power measurements) it is interesting to make an accurate measurement of the angular momentum carried by the wave, since this represents an indirect measurement of the spin of the photons.

Now in Carrara's experiment the torque is substantial and can be easily revealed. However, an accurate agreement with the theory has not yet been found, simply because an accurate theory is not yet available. The size of the screen is necessarily of the same order of magnitude as the wavelength. Accordingly, one cannot make use of the geometrical cross-section, but has to know the effective cross-section for angular momentum absorption. This was pointed out by the author ([3]), who evaluated the principal part of the angular momentum cross-section of a circular disc, whose radius is very small compared to the wavelength. But, in order to be able to carry out an accurate measurement, one ought to make sure that the first term approximation is valid for the disc employed. This can only be accomplished by evaluating some of the following terms of the series expansion of the cross-section.

It was also shown ([4]) that in order to assess the accuracy with which the polarization must be circular, one has to know the *imaginary* cross-section as well.

It is the purpose of the present paper to evaluate some terms beyond the first for both the real and the imaginary cross-sections. This will enable us to discuss the feasibility of an accurate experiment with a screen of small size.

2. – Recurrent integro-differential equations for the series expansion of the current.

The screen Σ will be assumed to be an infinitely thin disc of radius r, which has infinite conductivity parallel to a given direction and is perfectly insulating in the perpendicular direction.

We shall refer to a rectangular system of co-ordinates x, y, z with x in the direction of conductivity and z coincident with the axis of the disc. The unit vectors in the directions of x, y, z will be denoted by $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ respectively.

Let the incident wave have elliptic polarization and travel in a direction $\boldsymbol{s} = \boldsymbol{j}\sin\varphi + \boldsymbol{k}\cos\varphi$, which makes an angle φ with z and is parallel to the

([3]) G. Toraldo di Francia: *Nuovo Cimento*, **3**, 1276 (1956).
([4]) G. Toraldo di Francia: *Boll. Un. Mat. It.*, **11**, 332 (1956).

2011

plane yz. The wave may be split into two linearly polarized waves, one with the electric field parallel to x, and one with the electric field perpendicular to x. The second wave will not be scattered by Σ and, for the time being, we can dispense with its consideration. The first wave will be assumed to have the form

$$\boldsymbol{E}^i(P) = \boldsymbol{i} \exp[ik\boldsymbol{s}\cdot(P-O)] = \boldsymbol{i} \exp[ik(y \sin\varphi + z\cos\varphi)] , \tag{1}$$

where O represents the origin, $P(x, y)$ a variable point, $\boldsymbol{E}^i$ the electric field and $k = 2\pi/wavelength$. The electric field has unit intensity. The time factor $\exp[-i\omega t]$ will be understood.

It has been shown ([5]) that the induced current density $I(Q)$ (which, of course, has the direction of x) is a solution of the following integro-differential equation

$$\left(\frac{\partial^2}{\partial x_P^2} + k^2\right) \iint_{\Sigma} I(Q) G(P, Q)\, \mathrm{d}\Sigma_Q = \frac{ik}{Z} E_x^i(P) , \tag{2}$$

where *both P and Q are points of Σ*, Z is the intrinsic impedance of empty space ([6]) and

$$G(P, Q) = \frac{\exp[ik|P-Q|]}{4\pi|P-Q|} , \tag{3}$$

is the free-space Green function.

As is customary for integral equations of the first kind ([7]), we will attempt to solve (2) by expanding $I(Q)$ as a series of a convenient set of functions. This will be done in two successive steps.

First, since the wavelength will be assumed to be large compared to the other linear dimensions involved (radius of the disc), it is natural to put

$$I(Q) = I_0(Q) + ikI_1(Q) + (ik)^2 I_2(Q) + \dots , \tag{4}$$

where $I_0(Q)$, $I_1(Q)$, $I_2(Q)$ ... are independent of k. We will assume that this expansion is possible and is uniformly convergent in the domain Σ for a non-vanishing interval $0 \leqslant k \leqslant k_1$.

([5]) G. TORALDO DI FRANCIA: *Rend. Acc. Naz. Linc.*, **21**, 86 (1956).

([6]) The rationalized MKSA system will be employed throughout.

([7]) See for instance: P. M. MORSE and H. FESHBACH: *Methods of Theoretical Physics* (New York, 1953), vol. I, p. 925.

2012

For $z_P = 0$, we will put

$$\frac{4\pi ik}{Z} E_x^i(P) = ika_1 + (ik)^2 a_2 y_P + (ik)^3 a_3 y_P^2 + \dots , \tag{5}$$

where, by (1), the coefficients are found to be

$$a_n = \frac{4\pi}{Z} \frac{\sin^{n-1}\varphi}{(n-1)!} . \tag{6}$$

By substituting (4) and (5) into (2), by expanding $G(P, Q)$ as a power series of $ik|P-Q|$ and equating the coefficients of equal powers of ik on both sides, we find

$$\frac{\partial^2}{\partial x_P^2} \iint_\Sigma \frac{I_m(Q)}{|P-Q|} \, d\Sigma_Q = \tag{7}$$

$$= a_m y_P^{m-1} + \sum_0^{m-2} \left\{ \frac{1}{(m-n-2)!} \iint_\Sigma I_n(Q) |P-Q|^{m-n-3} \, d\Sigma_Q - \right.$$

$$\left. - \frac{1}{(m-n)!} \frac{\partial^2}{\partial x_P^2} \iint_\Sigma I_n(Q) |P-Q|^{m-n-1} \, d\Sigma_Q \right\} ,$$

where $a_0 = 0$ and the last sum is to be replaced by zero for $m < 2$.

Equation (7) represents a recurrent integro-differential equation for $I_m(Q)$, since the right side contains only the $I_n(Q)$'s with $n \leqslant m-2$, in addition to known quantities.

3. – Solution of the integro-differential equations for the first terms of the series expansion.

In order to solve the successive equations which are obtained from (7), we will once more apply the procedure of expressing the solution as a sum of a convenient set of functions. Before doing this, it is useful to discuss the behavior of the current $I(Q)$ at the rim of the screen Σ.

A boundary condition which seems to have general validity for an infinitely thin screen with ordinary (omnidirectional) conductivity is that the component of the current normal to the rim should vanish as $\sqrt{\varrho}$, where ϱ denotes the distance from the rim ([8-10]).

([8]) A. W. MAUE: *Zeits. f. Phys.*, **126**, 601 (1949).

([9]) D. S. JONES: *Quart. Journ. Mech.*, **3**, 420 (1950); **5**, 363 (1952); *Proc. Lond. Math. Soc.*, **2**, 440 (1952).

([10]) C. J. BOUWKAMP: *Philips Res. Rep.*, **5**, 401 (1950).

In our case, this seems to indicate that $I(Q)$ should vanish as $\sqrt{\varrho}$ at all points of the rim where the tangent is not parallel to the x-axis. At the same conclusion one can arrive by considering the scattering by a small ellipsoid with unidirectional conductivity [4] and making the ellipsoid to degenerate into a thin disc.

Tentatively, we will assume that for any given value of k in the interval $0 \leqslant k \leqslant k_1$, the current $I(Q)$ may be expressed in the form $I(Q) = F(Q) \cdot \sqrt{r^2 - (Q-O)^2}$, where $F(Q)$ is a function which is regular in the domain Σ including the rim, and can be expanded into a power series of x_Q, y_Q. Precisely, we will assume that the coefficients $I_m(Q)$ of (4) may be expressed in the form

$$I_m(Q) = \sum_{s+p+q=m-1} A_{spq} r^s x_Q^p y_Q^q \sqrt{r^2 - (Q-O)^2} \tag{8}$$

where the coefficients have the dimensions of a surface density of current and s, p, q are positive integers or zero. Position (8) satisfies the requirement that $I_m(Q)$ be homogeneous of order m with respect to the lengths r, x_Q, y_Q, as is necessary on account of (4). However, it will be emphasized that no better justification of position (8) is offered than the ultimate success of the procedure.

Upon substitution of (8) into (7), we obtain after some rearrangements

$$\sum_{s+p+q=m-1} A_{spq} r^s \frac{\partial^2}{\partial x_P^2} J_{pq0}(P) = \tag{9}$$

$$= a_m y_P^{m-1} + \sum_{u+v+w \leq m-3} A_{uvw} r^u \left[\frac{1}{(t-2)!} J_{vw(t-2)}(P) - \frac{1}{t!} \frac{\partial^2}{\partial x_P^2} J_{vwt}(P) \right],$$

where

$$t = m - u - v - w - 1 \tag{10}$$

and

$$J_{pqt}(P) = \iint_{\Sigma} x_Q^p y_Q^q |P-Q|^{t-1} \sqrt{r^2 - (Q-O)^2} \, \mathrm{d}\Sigma_Q . \tag{11}$$

The evaluation of the integrals $J_{pqt}(P)$ may be carried out as shown in the Appendix. It turns out that $J_{pqt}(P)$ is a polynomial of degree $p+q+t+2$ in r, x_P, y_P. Therefore, by substituting into (9) and equating the coefficients of equal powers, one obtains a recurrent system of linear equations for the coefficients A_{spq}. We do not want to enter into a general discussion about the solutions of these successive systems, which would be very involved. We will limit ourselves to saying that the solutions of the systems corresponding

2014

to the first few values of m have actually been determined and turn out to be unique.

Before presenting the results, we want to remind that our aim is the evaluation of the angular momentum absorbed by the screen. It will turn out that the terms in the expressions of $I_6(Q)$ and $I_8(Q)$ which are of odd degree in y_Q and the terms of the expression of $I_7(Q)$ which are of even degree in y_Q are not needed for our approximation. Accordingly, we have dispensed with their evaluation. Incidentally, we note that, on account of the symmetry of the problem, $I(Q)$ is an even function of x_Q.

We have obtained the following results:

$$I_0(Q) = 0\,, \tag{12}$$

$$I_1(Q) = -\frac{2}{\pi^2}\, a_1 \sqrt{r^2 - (Q-O)^2}\,, \tag{13}$$

$$I_2(Q) = -\frac{8}{3\pi^2}\, a_2 y_Q \sqrt{r^2 - (Q-O)^2}\,, \tag{14}$$

$$I_3(Q) = \frac{1}{45\pi^2}\left[(68a_1 + 14a_3)r^2 - (11a_1 + 8a_3)x_Q^2 - (59a_1 + 152a_3)y_Q^2\right] \sqrt{r^2 - (Q-O)^2}\,, \tag{15}$$

$$I_4(Q) = \left\{\frac{16}{9\pi^2}\, a_1 r^3 + \frac{4}{315\pi^2}\, y_Q\left[(130a_2 + 54a_4)r^2 - (33a_2 + 36a_4)x_Q^2 - (129a_2 + 324a_4)y_Q^2\right]\right\} \sqrt{r^2 - (Q-O)^2}\,, \tag{16}$$

$$I_5(Q) = \frac{1}{9\cdot 15\cdot 105\pi^2}\Bigg[(834a_5 - 910a_3 - 12\,802a_1)r^4 + (-528a_5 + 1\,435a_3 + 3\,679a_1)r^2 x_Q^2 + (15\,312a_5 + 29\,323a_3 + 28\,255a_1)r^2 y_Q^2 + 12\left(12a_5 - 11a_3 - \frac{203}{16}\, a_1\right) x_Q^4 - 144\left(78a_5 + 61a_3 + \frac{841}{32}\, a_1\right) x_Q^2 y_Q - 12\left(5\,748a_5 + 2\,321a_3 + \frac{15\,083}{16}\, a_1\right) y_Q^4\Bigg] \sqrt{r^2 - (Q-O)^2}\,, \tag{17}$$

$$I_6(Q) = \frac{8}{27\pi^3}\left[\left(\frac{6}{5}\, a_3 - \frac{196}{25}\, a_1\right) r^5 + a_1 r^3 x_Q^2 + \frac{29}{5}\, a_1 r^3 y_Q^2 + \text{odd terms in } y_Q\right] \sqrt{r^2 - (Q-O)^2}\,, \tag{18}$$

$$I_7(Q) = \left[-\frac{512}{2\,025\pi^3}\, r^5 a_2 y_Q + \text{even terms in } y_Q\right] \sqrt{r^2 - (Q-O)^2}\,, \tag{19}$$

2015

$$(20)\qquad I_8(Q)=\frac{1}{105\pi^3}\left[\left(16a_5-\frac{368}{9}a_3+\frac{576\,544}{2\,835}a_1\right)r^7+\left(\frac{56}{9}a_3-\frac{17\,128}{405}a_1\right)r^5x_Q^2+\right.$$

$$+\frac{8}{45}\left(203a_3-\frac{15\,941}{9}a_1\right)r^5y_Q^2+\frac{2}{135}a_1r^3(97x_Q^4+2\,514x_Q^2y_Q^2+7\,537y_Q^4)+$$

$$\left.+\text{ odd terms in }y_Q\right]\sqrt{r^2-(Q-O)^2}\,.$$

Strictly speaking, we can only say that, if the series expansion (4) exists and is uniformly convergent in a given interval $0\leqslant k\leqslant k_1$, and the coefficients can be expressed in the form (8), then the first coefficients are given by the expressions (12)–(20). It remains an open question whether the series (4) converges at all. However, the first terms found should at least represent a good approximation to the exact solution, when k is sufficiently small. In other words, they should represent the first terms of an asymptotic expansion.

The lengthy and tedious calculations which lead to the formulas (12)–(20) have been repeated several times by different persons, and one may be fairly confident that the results given are correct.

4. – Angular momentum absorbed by the screen.

Let us now consider the whole wave with elliptic polarization. To do this, we have to introduce, beside the wave (1), a second wave whose electric field is in the direction $\boldsymbol{s}\wedge\boldsymbol{i}$, i.e. perpendicular bo both the direction of propagation and the direction of conductivity. Precisely, the total field will now be expressed by

$$(21)\qquad \boldsymbol{E}^i(P)=(\boldsymbol{i}+ie\,\boldsymbol{s}\wedge\boldsymbol{i})\exp\left[ik\,\boldsymbol{s}\cdot(P-O)\right],$$

where e is a complex quantity which determines the shape of the elliptic polarization. By applying some standard formulas ([11]), it can be proved that the polarization is linear if $\mathrm{Re}\,e=0$ and is circular if $e=\pm 1$. It can also be shown that, if the polarization differs only very little from circular polarization, so that we can put $e=\pm(1+\varepsilon)$, with $|\varepsilon|$ very small, the ratio $a/b=1+\alpha$ of the axes of the ellipse differs from unity by a very small amount α and, apart from higher order infinitesimals, $|\alpha|=|\varepsilon|$. We shall call α the *ellipticity* of the polarization.

([11]) See, for instance: G. TORALDO DI FRANCIA: *Electromagnetic Waves* (New York, 1956), p. 152.

2016

If we define the *complex scattering cross section* σ of Σ by

$$\sigma = \frac{Z}{1 + ee^*} \iint_{\Sigma} I^*(Q) \exp\left[iks' \cdot (Q - O)\right] \mathrm{d}\Sigma_Q \,, \tag{22}$$

it can be shown ([12]) that the ordinary scattering cross-section $\bar{\sigma}$ of Σ for the wave (21) is equal to $\mathrm{Re}\,\sigma$ and that the average torque M about s experienced by the screen is expressed by

$$M = \frac{S}{\omega} \mathrm{Re}\,(e\sigma) \,, \tag{23}$$

where S represents the average Poynting vector of the wave (21).

It is seen from (23) that when the polarization is circular, ($e = \pm 1$), there results

$$|M| = \frac{S}{\omega} \bar{\sigma} \,. \tag{24}$$

Since the (non-orbital) angular momentum carried by the wave per unit time per unit surface is S/ω, we see that in this case *the angular momentum cross-section equals the ordinary scattering cross section.*

However, if the polarization is not circular, it is seen by (23) that, generally speaking, also the imaginary part of σ appears in the expression of M. If M_0 denotes the torque corresponding to circular polarization and M_α the torque corresponding to a small ellipticity α for one and the same value of S, the quantity $(M_\alpha - M_0)/M_0$ will be termed the relative error. It is readily found by (23) that the maximum absolute value of the relative error is given by

$$\left| \frac{M_\alpha - M_0}{M_0} \right|_{\max} = |\alpha| \sqrt{1 + \left(\frac{\mathrm{Im}\,\sigma}{\mathrm{Re}\,\sigma}\right)^2} \,. \tag{25}$$

Therefore, in order to determine the error which may be introduced in the measurement of the spin by an inaccurate production of circular polarization, we have also to know the imaginary part of the complex scattering cross-section σ. The error can be quite substantial, even for a very small $|\alpha|$, when $\mathrm{Im}\,\sigma$ is large compared to $\mathrm{Re}\,\sigma$. We shall see that this actually occurs when r is very small compared to the wavelength.

([12]) See ref. ([1]).

2017

5. – The first terms of the expansion of the complex scattering cross section.

By substituting (12)–(20) into (22), by expanding the exponential of (22) in a power series of $\boldsymbol{s}\cdot(Q-O)=iky_0\sin\varphi$ and remembering (6), it is found that the first three terms of the expression of the ordinary scattering cross-section $\bar{\sigma}$ are given by

$$(26)\qquad \bar{\sigma}=\operatorname{Re}\sigma=\frac{2}{1+ee^*}\,\frac{64}{27\pi}\,k^4r^6\left[1+\left(\frac{27}{25}-\frac{1}{5}\sin^2\varphi\right)(kr)^2+\right.$$
$$\left.+\left(\frac{4\,682}{6\,125}-\frac{1\,144}{7\,875}\sin^2\varphi+\frac{3}{175}\sin^4\varphi\right)(kr)^4+\ldots\right],$$

while the first three terms of the imaginary cross-section are given by

$$(27)\qquad \operatorname{Im}\sigma=\frac{2}{1+ee^*}\,\frac{8}{3}\,kr^3\left[1+\frac{1}{5}\left(3+\frac{1}{3}\sin^2\varphi\right)(kr)^2+\right.$$
$$\left.+\frac{1}{525}\left(139-\frac{4}{3}\sin^2\varphi-\sin^4\varphi\right)(kr)^4+\ldots\right].$$

In the particular case of normal incidence ($\varphi=0$) and linear polarization parallel to x ($e=0$), we have

$$(28)\qquad \bar{\sigma}=\frac{128}{27\pi}\,k^4r^6\left[1+\frac{27}{25}(kr)^2+\frac{4\,682}{6\,125}(kr)^4+\ldots\right],$$

$$(29)\qquad \operatorname{Im}\sigma=\frac{16}{3}\,kr^3\left[1+\frac{3}{5}(kr)^2+\frac{139}{525}(kr)^4+\ldots\right].$$

It is interesting to compare (28) with the expression

$$(30)\qquad \bar{\sigma}=\frac{16}{27\pi}\,k^4r^6\left[1+\frac{8}{25}(kr)^2+\frac{311}{6\,125}(kr)^4+\ldots\right],$$

which is valid for the scattering at normal incidence of an acoustic wave by a rigid disc ([13,14]), and with the expression

$$(31)\qquad \bar{\sigma}=\frac{128}{27\pi}\,k^4r^6\left[1+\frac{22}{25}(kr)^2+\frac{7\,312}{18\,375}(kr)^4+\ldots\right],$$

([13]) C. J. Bouwkamp: *Phys. Rev.*, **75**, 1608 (1949); *Physica*, **26**, 1 (1950).
([14]) H. Levine and J. Schwinger: *Phys. Rev.*, **74**, 958 (1948).

2018

which is valid for the scattering at normal incidence of an electromagnetic wave by a disc with ordinary conductivity ([15]). This comparison seems to suggest that when kr is small, the acoustic case is the most favourable for the representation of $\bar{\sigma}$ by the first few terms of the expansion, while the case treated in the present paper is the least favourable.

The three cases are shown in Fig. 1 where the ratio of the scattering cross-section to the geometric cross-section $\sigma_g = \pi r^2$ is plotted against kr.

In the case of grazing incidence ($\varphi = \pi/2$) and linear polarization parallel to x ($e = 0$), the cross-sections become

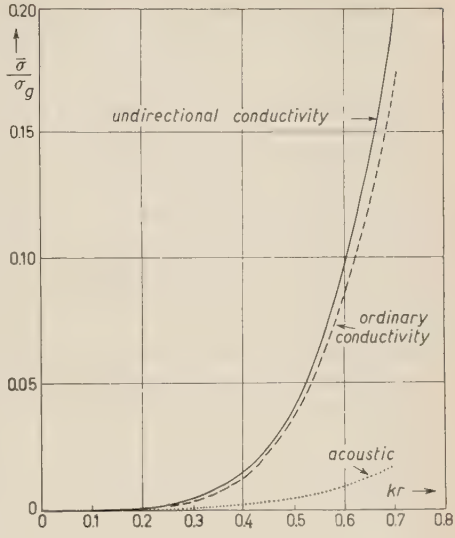

Fig. 1.

$$\bar{\sigma} = \frac{128}{27\pi} k^4 r^6 \left[1 + \frac{22}{25}(kr)^2 + \frac{1\,403}{2\,205}(kr)^4 + \ldots\right], \tag{32}$$

$$\operatorname{Im}\sigma = \frac{16}{3} kr^3 \left[1 + \frac{2}{3}(kr)^2 + \frac{82}{315}(kr)^4 + \ldots\right]. \tag{33}$$

Upon comparison of (32) and (28), we see that in the case of grazing incidence the scattering cross-section increases less rapidly than in the case of normal incidence.

Whatever the angle of incidence, when kr is small it is found $\operatorname{Im}\sigma/\bar{\sigma} = 9\pi/(2kr)^3$, apart from terms of smaller order. By substituting into (25), we can fix an upper limit for the values of $|\alpha|$ which are allowed if we do not want to exceed a certain maximum value for the relative error. It is found

$$|\alpha| < \frac{(2kr)^3}{9\pi} \left| \frac{M_\alpha - M_0}{M_0} \right|_{\max}. \tag{34}$$

For example, suppose that we want to make a measurement of the spin with $kr = \frac{1}{2}$ and with 1% accuracy. There follows from (34) that the ellipticity should not exceed .04%. This would be a very exacting requirement.

([15]) C. J. Bouwkamp: *Rep. Progr. Phys.*, **17**, 35 (1954).

20119

6. – Evaluation of the scattered field on the screen.

It is of interest to evaluate the field generated by the induced current, namely the scattered field, on the disc Σ. Especially the components E_y and H_z, which in our case are not necessarily equal and opposite to those of the incident field, are likely to give us some insight as to the different behavior of a screen with unidirectional conductivity, compared whith the behaviour of a screen with omnidirectional conductivity.

The incident wave will be assumed to have the form (1). By constructing the vector potential generated by the current $I(Q)$ and applying standard formulas, we find for the components E_x and E_y of the scattered field

$$E_x(P) = -\frac{Z}{ik}\left(k^2 + \frac{\partial^2}{\partial x_P^2}\right)\iint_{\Sigma} I(Q)G(P,Q)\,\mathrm{d}\Sigma_Q\,, \tag{35}$$

$$E_y(P) = -\frac{Z}{ik}\,\frac{\partial^2}{\partial x_P \partial y_P}\iint_{\Sigma} I(Q)\,G(P,Q)\,\mathrm{d}\Sigma_Q\,. \tag{36}$$

If $I(Q)$ represents the exact solution of the integro-differential equation (2), we obviously get from (35) $E_x(P) = -E_x^i(P)$. Now, equation (35) can be interpreted as a differential equation for the integral on the right side. Since $I(Q)$ is an even function of x_Q, it is readily proved that the integral is an even function of x_P. On the other hand, $E_x^i(P)$ depends solely on y_P. Therefore, we get upon integration of the differential equation

$$\frac{Z}{ik}\iint_{\Sigma} I(Q)G(P,Q)\,\mathrm{d}\Sigma_Q = \frac{1}{k^2}\,E_x^i(P) + F(y_P)\cos(kx_P)\,, \tag{37}$$

where $F(y_P)$ is an unknown function. By substituting (12)–(20) into (37), by expanding $G(P,Q)$ and $E_x^i(P)$ in power series, and utilizing the integrals $J_{pqt}(P)$ evaluated in the Appendix, one can derive the expression of $F(y_P)$. Its first derivative turns out to be [16]

$$F'(y) = \frac{Z}{4\pi}\left\{\frac{a_2}{ik} + y(a_1 + 2a_3) + ik\left[-\frac{2}{3}\,a_2 r^2 + \frac{3}{2}\,y^2(a_2 + 2a_4)\right] + \right. \tag{38}$$

$$\left. + (ik)^2\left[-\frac{1}{5}\,r^2 y(7a_1 + 6a_3) + \frac{1}{6}\,y^3(5a_1 + 12a_3 + 24a_5)\right] + \right.$$

[16] When no confusion is possible we simply write x, y for x_P, y_P.

2020

$$+ (ik)^3 \left[\frac{1}{315} r^4(130a_2 - 9a_4) - \frac{4}{3\pi} r^3 a_1 y - \frac{1}{7} r^2 y^2 (11a_2 + 12a_4) + \right.$$

$$\left. + \frac{5}{24} y^4 (5a_2 + 12a_4 + 24a_6) \right] + \ldots \Big\},$$

apart from terms of higher order in k.

Now, by substituting (37) into (36), we get

$$E_y = kF'(y) \sin(kx) . \tag{39}$$

By combining (39) with (38), we get the expression of E_y exact with respect to x and approximate up to the term in k^4 with respect to y. In the case of normal incidence ($\varphi = 0$) we obtain by recalling (6)

$$E_y = ky \left[1 + (kr)^2 \left(\frac{7}{5} - \frac{5}{6} \frac{y^2}{r^2} \right) + i \frac{4}{3\pi} (kr)^3 + \ldots \right] \sin(kx) , \tag{40}$$

and in the case of grazing incidence ($\varphi = \pi/2$)

$$E_y = -i \left[1 + 2iky + 2(kr)^2 \left(\frac{1}{3} - \frac{y^2}{r^2} \right) + 2i(kr)^3 \frac{y}{r} \left(1 - \frac{y^2}{r^2} \right) + \right.$$

$$\left. + (kr)^4 \left(\frac{257}{630} - \frac{4}{3} \frac{y}{r} - \frac{13}{7} \frac{y^2}{r^2} + \frac{3}{2} \frac{y^4}{r^4} \right) + \ldots \right] \sin(kx) . \tag{41}$$

It will be noted that E_y is of the order of $(kr)^2$ in the former case and of the order of kr in the latter case.

From the knowledge of $I(0)$ and the equation of continuity we can derive the value of the surface density of charge on Σ, then by a standard relation the value of E_z, which is equal and opposite on both sides of Σ. Precisely, we get

$$E_z = \pm \frac{Z}{2ik} \frac{\partial I}{\partial x} . \tag{42}$$

Upon substitution, we obtain in the case of normal incidence

$$E_z = \pm \frac{4x}{\pi \sqrt{r^2 - x^2 - y^2}} \left[1 + \frac{1}{30} (kr)^2 \left(30 - 11 \frac{x^2}{r^2} - 27 \frac{y^2}{r^2} \right) + i \frac{8}{9\pi} (kr)^3 + \ldots \right] , \tag{43}$$

and in the case of grazing incidence

$$E_z = \pm \frac{4x}{\pi \sqrt{r^2 - x^2 - y^2}} \left[1 + \frac{4}{3} iky + \frac{1}{6} (kr)^2 \left(7 - 3 \frac{x^2}{r^2} - 11 \frac{y^2}{r^2} \right) + \right.$$

$$\left. + i(kr)^3 \left(\frac{8}{9\pi} + \frac{62}{45} \frac{y}{r} - \frac{26}{35} \frac{x^2 y}{r^3} - \frac{58}{35} \frac{y^3}{r^3} \right) + \ldots \right] . \tag{44}$$

In both cases E_z becomes infinite at the rim of Σ.

As to the magnetic field, $H_x = 0$, because the current is in the direction of the x-axis, while $H_y = \mp I(Q)/2$ depending on which side of Σ is considered. Thus, for normal incidence it is found

$$(45) \qquad H_y = \mp \frac{4ik}{\pi Z}\sqrt{r^2 - x^2 - y^2}\cdot \cdot\left[1 + \frac{1}{90}(kr)^2\left(68 - 11\frac{x^2}{r^2} - 59\frac{y^2}{r^2}\right) + \frac{8}{9\pi}i(kr)^3 + \ldots\right],$$

and for grazing incidence

$$(46) \qquad H_y = + \frac{4ik}{\pi Z}\sqrt{r^2 - x^2 - y^2}\left[1 + \frac{4}{3}iky + \frac{1}{90}(kr)^2\left(75 - 15\frac{x^2}{r^2} - 135\frac{y^2}{r^2}\right) - \right. \\ \left. + i(kr)^3\left(\frac{8}{9\pi} + \frac{278}{315}\frac{y}{r} - \frac{26}{105}\frac{yx^2}{r^3} - \frac{122}{105}\frac{y^3}{r^3}\right) + \ldots\right].$$

In all cases, H_y vanishes at the rim of Σ.

Finally, the value of H_z can be derived from (40), (41) by applying the second Maxwell equation. For normal incidence it is found

$$(47) \qquad H_z = -\frac{ik}{Z}y\left[1 + (kr)^2\left(\frac{7}{5} - \frac{5}{6}\frac{y^2}{r^2}\right) + i\frac{4}{3\pi}(kr)^3 + \ldots\right]\cos(kx),$$

and for grazing incidence

$$(48) \qquad H_z = -\frac{1}{Z}\left[1 + 2iky + 2(kr)^2\left(\frac{1}{3} - \frac{y^2}{r^2}\right) + 2i(kr)^3\frac{y}{r}\left(1 - \frac{y^2}{r^2}\right) + \right. \\ \left. + (kr)^4\left(\frac{257}{630} - \frac{4}{3\pi}\frac{y}{r} - \frac{13}{7}\frac{y^2}{r^2} + \frac{3}{2}\frac{y^4}{r^4}\right) + \ldots\right]\cos(kx).$$

Thus, we have completed the evaluation of the scattered field over the screen Σ.

An interesting result which we want to stress is that *the components E_y, H_z of the scattered field are, on Σ, periodic with respect to x with the periodicity equal to the wavelength*. It is rather puzzling to note that this result is readily proved to be valid for any shape of Σ (provided its rim is met not more than twice by any line parallel to the x-axis, and the incident field does not depend on x). In the particular case when Σ is circular (or, more generally, symmetric with respect to the y-axis), the straight lines $x = 0$, $x = \pm\lambda/2$, $x = \pm 2\lambda/2$ etc. are nodal lines for E_y and anti-nodal lines for H_z.

7. – Conclusion.

As a result of our analysis, we may conclude that an accurate measurement of the spin of a circularly polarized wave by means of a small screen Σ is not an easy matter. If kr is very small, the real cross section becomes negligible compared to the imaginary cross section. We have seen that in this case *even an extremely small degree of ellipticity of the polarization may bring about a considerable error in the measurement.* On the other hand, if kr approaches or exceeds unity, the theory developed breaks down, because the series (26) converges only very slowly (if at all). At present, the best way to proceed should consist in making kr very small and repeating the measurement at several different azimuths, so that the effect of ellipticity averages to zero.

The experiment may be carried out by suspending the disc and measuring the deflection produced by the torque. The direction of propagation of the wave is along the vertical, while the plane of the disc may be either horizontal or vertical. Comparison of (26) and (32) seems to suggest that our approximation is a little better in the case of the vertical disc.

In any case, it is desirable that the problem be tackled also from the direction opposite to that of this paper, namely starting from large values of kr and setting up an asymptotic expansion.

Some byproducts of our investigation are worth mentioning. For instance, it turns out that, at least for a given range of values of kr, *the scattering cross section is larger for unidirectional conductivity than for omnidirectional conductivity* (see Fig. 1). This suggests the intriguing problem of determining what electromagnetic properties should be given to Σ for obtaining maximum cross section. Another interesting result is the exact form of the dependence on x of the components E_y, H_z of the scattered field over Σ, expressed by the formulas (40), (41), (47), (48). These components characterize the difference from the case of omnidirectional conductivity.

* * *

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A considerable amount of planning and of organization was necessary to carry out the long and bulky calculations required. For all this work as well as for several constructive suggestions about this paper, the author is indebted to Mrs. L. Abbozzo Ronchi. Thanks are due also to Miss V. Russo

2023

and to Mrs. A. M. Scheggi Verga, who shared the heavy task of carrying out the many integrations and checking several times the results.

Appendix

The integrals $J_{pq}(P)$ of equation (11) may be evaluated as follows.

First we distinguish two cases, according as t is odd or even. If t is odd and we put $t = 2h + 1$, we get

$$(49) \qquad J_{pq}(P) = \iint_{\Sigma} x_Q^p y_Q^q (P-Q)^2 \cdot \sqrt{r^2 - (Q-O)^2}\, \mathrm{d}\Sigma_Q =$$

$$= \iint_{\Sigma} x_Q^p y_Q^q [(P-O)^2 + (Q-O)^2 - 2(P-O)\cdot(Q-O)]^h \sqrt{r^2 - (Q-O)^2}\, \mathrm{d}\Sigma_Q .$$

By introducing polar co-ordinates with $x_Q = \varrho \cos\theta$, $y_Q = \varrho \sin\theta$, we have

$$(50) \qquad J_{pq}(P) = \int_0^{2\pi} \cos^p\theta \sin^q\theta \, \mathrm{d}\theta \cdot$$

$$\cdot \int_0^r \varrho^{p+q+1} [x_P^2 + y_P^2 + \varrho^2 - 2\varrho(x_P \cos\theta + y_P \sin\theta)]^h \sqrt{r^2 - \varrho^2}\, \mathrm{d}\varrho .$$

The integration is straightforward for every value of h. We list some of the results for this case, which have been applied for the solution of the integro-differential equations (7):

$$J_{001} = \frac{2}{3}\pi r^3 ,$$

$$J_{021} = \frac{2}{15}\pi r^5 , \qquad J_{003} = \frac{2}{15}\pi r^3 [5(x^2 + y^2) + 2r^2] ,$$

$$J_{013} = -\frac{4}{15}\pi r^5 y ,$$

$$J_{041} = \frac{2}{35}\pi r^7 , \qquad J_{221} = \frac{2}{105}\pi r^7 ,$$

$$J_{203} = \frac{2}{15}\pi r^5\left[x^2 + y^2 + \frac{4}{7}r^2\right],$$

$$J_{005} = \frac{2}{3}\pi r^3\left[(x^2 + y^2)^2 + \right.$$
$$\left. + \frac{8}{5}r^2(x^2 + y^2) + \frac{8}{35}r^4\right].$$

It turns out in this case that $J_{qpt} = J_{pqt}$.

When t is even, the integration is a little less straightforward. It is expedient to introduce for a moment polar co-ordinates $|Q - P|$, φ with the origin at P ([17]) and to put $\mathrm{d}\Sigma = |Q - P|\,\mathrm{d}\varphi\,|\mathrm{d}Q|$ (Fig. 2), with the understanding that $\mathrm{d}Q$ has the same direction as $Q - P$. By putting $t = 2h$, we may write

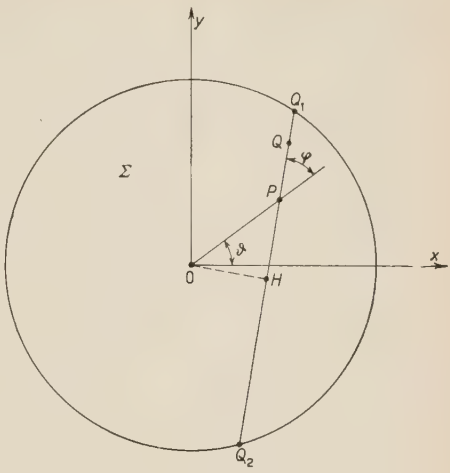

Fig. 2.

$$(51)\qquad J_{pqt}(P) = \int_0^{2\pi}\mathrm{d}\varphi\int_P^{Q_1} x_Q^p y_Q^q (P - Q)^{2h}\sqrt{r^2 - (Q - O)^2}\,|\mathrm{d}Q|\,.$$

We may add to the integral with respect to Q its value for the angle $\varphi + \pi$, obtaining in this way the integral evaluated from Q_2 to Q_1. Accordingly, φ will vary only from 0 to π. Further we note that

$$(52)\quad r^2 - (Q - O)^2 = (Q_1 - O)^2 - (H - O)^2 - (Q - H)^2 = (Q_1 - H)^2 - (Q - H)^2\,.$$

Thus, we obtain

$$(53)\qquad J_{pqt}(P) = \int_0^{\pi}\mathrm{d}\varphi\int_{Q_2}^{Q_1} x_Q^p y_Q^q (P - Q)^{2h}\sqrt{(Q_1 - H)^2 - (Q - H)^2}\,|\mathrm{d}Q|\,.$$

Next, if we put $|P - O| = \varrho$ and denote by u the distance HQ, counted positive upwards and negative downwards, we can write

$$(54)\qquad (P - Q)^2 = (P - H)^2 + (Q - H)^2 - 2(P - H)\cdot(Q - H) =$$
$$= \varrho^2\cos^2\varphi + u^2 - 2u\varrho\cos\varphi\,,$$

and

$$(55)\qquad x_Q = u\cos(\theta + \varphi) + \varrho\sin\varphi\sin(\theta + \varphi)\,,$$

$$(56)\qquad y_Q = u\sin(\theta + \varphi) - \varrho\sin\varphi\cos(\theta + \varphi)\,.$$

([17]) H. A. Bethe: *Phys. Rev.*, **66**, 163 (1944).

20235

Finally, since $(Q_1 - H)^2 = r^2 - \varrho^2 \sin^2\varphi$, the integral (53) becomes

$$(57) \qquad J_{pq}(P) = \int_0^{\pi} d\varphi \int_{-\sqrt{r^2-\varrho^2\sin^2\varphi}}^{\pi+\sqrt{r^2-\varrho^2\sin^2\varphi}} [u\cos(\theta+\varphi) + \varrho\sin\varphi\sin(\theta+\varphi)]^p \cdot$$
$$\cdot [u\sin(\theta+\varphi) - \varrho\sin\varphi\cos(\theta+\varphi)]^q \cdot$$
$$\cdot [\varrho^2\cos^2\varphi + u^2 - 2u\varrho\cos\varphi]^{\frac{1}{2}} \sqrt{r^2 - \varrho^2\sin^2\varphi - u^2}\, du\,.$$

Upon expansion, this transforms into a sum of elementary integrals. Some of the results obtained, which have been applied for the solution of the integro-differential equations (7) are listed below:

$$J_{000} = \frac{\pi^2}{2}\left[r^2 - \frac{1}{2}(x^2+y^2)\right],$$

$$J_{010} = \frac{\pi^2}{4}\,y\left[r^2 - \frac{3}{4}(x^2+y^2)\right],$$

$$J_{020} = \frac{\pi^2}{16}\left[r^4 + \frac{1}{2}r^2(5y^2-x^2) - \frac{1}{8}(19y^4 + 18x^2y^2 - x^4)\right],$$

$$J_{002} = \frac{\pi^2}{8}\left[r^4 + r^2(x^2+y^2) - \frac{1}{8}(x^2+y^2)^2\right],$$

$$J_{030} = \frac{\pi^2}{64}\,y\left[3r^4 + r^2(7y^2-3x^2) + \frac{5}{16}(x^2+y^2)(3x^2-25y^2)\right],$$

$$J_{210} = \frac{\pi^2}{64}\,y\left[r^4 - r^2(y^2-9x^2) + \frac{5}{16}(x^2+y^2)(y^2-27x^2)\right],$$

$$J_{012} = -\frac{\pi^2}{16}\,y\left[r^4 - \frac{1}{2}r^2(x^2+y^2) + \frac{1}{8}(x^2+y^2)^2\right],$$

$$J_{040} = \frac{\pi^2}{128}\left[3r^6 - \frac{3}{2}r^4(x^2-3y^2) + \frac{1}{16}r^2(9x^4 - 102x^2y^2 + 169y^4) - \right.$$
$$\left. - \frac{3}{32}(x^2+y^2)(x^4 - 26x^2y^2 + 141y^4)\right],$$

$$J_{220} = \frac{\pi^2}{128}\left[r^6 + \frac{1}{2}r^4(x^2+y^2) - \frac{1}{16}r^2(17x^4 - 246x^2y^2 + 17y^4) + \right.$$
$$\left. + \frac{1}{32}(x^2+y^2)(13x^4 - 478x^2y^2 + 13y^4)\right],$$

2026

$$J_{022} = \frac{\pi^2}{32}\left[r^6 + \frac{1}{4}\,r^4(3x^2 + y^2) - \frac{1}{8}\,r^2(x^2 + y^2)(x^2 - 3y^2) + \frac{1}{64}\,(x^2+y^2)^2(x^2-9y^2)\right],$$

$$J_{004} = \frac{\pi^2}{16}\left[r^6 + \frac{9}{2}\,r^4(x^2 + y^2) + \frac{9}{8}\,r^2(x^2 + y^2)^2 - \frac{1}{16}\,(x^2 + y^2)^3\right],$$

$$J_{032} = -\frac{\pi^2}{128}\,y\left[3r^6 - \frac{1}{2}\,r^4(3x^2 + y^2) + \frac{1}{16}\,r^2(x^2 + y^2)(9x^2 - 11y^2) - \right.$$
$$\left. - \frac{1}{32}\,(x^2 + y^2)^2(3x^2 - 11y^2)\right],$$

$$J_{212} = -\frac{\pi^2}{128}\,y\left[r^6 + \frac{1}{2}\,r^4(x^2 - y^2) - \frac{1}{16}\,r^2(x^2 + y^2)(17x^2 - 3y^2) + \right.$$
$$\left. + \frac{1}{32}\,(x^2 + y^2)^2(13x^2 - y^2)\right],$$

$$J_{014} = -\frac{3\pi^2}{32}\,y\left[r^6 + \frac{3}{4}\,r^4(x^2 + y^2) - \frac{1}{8}\,r^2(x^2 + y^2)^2 + \frac{1}{64}\,(x^2 + y^2)^3\right].$$

It is to be noted that when t is even, J_{qpt} may be obtained from J_{pqt} by interchanging x and y. Accordingly, we have dispensed with giving the expression of J_{qpt} when J_{pqt} was already given.

RIASSUNTO

Lo spin del campo elettromagnetico può essere rivelato per mezzo del momento di torsione esercitato da micro-onde polarizzate circolarmente su uno schermo capace di condurre la corrente soltanto in una direzione. Ma per effettuare una precisa misura quantitativa si deve conoscere la sezione efficace di assorbimento di momento angolare da parte dello schermo. In questo lavoro vengono valutati alcuni primi termini della sezione efficace nell'ipotesi che lo schermo sia un disco circolare di diametro piccolo rispetto alla lunghezza d'onda. Allo scopo di determinare la precisione con la quale la polarizzazione deve essere circolare, viene valutata anche la parte immaginaria della sezione efficace. Espressioni esplicite vengono date per il campo sullo schermo. Viene discussa la possibilità di effettuare con questa esperienza una misura precisa.

12*

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Evaluation of Nuclear Fusion Probability in Mesic Molecules.

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(ricevuto il 19 Aprile 1957)

Summary. — The binding energies of the mesic hydrogen-like molecular ions have been calculated together with the probability of nuclear fusion in the molecular ground state.

1. – Introduction.

Recently L. W. Alvarez and co-workers [1] put forward the hypothesis that nuclear fusion might be catalyzed through the formation of mesic molecules. In such reactions the following processes may occur:

i) fusion with nucleon emission — e.g. (d, d) reaction,

ii) { radiative fusion - γ emission / conversion fusion - muon rejuvenation } — e.g. (p, d) reaction.

We have studied the reactions (p, d), (p, t) of the second type and (d, d), (d, t), (t, t) of the first type.

[1] L. W. Alvarez, H. Bradner, F. S. Crawford Jr., J. A. Crawford, P. Falk-Vairant, M. L. Good, J. D. Gow, A. H. Rosenfeld, F. Solmitz, M. L. Stevenson, H. K. Ticho and R. D. Tripp: *The Catalysis of Nuclear Reactions by Mesons*. UCRL-3620, December 10, 1956; *Phys. Rev.*, **105**, 1127 (1957).

2028

On the ground of E. TELLER's paper ([2]) on the hydrogen molecular ion and approximating the internuclear potential with a Morse potential, we have calculated the binding energies of the various mesic molecules and evaluated their mean lives for nuclear fusion. This latter calculation was performed in such a way as to make use of the experimental fusion cross-sections.

In the (p, d) case we found a mean life for radiative fusion of the same order as that of the muon. In the other cases the mean lives are much shorter.

2. – The wave function of the system consisting of a μ-meson and two nuclei may be written approximately in the following way:

$$\Phi(R_1, R_2, r_\mu) = \chi(R_1, R_2) U(R_1, R_2, r_\mu) , \tag{1}$$

where χ is the wave function of the nuclei, U, parametrically dependent on the position co-ordinates R_1, R_2, of the nuclei, is the wave function of the μ-meson. The approximation made in (1) consists in neglecting terms of the order $(m/\overline{M})^{\frac{5}{4}}$ in the Schrödinger equation of the system ([3]), m is the mass of the μ-meson, $\overline{M}$ the average mass of the nuclei. In this approximation E. TELLER ([2]) has calculated the internuclear potential for the hydrogen molecular ion which, for the configuration of lowest energy, is sketched in Fig. 1, the Bohr radius and the hydrogen atom ground state energy being taken as units of length and energy. The same curve represents the potential for the cases under consideration by a mere change of scale, that is to say by using the corresponding units of the mesic atom ($\hbar^2/me^2 = 2.55 \cdot 10^{-11}$ cm, $me^4/2\hbar^2 = 2.81$ keV), which we will do throughout the following.

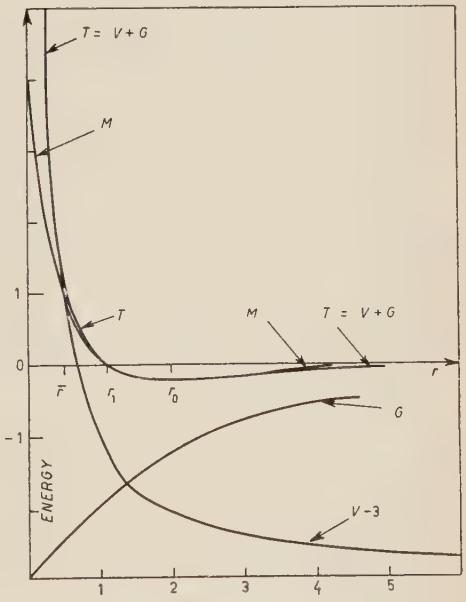

Fig. 1. – Sketch of the potentials involved in the calculations.

([2]) E. TELLER: *Zeits. f. Phys.*, **61**, 438 (1930).
([3]) M. BORN and R. OPPENHEIMER: *Ann. der Phys.*, **84**, 457 (1927).
([4]) P. MORSE: *Phys. Rev.*, **34**, 57 (1929).

The molecular potential may be approximated by a Morse potential

$$M(r) = D\{\exp[-2a(r-r_0)] - 2\exp[-a(r-r_0)]\}\,, \tag{2}$$

with $r_0 = 2$, $D = 0.204$, $a = 0.796$.

The corresponding energy levels are given by the formula:

$$E_n = -D\,\frac{b_n^2}{4d^2}\,, \tag{3}$$

where, in our units, $d = (DM_r/m)^{\frac{1}{2}}(1/a)$; $b_n = 2d - 1 - 2n$, and M_r is the reduced mass of the nuclei.

The condition for the existence of at least one bound state is $2d > 1$ and is satisfied in all the cases examined. In Table I the binding energies of the ground states are given.

The condition for the existence of a second vibrational state, again for $l = 0$, is $2d > 3$. It is satisfied only in the last three cases, the corresponding binding energies being very small. Very loosely bound rotational states ($l = 1$) are also possible.

3. – Considering processes of type i) and ii) radiative, the probability of fusion per unit time in the molecular ground state is given by the usual perturbation theory formula:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\,\frac{1}{N_{i'}}\sum_i\sum_f |H_{if}|^2 \varrho\,, \tag{4}$$

where i' is the ground state of the molecule, $N_{i'}$ its spin multiplicity. Similarly the fusion cross-section for the free collision is:

$$\sigma = v^{-1}\,\frac{2\pi}{\hbar}\,\frac{1}{N_i}\sum_i\sum_f |H_{if}|^2 \varrho_f\,, \tag{5}$$

where v is the relative velocity, i corresponds to an incident wave of unit amplitude, f indicates the final states which are the same as those appearing in (4).

We now remark (Fig. 1) that the internuclear potential $T(r)$ is the sum of a Coulomb potential $V(r) = .2/r$ and an attractive potential $G(r)$ which has the value -3 in the origin. Then, for very small r, the equation of motion for the nuclei bounded in the molecular ground state is the same as that for the free «equivalent» collision with relative energy $E_{eq} = 3 + E_0 \sim$ ~ 8 keV and zero angular momentum ($E_0 \sim -0.2$ keV being the binding energy of the molecule).

2030

In such conditions the corresponding wave functions differ only in amplitude near the origin, and, as at the low energies involved only S waves contribute, we may write:

$$H_{if} = AH_{if}^{\cdot}, \tag{6}$$

where A is the amplitude ratio at the origin of the molecular wave function and the normalized Coulomb function which enters into the matrix element of (5).

When the molecule has two identical nuclei some spin states have to be excluded for symmetry reasons so that $N_{i'}$ will be different from N_i.

We may finally write:

$$\frac{1}{\tau} = \frac{N_i}{N_{i'}} |A|^2 v\sigma . \tag{7}$$

4. – To perform the computations we have approximated the molecular potential $T(r)$ (Fig. 1) with $T'(r)$ defined as follows:

$$T'(r) = \begin{cases} V(r) - 3 & r < \bar{r} , \\ M(r) & r > \bar{r} , \end{cases} \tag{8}$$

where $\bar{r}$ is given by $M(\bar{r}) = V(\bar{r}) - 3$.

An approximate solution for the T' potential may be obtained by matching at $\bar{r}$ only the value of the Morse wave function ψ with that of the S component φ_c^S of a properly normalized Coulomb function (*), which gives:

$$\psi(\bar{r}) = A\varphi_c^S(\bar{r}); \qquad \psi(r) = \frac{1}{r}\left(\frac{a}{4\pi\Gamma(2d-1)}\right)^{\frac{1}{2}} (2d)^{d-\frac{1}{2}} \cdot \exp\left[-d(u+e^u) + \frac{u}{2}\right], \tag{9}$$

with: $u = a(r - r_0)$.

The resulting mismatch in the logarithmic derivatives is about 20% in the (p, d) case, and gives an idea of the error thus introduced. The A values obtained may then be corrected to take into account the difference $T - T'$. Simple considerations of barrier transparency coefficients lead to the correction factor

$$R = \exp\left[-\sqrt{\frac{M_r}{m}} \int_0^{r_1} (\sqrt{T - E_0} - \sqrt{T' - E_0})\, dr\right], \tag{10}$$

r_1, being given by $M(r_1) = 0$.

(*) The Coulomb function is normalized to unit amplitude at infinity. Tables of this function are given by: J. Bloch, M. H. Hull jr., A. A. Boyles, W. G. Bouricius, B. E. Freeman and G. Breit: *Rev. Mod. Phys.*, **23**, 147 (1951).

2021

TABLE I.

	Binding Energy of Mol. Ground State (keV)	Equivalent Collision Energy in Lab. Syst. (keV)	Relative Equivalent Velocity (cm/s)	A ($cm^{-\frac{3}{2}}$)	R	$N_i/N_{i'}$	$\frac{N_i'/N_i}{\lvert A\rvert^2 R\, v}$ ($cm^3\,s$)	σ (cm^2)	τ (s)
(p, d)	0.23	12.3	$1.5\cdot10^8$	$1.51\cdot10^{15}$	0.78	1	$4.69\cdot10^{-39}$	$1.25\cdot10^{-33}$	$3.8\cdot10^{-6}$
(p, t)	0.25	10.9	$1.4\cdot10^8$	$1.52\cdot10^{15}$	0.77	1	$4.95\cdot10^{-39}$	—	—
(d, d)	0.28	16.3	$1.2\cdot10^8$	$1.55\cdot10^{15}$	0.74	9/6	$4.03\cdot10^{-39}$	$2.1\cdot10^{-28}$	$1.9\cdot10^{-11}$
(d, t)	0.30	13.5	$1.1\cdot10^8$	$1.52\cdot10^{15}$	0.72	1	$7.23\cdot10^{-39}$	$0.9\cdot10^{-26}$	$8.0\cdot10^{-13}$
(t, t)	0.33	16.2	$1.0\cdot10^8$	$1.45\cdot10^{15}$	0.69	4/1	$2.43\cdot10^{-39}$	—	—

5. – The numerical results are given in Table I (*). The headlines are: the binding energy in the molecular ground state, the energy in the laboratory system corresponding to E_{eq}, the equivalent collision velocity v, the amplitude factor A, the correction factor R, the statistical coefficient $N_i/N_{i'}$, the fusion cross-section at the proper energies, and finally the mean life. In the (p, d) and (p, t) cases the computed $1/\tau$ values are the partial probabilities for purely radiative fusion, the actual mean life will be further shortened due to the competitive process of conversion fusion. An order of magnitude evaluation of the conversion coefficient (for $E2$ radiation) gives $\alpha(E2) \sim 0.25$.

* * *

We are indebted to prof. P CALDIROLA and prof. B. FERRETTI for useful discussions on this subject.

(*) The values of the fusion cross-sections are taken from the bibliography given by RICHARD F. POST: *Rev. Mod. Phys.*, **28**, 338 (1956).

Note added in proof. – While this work was in print the paper by J. D. JACKSON came to our notice (*Phys. Rev.*, **106**, 330 (1957)). The paper gives an extensive theoretical treatment of meson catalyzed reactions. The method followed by the author for calculating the mean life for nuclear fusion in mesic molecular ions differs from ours. The results are however in agreement.

RIASSUNTO

Si sono calcolate le energie di legame degli ioni molecolari mesici idrogenidi, ed è stata valutata la probabilità di fusione nucleare nello stato fondamentale della molecola.

Contribution to Elastic Scattering from Inelastic Processes: Application to High-Energy Electron Scattering from Hydrogen and Deuterium (*).

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(ricevuto il 19 Aprile 1957)

Summary. — An expression is derived in first Born approximation for the correction to high-energy elastic electron-nucleus scattering arising from the existence of non-radiative inelastic processes. The magnitude of this correction is estimated for electron-proton and electron-deuteron scattering and is shown to be a very small but rapidly increasing function of energy over the range of incident electron energies from 150 to 450 MeV. The correction to electron-deuteron scattering also shows a marked angular dependence increasing with increasing scattering angle.

1. – Introduction.

Inelastic scattering of electrons from nuclei can occur in many different ways, some of which are: 1) Excitation to bound nuclear levels; 2) Excitation to unbound nuclear levels (electro-disintegration); 3) π-meson production; 4) Inverse β-decay; 5) Radiative effects: Bremsstrahlung or electron-positron pair production. Of these possibilities, we shall confine our attention to 1) through 3). This choice is justified by the fact that the inverse β-decay process has an exceedingly small cross section of the order of 10^{-44} cm^2, while the lowest order radiative (Bremsstrahlung) corrections can be made by the usual Schwinger formula ([1-3]).

(*) Supported in part by a joint program of the O.S.R. and O.N.R.

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(1) J. Schwinger: *Phys. Rev.*, **75**, 898 (1949).

(2) H. Suura: *Phys. Rev.*, **99**, 1020 (1955).

(3) D. R. Yennie and H. Suura: *Phys. Rev.*, **105**, 1378 (1957).

It is evident that the existence of any of the inelastic modes of scattering listed above will alter the corresponding elastic cross section. In most cases these inelastic processes involve corrections which are quite small and may be safely neglected; however, with the increasing accuracy of the high-energy electron scattering experiments now underway, it is of some interest to derive an expression for such corrections and make rough estimates in the particular examples of scattering from hydrogen and deuterium.

Since neither the proton nor the deuteron possesses a bound excited state, only 2) and 3) will give rise to corrections.

The entire treatment is phenomenological in that the values of these corrections are obtained not from fundamental concepts (this would involve a meson field theoretic approach which is hardly justified considering the smallness of the corrections) but from comparison with the results of experiments by means of a formula expressing the correction to the differential elastic cross section in terms of the total inelastic cross section.

2. – Inelastic corrections to the elastic cross section.

As in the theory of nuclear reactions, a scattering process in which the incident particle fails to be re-emitted into the entrance channel can be described phenomenologically by the addition of a suitable imaginary term to the usual interaction Hamiltonian governing the elastic process. Applying such a formalism to high-energy electron scattering, we can write the interaction Hamiltonian between the electron and the target nucleus as

$$\mathcal{H} = \mathcal{H}_{\mathrm{el}} + i\mathcal{H}_{\mathrm{in}} , \tag{1}$$

where $\mathcal{H}_{\mathrm{el}}$ and $\mathcal{H}_{\mathrm{in}}$ determine the elastic and inelastic components of the scattering, respectively. The electron wave function associated with an elastic scattering state of this Hamiltonian can then be written in first Born approximation as ($|\boldsymbol{k}_i| = |\boldsymbol{k}_f| = k$)

$$\Psi = \exp[i\boldsymbol{k}_i \cdot \boldsymbol{r}]u_i - \frac{\exp[ikr]}{r}\,\frac{(\boldsymbol{\alpha}\cdot\boldsymbol{k}_f + k)}{4\pi\hbar c}\int \exp[i\boldsymbol{q}\cdot\boldsymbol{r}']\,\mathcal{H}u_i(\mathrm{d}\boldsymbol{r}') , \tag{2}$$

where $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ are the initial and final electron propagation vectors in the c.m. system; $\hbar\boldsymbol{q} = \hbar(\boldsymbol{k}_i - \boldsymbol{k}_f)$ is the momentum transfer; and u_i is the Dirac spinor for the initial state. The co-ordinate $\boldsymbol{r}$ is the position vector of the electron relative to the center of mass of the nucleus. The scattered amplitude, from Eq. (2), is given by

$$f(\boldsymbol{q}) = ((u_f^+, \mathcal{F}(q)) = -\left(u_f^+, \frac{\boldsymbol{\alpha}\cdot\boldsymbol{k}_f + k}{4\pi\hbar c}\int \exp[i\boldsymbol{q}\cdot\boldsymbol{r}']\,\mathcal{H}u_i(\mathrm{d}\boldsymbol{r}')\right) . \tag{3}$$

The differential elastic cross section can be written down directly in terms of Eq. (3) as [4]

$$\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega = |f(\boldsymbol{q})|^2 = (k/2\pi\hbar c)^2 \cos^2(\theta/2)\,|(f|\mathcal{H}|i)|^2 \,. \tag{4}$$

where we have averaged over initial and summed over final spin states. Here $\cos\theta = \boldsymbol{k}_i \cdot \boldsymbol{k}_f / k^2$ and

$$(f|\mathcal{H}|i) = \int \exp[i\boldsymbol{q}\cdot\boldsymbol{r}']\,\mathcal{H}(\mathrm{d}\boldsymbol{r}') = \tag{5}$$

$$= \int \exp[i\boldsymbol{q}\cdot\boldsymbol{r}']\,\mathcal{H}_{\mathrm{el}}(\mathrm{d}\boldsymbol{r}') + i\int \exp[i\boldsymbol{q}\cdot\boldsymbol{r}']\,\mathcal{H}_{\mathrm{in}}(\mathrm{d}\boldsymbol{r}') \,.$$

Both integrals of Eq. (5) are real in first Born approximation, so that Eq. (4) becomes

$$\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega = (k/2\pi\hbar c)^2 \cos^2(\theta/2)[\,|(f|\mathcal{H}_{\mathrm{el}}|i)|^2 + |(f|\mathcal{H}_{\mathrm{in}}|i)|^2] \,. \tag{6}$$

Eq. (6) may be further simplified by noting that $(f|\mathcal{H}_{\mathrm{in}}|i)$ can be put in the form

$$(f|\mathcal{H}_{\mathrm{in}}|i) = (f|\mathcal{H}_{\mathrm{in}}|i)_{q=0}\, g(\boldsymbol{q}) \,, \tag{7}$$

where $g(\boldsymbol{q})$ is a function of the momentum transfer such that $g(0) = 1$; moreover, if $\mathcal{H}_{\mathrm{in}}$ manifests itself strongly only within a range $r \leqslant a$, then $g(\boldsymbol{q}) \ll 1$ for $q > \pi a^{-1}$. We can now make use of a theorem relating the total (inelastic and elastic) cross section $\sigma_{\mathrm{in}} + \sigma_{\mathrm{el}}$ to the imaginary part of the non-spin-flip elastic forward scattered amplitude, viz.:

$$\sigma_{\mathrm{in}} + \sigma_{\mathrm{el}} = (4\pi/k)\,\mathcal{I}m\langle(u_i^{\dagger},\, \mathcal{F}(0))\rangle \,, \tag{8}$$

where the angular brackets indicate an average over initial positive energy spin states. Substituting Eq. (3) into Eq. (8), using (5), and performing the indicated operations, we obtain for the total inelastic cross section:

$$\sigma_{\mathrm{in}} = -(2/\hbar c)(f|\mathcal{H}_{\mathrm{in}}|i)_{q=0} \,. \tag{9}$$

Using Eqs. (9) and (7) in Eq. (6), we find

$$\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega = (k/2\pi\hbar c)^2 \cos^2(\theta/2)[\,|(f|\mathcal{H}_{\mathrm{el}}|i)|^2 + (\sigma_{\mathrm{in}}^2\hbar^2c^2/4)\,|g(\boldsymbol{q})|^2|] \,. \tag{10}$$

[4] Since we are estimating a small correction, we shall neglect recoil effects throughout the paper. This is not strictly valid for the proton; however, it will not change the order of magnitude of the results.

2035

It is more convenient to express Eq. (10) in terms of a factor multiplying the uncorrected elastic scattering cross section

$$(\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega)_0 = (k/2\pi\hbar c)^2 \cos^2(\theta/2)\,|(f|\mathscr{H}_{\mathrm{el}}|i)|^2 ,$$

$$\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega = (\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega)_0\,(1 + C) , \tag{11}$$

where

$$C = \sigma_{\mathrm{in}}^2(\hbar^2c^2/4)\,|g(\boldsymbol{q})|^2/|(f|\mathscr{H}_{\mathrm{el}}|i)|^2 , \tag{12}$$

$$= \sigma_{\mathrm{in}}^2(k/4\pi)^2\,|g(\boldsymbol{q})|^2\cos^2(\theta/2)/(\mathrm{d}\sigma/\mathrm{d}\Omega)_0 , \tag{12a}$$

$$= \frac{\sigma_{\mathrm{in}}^2(k/4\pi)^2\,|g(\boldsymbol{q})|^2\cos^2(\theta/2)}{\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega - \sigma_{\mathrm{in}}^2(k/4\pi)^2\,|g(\boldsymbol{q})|^2\cos^2(\theta/2)} . \tag{12b}$$

It is to be emphasized that Eqs. (10) through (12*b*) are quite general relations yielding the corrections to $(\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega)_0$, arising from the presence of σ_{in}, for *any* type of scattering and depending for their validity *only* on the validity of the Born approximation in estimating $(\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega)_0$.

In general, $(\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega)_0$ decreases more rapidly than $\sigma_{\mathrm{in}}^2\,|g(\boldsymbol{q})|^2$ with increasing momentum transfer, so that for a given angle the correction factor C (as a function of energy) can take on all values from much less to much greater than unity; in the latter case:

$$\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega = (\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}\Omega)_0 C \tag{12c}$$

$$= (k/4\pi)^2\cos^2(\theta/2)\,|g(\boldsymbol{q})|^2\sigma_{\mathrm{in}}^2 .$$

Eq. (12*c*) describes the extreme high incident energy situation where all the elastic scattering is of the diffraction or shadow type and the total inelastic and elastic cross sections are equal so that

$$\int \cos^2(\theta/2)\,|g(\boldsymbol{q})|^2\,\mathrm{d}\Omega = (4\pi/k)^2/\sigma_{\mathrm{in}} . \tag{12d}$$

Since under these circumstances $q \approx k\theta$ and since $g(\boldsymbol{q}) \ll 1$ for $q > \pi a^{-1}$, Eq. (12*d*) yields

$$\sigma_{\mathrm{in}} \approx \sigma_{\mathrm{el}} \approx \pi(4/\pi)^2 a^2 ,$$

as an interesting limiting relation.

Of the quantities appearing in Eq. (12*b*), only $g(\boldsymbol{q})$ cannot be uniquely determined in our phenomenological treatment; this reflects our ignorance of

the precise form of $\mathcal{H}_{in}$. Because of this ignorance, the best we are able to do is select some reasonable function for $g(\boldsymbol{q})$ satisfying the previously stated boundary conditions and corresponding to a range of interaction consistent with the inelastic effect under consideration. However, an upper limit for the correction can be obtained by setting $g(\boldsymbol{q})$ equal to its maximum value of unity. The last procedure, of course, has the benefit of being free of any particular assumption concerning the nature of the factor $g(\boldsymbol{q})$; however, this will on occasion lead to a value considerably greater than the physically observed cross section and hence yield no significant information.

3. – Correction to electron-proton scattering.

Recent experimental ([5-7]) and theoretical ([8]) investigations show the possibility of direct pion production when electrons are scattered from nuclei, and, in particular, from hydrogen. Such inelastic processes will give contributions to the electron-proton elastic cross section of the type discussed above. In order to estimate the effect of these processes using Eq. (12*b*), it is necessary to make an approximate theoretical calculation of the total electron-pion production cross section. In accordance with the results of the experiments of PANOFSKY *et al.* ([5-7]), a rough approximation is obtainable directly from the corresponding photo-pion production cross section if we employ the technique of WEIZSÄCKER ([9]) and WILLIAMS ([10]). In this classical approximation, the electromagnetic field of the incident electron is considered as being equivalent to a spectrum of virtual photons any one of which may interact to bring about the photopion production reactions. Thus in this picture, the reactions:

$$e^- + p \to e^- + p + \pi^0 ,$$

$$e^- + p \to e^- + n + \pi^+ ,$$

are brought about indirectly through the corresponding reactions:

$$\gamma + p \to p + \pi^0 ,$$

$$\gamma + p \to n + \pi^+ .$$

([5]) W. K. H. PANOFSKY, C. M. NEWTON and G. B. YODH: *Phys. Rev.*, **98**, 751 (1955).

([6]) W. K. H. PANOFSKY, W. M. WOODWARD and G. B. YODH: *Phys. Rev.*, **102**, 1392 (1956).

([7]) G. B. YODH and W. K. H. PANOFSKY: *Phys. Rev.*, **105**, 731 (1957).

([8]) R. B. CURTIS: *Phys. Rev.*, **104**, 211 (1956).

([9]) C. F. v. WEIZSÄCKER: *Zeits. f. Phys.*, **88**, 612 (1934).

([10]) E. J. WILLIAMS: *Kgl. Danske Videnskab. Selskab., Mat.-Fys Medd.*, **13**, No. 4 (1935).

A simple calculation gives for the direct pion production cross section:

$$\sigma_e^{\pi}(E_0) \cong \frac{2\alpha}{\pi}\int_{E_{th}}^{E_0 - mc^2} (dk'/k') \ln (2E_0^2/k'mc^2)\sigma_\gamma^{\pi}(k') \,, \tag{13}$$

where the integral over virtual photon energies k' extends from the threshold energy for photopion production, E_{th}, to the kinetic energy of the incident electron $E_0 - mc^2$, and $\sigma_\gamma^{\pi}(k')$ is the total photopion production cross section at the energy k'. Although Eq. (13) would be satisfactory in our rough computations, it is somewhat better to employ a less qualitative expression such as the one derivable from Eq. (23) of CURTIS [8] under the assumption that the rest energy of the nucleon is much greater than the other energies involved. This gives the familiar magnetic dipole absorption expression

$$\sigma_e^{\pi}(E_0) \cong \frac{\alpha}{\pi}\int_{E_{th}}^{E_0 - mc^2} (dk'/k') \left[1 + \left(1 - \frac{k'}{E_0}\right)^2\right] \ln \left[\frac{2E_0(E_0 - k')}{k'mc^2}\right] \sigma_\gamma^{\pi}(k') \,, \tag{14}$$

which reduces to Eq. (13) if the principal contribution to the integrand comes from a region of photon energies such that $E_0 \gg k'$.

Eqs. (13) and (14) have been evaluated for several incident electron energies using the values of $\sigma_\gamma^{\pi}(k')$ for positive pion production in hydrogen as given in BETHE and DE HOFFMANN [11]. The threshold has been taken to be 150 MeV in the laboratory. The results are summarized in Table I. It is evident that Eq. (13) gives a cross section several times larger than that predicted by Eq. (14) but not sufficiently different to change the order of magnitude of the contribution. Again we see that as the incident electron energy E_0 becomes much greater than the resonance energy for photopion production ($\sim$ 300 MeV lab.) the results of the two formulae become equivalent. In using either of these expressions, we neglect terms not directly dependent on $\sigma_\gamma^{\pi}(k')$ (the longitu-

TABLE I. – *Total cross sections for positive pion production.*

E_0 (lab) MeV	E_0 (c.m.) MeV (*)	$\sigma_e^{\pi^+}$ (13) (cm^2)	$\sigma_e^{\pi^+}$ (14)(cm^2)
150	131	$0 \cdot (\alpha/\pi) \cdot 10^{-29}$	$0 \cdot (\alpha/\pi) \cdot 10^{-29}$
250	202	56	18
300	234	109	41
350	265	156	66
400	294	188	85
450	322	210	102

(*) c.m. refers to the electron-proton barycentric system.

2038

dinal terms); these, however, have been shown [8] to constitute only a small fraction of the cross section, and are thus negligible in our calculations.

So far we have ignored the presence of neutral pion production which is also possible. This effect has been computed in the same manner as the charged pion production by fitting a curve to the MIT and CIT data as summarized in BETHE and DE HOFFMANN [11], and making a graphical integration of Eq. (14). Table II shows the results of this procedure along with the corresponding values for the total direct pion production cross section.

TABLE II. – *Cross sections for neutral and total pion production.*

E_0 (c.m.) MeV	$\sigma_e^{\pi^0}$ (14) (cm^2)	σ_{in} (cm^2)
131	$0\cdot(\alpha/\pi)\cdot 10^{-29}$	$0.0\cdot 10^{-31}$
202	7	5.8
234	20	14.2
265	43	25.3
294	64	34.6
322	77	41.6

TABLE III. – *Values of the correction factor C for hydrogen at $\theta = 60°$ and $\theta = 120°$.*

E_0 (c.m.) MeV	C (60°)	$C_{u.l.}$ (60°)	C (120°)	$C_{u.l.}$ (120°)
131	$0.00\cdot 10^{-6}$	$0.00\cdot 10^{-6}$	$0.00\cdot 10^{-6}$	$0.00\cdot 10^{-6}$
202	0.01	0.12	0.01	0.78
234	0.09	1.33	0.11	9.40
265	0.35	7.53	0.39	$5.47\cdot 10^{-5}$
294	0.79	$2.35\cdot 10^{-5}$	0.82	$1.67\cdot 10^{-4}$
322	1.31	5.26	1.33	3.84

We now calculate the correction factor C in Eq. (12*b*) by using the values of $d\sigma_{el}/d\Omega$ from the Stanford experiments on elastic electron-proton scattering [12-13] and the results for σ_{in} listed in Table II, and taking $g(\boldsymbol{q}) \cong g(0) = 1$. This gives the values for the upper limit $C_{u.l.}$ shown in Table III; also given in the table are more realistic values for C obtained by using the factor

$$g(\boldsymbol{q}) = (1 + a^2 q^2)^{-1}\,, \tag{15}$$

corresponding to a Yukawa type interaction: $\mathcal{H}_{in} \sim \exp[-r/a]/r$ with a characteristic range $a = 1.4$. fermi = pion Compton wavelength. As anticipated,

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[11] H. A. BETHE and F. DE HOFFMANN: *Mesons and Fields* (Evanston, 1955), vol. 2, pp. 165 ff.

[12] R. W. MCALLISTER and R. HOFSTADTER: *Phys. Rev.*, **102**, 851 (1956).

[13] E. E. CHAMBERS and R. HOFSTADTER: *Phys. Rev.*, **103**, 1454 (1956).

the results of Table III show the inelastic corrections from pion production to be increasing but negligible functions of energy and angle throughout the extent of experimental energies and angles now employed.

4. – Correction to electron-deuteron scattering.

The extension of the above formalism to the deuteron is straightforward and entirely analogous except for the modifications introduced by the fact that the two nucleons are now capable of internal motions and that the bond between them may be broken up by the incident electron (electro-disintegration). Since the latter inelastic process is incoherent with the previously considered pion production we must add the two total cross sections to obtain σ_{in}.

We examine first the pion contributions. Since we are interested in the region of high energies, it is sufficient to neglect any Coulomb effects between the individual nucleons and pions, and consider the neutron and proton to act as separate sources of pions through the reactions:

(A)
$$\begin{cases} e^- + p \to e^- + p + \pi^0 , \\ e^- + p \to e^- + n + \pi^+ . \end{cases}$$

(B)
$$\begin{cases} e^- + n \to e^- + n + \pi^0 , \\ e^- + n \to e^- + p + \pi^- . \end{cases}$$

Although, within our approximation, we may take the total cross section for (A) to be the same as that for (B), we are not justified in taking the cross section for (A) to be that for a free nucleon. It must be remembered that the motions of the individual particles, as represented by the ground state wave function of the deuteron, tend to reduce the cross sections from their free particle values. This manifests itself most strongly in charged pion production where there is an exclusion principle inhibition against producing di-proton and di-neutron states.

Again we can employ the experimental data on photopion production. As expected from the above discussion, experimental evidence [11] indicates that the following relations hold approximately between the total photoproduction cross sections in deuterium and hydrogen:

$$\sigma_\gamma^{\pi^+}(d) \cong 0.75\sigma_\gamma^{\pi^+}(p) ,$$
$$\sigma_\gamma^{\pi^0}(d) \cong 2\sigma_\gamma^{\pi^0}(p) ,$$
$$\sigma_\gamma^{\pi^-}(d) \cong \sigma_\gamma^{\pi^+}(d) .$$

Consistent with these results, we shall assume

$$\sigma_\gamma^{\pi^+}(\mathrm{p})_{\mathrm{bnd.}} \cong \sigma_\gamma^{\pi^-}(\mathrm{n})_{\mathrm{bnd.}} \cong 0.75\sigma_\gamma^{\pi^+}(\mathrm{p})_{\mathrm{free}} ,$$

$$\sigma_\gamma^{\pi^0}(\mathrm{p})_{\mathrm{bnd.}} \cong \sigma_\gamma^{\pi^0}(\mathrm{n})_{\mathrm{bnd.}} \cong \sigma_\gamma^{\pi^0}(\mathrm{p})_{\mathrm{free}} .$$

The latter equality is, according to our qualitative argument about motional reduction, probably an overestimate.

The corresponding direct pion production cross sections may now be obtained from the values given in Tables I and II; the results are shown in Table IV.

TABLE IV. – *Total pion cross sections in deuterium.*

E_0 (c.m.) MeV (*)	$\sigma_e^{\pi^+}$ (p)$_{\mathrm{bnd.}}$ (cm^2)	$\sigma_e^{\pi^0}$ (p)$_{\mathrm{bnd.}}$ (cm^2)	σ_e^{π} (d) (μb)
139	$0\cdot(\alpha/\pi)\cdot 10^{-23}$	$0\cdot(\alpha/\pi)\cdot 10^{-23}$	0.00
222	13	7	0.93
261	31	20	2.37
299	49	43	4.28
335	64	64	5.95
370	76	77	7.11

(*) c.m. refers to the electron-deuteron barycentric system.

The evaluation of the remaining part of the total inelastic cross section arising from the electrodisintegration of the deuteron is not as simple as that for pion production, primarily because of the fact that even at high energies the longitudinal part of the electron's electromagnetic field is expected to give a sizable contribution to the electrodisintegration ([14]); this means that the simple Weizsäcker-Williams relation between the electro- and photocross sections no longer holds, and that the wealth of experimental data on deuteron photodisintegration is of no immediate use. JANKUS ([14]), however, has obtained expressions for the electrodisintegration cross section using a Møller potential to describe the electron interacting with point nucleons in the deuteron. We have made use of his Eq. (7) for large momentum transfer with a deuteron form factor corresponding to a Hulthén potential with triplet effective range 1.7 fermi and extrapolated the cross section there given to forward scattering; this has then been integrated over all solid angles to obtain a reasonable approximation to the total cross section. Further, since, as indicated by the most recent experiments the proton may be thought of as having

([14]) V. Z. JANKUS: *Phys. Rev.*, **102**, 1586 (1956).

a finite charge distribution ([12,13,15]) the Jankus expression was multiplied by the square of an appropriate proton form factor ($\exp[-q^2 r_e^2/6]$, $r_e = 0.8$ fermi) before the calculations were performed. In this way we find the values of the electrodisintegration cross section listed in the second column of Table V.

TABLE V. – *Total electrodisintegration cross sections for deuterium.*

E_0 (c.m.) MeV	$\sigma_{\text{el-d.}}$ (μb)	$\sigma^{\pi}_{\text{el-d.}}$ (μb)	$\sigma^{\text{tot}}_{\text{el-d.}}$ (μb)
139	46	1.1	47
222	33	2.3	35
261	28	2.8	31
299	24	3.1	27
335	21	3.4	24
370	18	3.7	22

However, these values neglect any virtual pion contributions to the electrodisintegration which may be expected to occur at higher energies (theory ([16]) indicates that it is these virtual pion contributions which give rise to the relative maximum in the photodisintegration cross section at about 250 MeV lab.). A rough estimate of these contributions can be found by using the corresponding photodisintegration data and the Weizsäcker-Williams approximation. For this purpose, we have subtracted the theoretical Marshall-Guth ([17]) photo-cross section (calculated with neglect of virtual pion effects) from the experimental photo-cross section summarized in the papers of LEVINGER ([18]) and DIXON and BANDTEL ([19]); the remaining part of the photo-cross section, taken to be zero below 50 MeV, was then inserted into the Weizsäcker formula to obtain the final virtual pion correction $\sigma^{\pi}_{\text{el-d.}}$ listed in the third column of Table V. This correction, when added to the modified

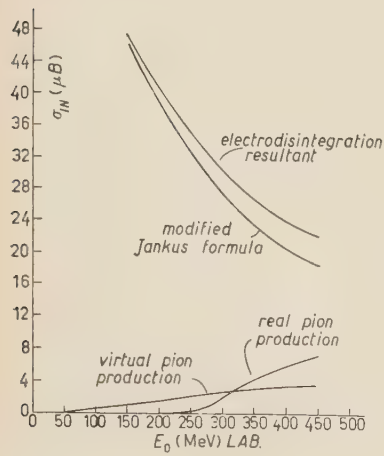

Fig. 1. – Contributions to the total inelastic cross section for deuterium.

([15]) R. HOFSTADTER: *Rev. Mod. Phys.*, **28**, 214 (1956).
([16]) F. ZACHARIASEN: *Phys. Rev.*, **101**, 371 (1956).
([17]) J. F. MARSHALL and E. GUTH: *Phys. Rev.*, **78**, 738 (1950).
([18]) J. S. LEVINGER: *Phys. Rev.*, **97**, 970 (1955).
([19]) D. R. DIXON and K. C. BANDTEL: *Phys. Rev.*, **104**, 1730 (1956).

2042

Jankus' result, gives the total electrodisintegration cross section $\sigma_{el\text{-}d}^{tot.}$ (fourth column of Table V). The behaviour of these contributions to the inelastic cross section is illustrated in Fig. 1. This shows that at the present experimental energies, electrodisintegration may be expected to give the primary correction to the differential elastic cross section; however, at higher energies, sufficiently above the resonance energy of 300 MeV, the pion contribution may be expected to dominate. The values of the total inelastic cross section σ_{in} may now be found by taking the sum of the values in the fourth columns of Tables IV and V.

In order to compute values of the correction factor C from Eq. (12*b*), we use these values of σ_{in} and again take $d\sigma_{el}/d\Omega$ from the Stanford experiments on elastic electron-deuteron scattering ([20-22]) (except for the very highest value of the momentum transfer where an extrapolation of the experimental results is used). As before the upper limit values have been obtained by setting the inelastic « form factor » $g(\boldsymbol{q})$ equal to unity. In this manner we arrive at the values shown in Table VI. For comparison, the factor from Eq. (15) has been used to compute the values in columns two and four. The upper limit at $\theta = 120°$ has not been computed since it would not lead to meaningful results yielding unreasonably large values ($C \geqslant d\sigma_{el}/d\Omega$).

TABLE VI. – *Values of the correction factor C for deuterium at* $\theta = 60°$ *and* $\theta = 120°$.

E_0 (c.m.) MeV	C (60°)	$C_{u.l.}$ (60°)	C (120°)
139	$0.79 \cdot 10^{-4}$	$3.12 \cdot 10^{-4}$	$4.60 \cdot 10^{-4}$
222	2.05	$2.56 \cdot 10^{-3}$	$1.66 \cdot 10^{-3}$
261	3.10	6.30	3.19
299	4.41	$1.38 \cdot 10^{-2}$	6.02
335	6.37	2.91	$1.17 \cdot 10^{-2}$
370	9.15	5.90	2.30

The results of Table VI again indicate that C is a very small but rapidly increasing function of energy as in the case of hydrogen; however, the values are considerably larger percentagewise, and, contrary to electron-proton scattering, show a marked angular dependence increasing sharply with increasing angle of scattering. This is a manifestation of the decrease in the uncorrected elastic cross section caused by the presence of the additional form factor for the deuteron ground state.

([20]) J. A. McINTYRE and R. HOFSTADTER: *Phys. Rev.*, **98**, 158 (1955).

([21]) J. A. McINTYRE: *Phys. Rev.*, **103**, 1464 (1956).

([22]) J. A. McINTYRE and SOBHANA DHAR (to be published). The author would like to thank Dr. McINTYRE for sending him a preprint of this paper.

5. – Discussion.

In summary, we can state, as expected, that the corrections to elastic scattering arising from the presence of the inelastic scattering for the proton and deuteron systems are sufficiently small to be neglected in any comparison of observed and calculated elastic cross sections at the presently available energies and angles of scattering. Indeed, at 450 MeV (lab) and $\theta = 120°$ and 60°, respectively, the upper limits $C_{u.l.}$ give a correction of only 0.04% for the proton, and 6% for the deuteron, and these probably overestimate the actual corrections by at least an order of magnitude. However, for larger momentum transfers, and for heavier nuclei possessing additional modes of inelastic scattering, these corrections could become significant. That corrections to the elastic scattering arising from inelastic effects might become important in heavier nuclei is emphasized by the rather large increase in C in going from a single nucleon to a compound system like the deuteron, and in the latter case from $\theta = 60°$ to $\theta = 120°$.

In cases where C is appreciable, the correction $(\mathrm{d}\sigma_{el}/\mathrm{d}\Omega)_0 C$ would have the additional consequence of smoothing out any dips occuring in a calculated $(\mathrm{d}\sigma_{el}/\mathrm{d}\Omega)_0$; so that if, for example, a uniform nuclear charge distribution were assumed, the zeros associated with the Born approximation calculation of $(\mathrm{d}\sigma_{el}/\mathrm{d}\Omega)_0$ would not appear in the Born approximation calculation of $\mathrm{d}\sigma_{el}/\mathrm{d}\Omega$.

It should be noted, however, that the present treatment of the proton and deuteron would be expected to be inadequate in the region where inelastic effects constitute an appreciable portion of the elastic scattering, since many of the approximations made here would cease to be valid. For example, the proton has been considered non-relativistically, this would surely be incorrect at higher energies; again, multiple pion production ([23]) which becomes energetically possible around 310 MeV (lab) could be expected to become an important part of the photopion cross section and hence of the electropion cross section.

* * *

The author would like to express his thanks to Professor H. PRIMAKOFF and Professor B. J. MALENKA for continued interest and guidance.

([23]) R. M. FRIEDMAN and K. M. CROWE: *Phys. Rev.*, **105**, 1369 (1957).

2044

Appendix [24]

We wish to indicate here a qualitative procedure by which the Hamiltonian Eq. (1) may be deduced from slightly more basic considerations. Although the method is quite general, in order to fix our ideas, we confine our discussion to the proton.

Let

$$(\mathcal{H}_p + \mathcal{H}_e + \mathcal{H})\Psi = E\Psi\,, \tag{A.1}$$

represent the wave equation for the electron-proton system. Here $\mathcal{H}_p$ is the physical proton plus pion field Hamiltonian involving both bare nucleon and associated pion co-ordinates; $\mathcal{H}_e$ is the electron Hamiltonian; and $\mathcal{H}$ is the interaction Hamiltonian describing the coupling between the electron and the physical proton.

Consider Ψ to be expanded in terms of the complete set of eigenfunctions φ_n of $\mathcal{H}_p$, where

$$\mathcal{H}_p\varphi_n = E_n\varphi_n\,, \tag{A.2}$$

then

$$\Psi = \sum_n \psi_n\varphi_n\,, \tag{A.3}$$

the ψ_n depending on the electron co-ordinates.

Substituting Eq. (A.3) into Eq. (A.1), we find

$$\sum_n (\mathcal{H}_e + E_u + \mathcal{H})\psi_n\varphi_n = \sum_n E\psi_n\varphi_n\,. \tag{A.4}$$

Since we are interested in elastic scattering where the physical proton plus pion field remains in its original state described by φ_0: one physical proton of specified energy, no pions, we multiply Eq. (A.4) by φ_0^* and integrate over the bare nucleon and pion co-ordinates, obtaining

$$(\mathcal{H}_e - E_e)\psi_0 + \sum_n \langle 0\,|\,\mathcal{H}\,|\,n\rangle\psi_n = 0\,, \tag{A.5}$$

where $E_e = E - E_0$ and

$$\langle 0\,|\,\mathcal{H}\,|\,n\rangle \equiv \int \varphi_0^+ \mathcal{H}\varphi_n \,\mathrm{d}\tau\,.$$

Let now Eq. (A.5) be rewritten as

$$(\mathcal{H}_e - E^e + \langle 0\,|\,\mathcal{H}\,|\,0\rangle + w + i\mathcal{H}^{(1)})\psi_0 = 0\,, \tag{A.6}$$

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[24] H. Primakoff: private communication.

where w and $\mathcal{H}^{(1)}$ are Hermitian operators such that

$$(w + i\mathcal{H}^{(1)})\psi_0 \equiv \sum_{n\neq 0} \langle 0|\mathcal{H}|n\rangle\psi_n ,$$

(two operators are required since $\sum_{n\neq 0}\langle 0|\mathcal{H}|n\rangle\psi_n$ is, in general, complex). It can be seen that Eq. (A.6) is of the form of an electron wave equation, where $\langle 0|\mathcal{H}|0\rangle$ is the electromagnetic interaction of the electron with the physical proton averaged over the latter's ground state, while w may be interpreted as a further electron-physical proton interaction associated with the possibility of the latter's virtual excitation ([25]). The remaining non-Hermitian operator $i\mathcal{H}^{(1)}$ describes the effect on the elastic scattering of the inelastic pion processes considered in this paper.

Thus

$$\mathcal{H}_{\rm el} \equiv \langle 0|\mathcal{H}|0\rangle + w ,$$

$$i\mathcal{H}_{\rm in} \equiv i\mathcal{H}^{(1)} ,$$

and Eq. (A.6) becomes

$$(\mathcal{H}_{\rm e} - E_{\rm e} + \mathcal{H}_{\rm el} + i\mathcal{H}_{\rm in})\psi_0 = 0 ,$$

which is consistent with the interaction Hamiltonian of Eq. (1) .

([25]) Thus, for example, w will contain those interactions associated with electron-induced polarization of the proton charge and current cloud. Such effects have been recently examined by S. D. Drell and M. A. Ruderman [*Phys. Rev.*, **106**, 561 (1957)] who estimate this contribution to be of the order of one percent.

RIASSUNTO (*)

In prima approssimazione di Born si deriva un'espressione per la correzione dello scattering elastico di alta energia che nasce dall'esistenza di processi anelastici non radiativi. Si stima la grandezza di questa correzione per lo scattering elettrone-protone ed elettrone-deutone e si dimostra essere, per energie dell'elettrone incidente da 150 a 450 MeV, una funzione molto piccola ma rapidamente crescente dell'energia. Anche la correzione dello scattering elettrone-deutone mostra una marcata dipendenza angolare crescente con l'angolo di scattering.

(*) *Traduzione a cura della Redazione.*

« Handed » Particle of Spin $\frac{1}{2}$ with Finite Mass.

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(ricevuto il 23 Aprile 1957)

Summary. — It is possible to formulate a four-component spinor theory for a particle of spin $\frac{1}{2}$ with a definite chirality (handed-ness), which has a finite mass. Interaction between an achiral (nonhanded) spinor and a chiral spinor requires an eight-component frame-work. Such an interaction manifests a parity-violating property. Two formulations are possible: 1) The theory can be made invariant separately for time-reversal and for the combination of space-inversion and charge-conjugation, or 2) it can be made invariant for space-*and*-time-inversion and for charge-conjugation. The present theory may have applications for hyperons. The Salam-Lee-Yang-Landau type [1] of neutrino theory can be derived from the present formalism for the case $m = 0$. (See Appendix).

1. – Chirality operator for particle with finite mass.

In the two-component neutrino theory [1], the useful two components can be extracted from the usual four-component spinor by the use of the chirality operator γ_5. In the same way, in the present theory, we shall first consider an eight-component spinor, from which we shall extract four components by the help of a chirality operator suitably defined.

According to the purely mathematical definition of a spinor, i.e., without the field-theoretical re-interpretation of time-reversal, the simple reflection of the x_μ-co-ordinate ($\mu = 1, 2, 3, 4$) is represented by $\gamma_\mu\gamma_5$. The chirality operator γ_5 in the case of the neutrino theory is the simplest operator that anti-

(1) A. Salam: *Nuovo Cimento*, **5**, 299 (1957); T. D. Lee and C. N. Yang: *Phys. Rev.*, **105**, 1671 (1957); L. Landau: preprint; W. Pauli: *Handbuch der Physik* (Berlin, 1933), vol. 24, p. 226.

commutes with each of $\gamma_\mu\gamma_5$ and commutes with the hamiltonian. This fact will give a clue in defining the chirality operator in the present case.

We define 8×8-matrices Γ_μ ($\mu=1,2,3,4$) and Γ_5 exactly in the same way as the usual γ's.

$$[\Gamma_\mu, \Gamma_\nu]_+ = 2\delta_{\mu\nu}\,, \qquad \Gamma_5 = \Gamma_1\Gamma_2\Gamma_3\Gamma_4\,. \tag{1.1}$$

The hamiltonian is given by

$$H = i\Gamma_4\Gamma_a p_a + \Gamma_4 m\,. \qquad (a=1,2,3). \tag{1.2}$$

We cannot use Γ_5 as the chirality operator since it does not commute with H unless $m=0$. We shall presently see the way to overcome this difficulty.

In the system of 4×4-matrices, a matrix that commutes with all four γ_μ ($\mu=1,2,3,4$) is the unity matrix multiplied by an ordinary number. However, in the system of 8×8-matrices, a unitary hermitian matrix Λ exists which without being a multiple of the unity matrix commutes with all four Γ_μ ($\mu=1,2,3,4$).

$$[\Lambda, \Gamma_\mu]_- = 0\,, \quad (\mu=1,2,3,4)\,, \quad [\Lambda, \Lambda_5]_- = 0\,, \tag{1.3}$$

$$\Lambda^2 = \overline{\Lambda}\Lambda = 1\,. \tag{1.4}$$

According to the usual definition of spinors, the simple reflection with respect to a plane whose normal is n_μ ($n_\mu n_\mu = \pm1$) is represented by the transformation matrix

$$S = n_\mu\Gamma_\mu\Gamma_5\,. \tag{1.5}$$

We can now change this to

$$S = n_\mu\Gamma_\mu\Gamma_5\Lambda\,, \tag{1.6}$$

without in any way altering the transformation properties of the physical quantities on account of Eq. (1.3). Eq. (1.4) shows that the eigenvalues of Λ are ±1. If we limit the wave-functions to those which are eigenfunctions of Λ corresponding to one of the eigenvalues, then the transformation rule Eq. (1.6) resumes the familiar form of Eq. (1.5).

Passing to the general case, if S_0 is the transformation matrix according to the customary definition based on Eq. (1.5), then the one we use here can be written

$$S = \Lambda^\nu S_0\,, \tag{1.7}$$

where v is the number of single reflections into which the transformation can be decomposed. Since $\Lambda^2 = 1$, we have $S = S_0$ for rotations and $S = \Lambda S_0$ for inversions.

The chirality operator X must, besides being commutative with H, satisfy

$$[X, \Gamma_\mu \Gamma_5 \Lambda]_+ = 0 . \qquad (\mu = 1, 2, 3, 4). \tag{1.8}$$

It can easily be shown that there exists in the system of 8×8-matrices a unitary hermitian matrix which cannot be expressed in terms of Γ_μ and which anti-commutes with each of four Γ_μ and with Λ.

$$[\Theta, \Gamma_\mu]_+ = 0 , \qquad (\mu = 1, 2, 3, 4) , \qquad [\Theta, \Lambda]_+ = 0 , \tag{1.9}$$

$$\Theta^2 = \overline{\Theta}\Theta = 1 . \tag{1.10}$$

Θ then necessarily commutes with Γ_5:

$$[\Theta, \Gamma_5]_- = 0 . \tag{1.11}$$

We can now define the chirality operator X by

$$X = \Gamma_5 \Theta , \tag{1.12}$$

which obviously satisfies Eq. (1.8). X is unitary and hermitian, and commutes with any Γ_μ and Γ_5.

$$[X, \Gamma_\mu]_- = 0 , \qquad [X, \Gamma_5]_- = 0 . \tag{1.13}$$

Thus, X commutes with any operator that contains any number of Γ's, including the hamiltonian. If the wave-function satisfies $\Theta\Psi = \Psi$, then our definition (1.12) reduces to Γ_5. Θ, however, does not commute with the hamiltonian unless $m = 0$.

Now, in solving the Schrödinger equation with the hamiltonian given in Eq. (1.2), we can characterize eight eigenfunctions by the signs of H, X, and another operator that commutes with these two, for instance, the helicity operator:

$$\eta = i\Gamma_4 \Gamma_5 \Gamma_a p_a / |p| , \qquad (a = 1, 2, 3). \tag{1.14}$$

Any wave-function Ψ can be decomposed into two parts, each of which is an eigenfunction of X.

$$\Psi = [(1 + X/2]\Psi + [(1 - X)/2]\Psi = \Psi_1 + \Psi_2 , \tag{1.15}$$

where

$$X\Psi_1 = +\Psi_1 , \tag{1.16}$$

$$X\Psi_2 = -\Psi_2 . \tag{1.17}$$

If the wave-function of a certain kind of particle is by nature one or the other of the eigenfunctions of X, we can say that the particle is a chiral particle with chirality $+1$ or -1 according as the case may be. In such a case, using the representation in which X is diagonal, the wave function becomes a four-component quantity. Therefore, a four-component theory is possible for a particle of spin $\frac{1}{2}$ with a definite chirality even if its mass is finite.

For space-inversion (mirage), we have according to Eq. (1.7) the transformation operator:

$$P = \Gamma_4 \Lambda , \tag{1.18}$$

which interchanges Eq. (1.16) with Eq. (1.17). If a particle has chirality $+1$ in the right-handed co-ordinate system, the same particle is described as having chirality -1 in the left-handed co-ordinate system.

The charge-conjugation matrix K is to be defined as usual by

$$K\Gamma_\mu K^{-1} = -\Gamma_\mu^T , \quad K\Gamma_5 K^{-1} = \Gamma_5^T , \quad K^T = -K , \quad \overline{K} = K^{-1} . \tag{1.19}$$

On account of the commutativity of Λ with Γ_μ, if K_1 satisfies Eq. (1.19), then K_2 defined by

$$K_2 = K_1 \Lambda , \tag{1.20}$$

also satisfies Eq. (1.19). It is easy to construct a K that commutes with Θ. Calling such a K K_1, we obtain

$$K_1 X K_1^{-1} = X^T , \tag{1.21}$$

$$K_2 X K_2^{-1} = -X^T . \tag{1.22}$$

in virtue of Eqs. (1.3), (1.9) and (1.12).

The wave-function Ψ is transformed by space-inversion (mirage) as

$$M: \qquad \Psi \to P\Psi = \Gamma_4 \Lambda \Psi , \tag{1.23}$$

and by charge-conjugation as (using a bar to indicate hermitian conjugate)

$$C: \qquad \Psi \to \overline{\Psi} \Gamma_4 K , \tag{1.24}$$

2050

where K can be either K_1 or K_2. Further, by time-reversal, Ψ becomes

$$R: \qquad \Psi \to \overline{\Psi} \Gamma_5 \Lambda K . \tag{1.25}$$

Insertion of Λ in the last expression is in accord with the field-theoretical formulation of time-reversal, in which time-reversal is achieved by the combination of charge-conjugation and spinor-theoretical time-reversal which is given by $\Lambda \Gamma_4 \Gamma_5$.

From transformation rules, (1.23), (1.24) and (1.25) follows the following significant rule. If one chooses $K = K_1$, then the defining equations of chirality, Eqs. (1.16) and (1.17), remain unchanged by space-*and*-time-inversion and by charge-conjugation. If one chooses $K = K_2$, these equations remain unchanged by the combination of space-inversion and charge-conjugation and by time-reversal. Naturally, one feature is common to both choices, namely, the theoretical formulation is not invariant for space-inversion only.

Just as X commutes with all Γ's, Λ also commutes with all Γ's. If the wave-function is one or the other of the two-eigenfunctions of Λ, i.e., if we have either

$$\Lambda \Psi = + \Psi \qquad \text{or} \qquad \Lambda \Psi = - \Psi , \tag{1.26}$$

the number of components is reduced to four. The part $\Lambda = 1$ and the part $\Lambda = -1$ are essentially equivalent. The only difference is the relative parity which can be seen in Eq. (1.6) by putting $\Lambda = +1$ and $\Lambda = -1$. We can safely adopt $\Lambda = +1$ to describe an achiral (non-handed) particle. Then the theory is completely identical to the usual four-component theory. The difference between K_1 and K_2 disappears in either one of the two possibilities in Eq. (1.26), since the sign of K has no meaning. Needless to say that the four-component theory of achiral particles is invariant for each of the three transformations, M, C and R, separately.

2. – Chiral and achiral representation.

We introduced several new matrices in the last section, such as Θ, Λ, etc. We shall first give the expressions of the major matrices in the (achiral) representation in which Λ is diagonal.

$$\left\{ \begin{array}{lll} \Gamma_\mu = \begin{pmatrix} \gamma_\mu & 0 \\ 0 & -\gamma_\mu \end{pmatrix}, & \Gamma_5 = \begin{pmatrix} \gamma_5 & 0 \\ 0 & \gamma_5 \end{pmatrix}, & \Lambda = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \\ \Theta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, & X = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix}, & P = \begin{pmatrix} \gamma_4 & 0 \\ 0 & \gamma_4 \end{pmatrix}, \\ K_1 = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix}, & K_2 = \begin{pmatrix} C & 0 \\ 0 & -C \end{pmatrix}, & \end{array} \right. \tag{2.1}$$

2051

where C is the usual charge-conjugation matrix such that

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T, \quad C\gamma_5 C^{-1} = \gamma_5^T, \quad C^T = -C, \quad \overline{C} = C^{-1}. \tag{2.2}$$

If we take the upper (or lower) half of the wave-function in this representation, we obtain exactly the usual four-component theory.

We can now pass from this representation to the (chiral) representation in which X is diagonal by the transformation T given by

$$T = (1/2^{\frac{5}{2}})(\Gamma_4 + \Gamma_5)[(1 - \Gamma_4\Lambda)(1 - \Gamma_5)\Theta - (1 + \Gamma_4\Lambda)(1 + \Gamma_5)](\Theta + \Lambda), \tag{2.3}$$

$$\overline{T} = T^{-1} = (1/2^{\frac{5}{2}})(\Theta + \Lambda)[\Theta(1 - \Gamma_5)(1 - \Gamma_4\Lambda) - (1 + \Gamma_5)(1 + \Gamma_4\Lambda)](\Gamma_4 + \Gamma_5). \tag{2.4}$$

The resulting matrices in this new representation are

$$\left\{\begin{array}{l} \Gamma_a' = \begin{pmatrix} \gamma_a & 0 \\ 0 & \gamma_a \end{pmatrix}, \quad \Gamma_4' = \begin{pmatrix} \gamma_4 & 0 \\ 0 & -\gamma_4 \end{pmatrix}, \quad \Gamma_5' = \begin{pmatrix} \gamma_5 & 0 \\ 0 & -\gamma_5 \end{pmatrix}, \quad \Lambda' = \begin{pmatrix} 0 & \gamma_4\gamma_5 \\ \gamma_5\gamma_4 & 0 \end{pmatrix}, \\ \Theta' = \begin{pmatrix} \gamma_5 & 0 \\ 0 & \gamma_5 \end{pmatrix}, \quad X' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P' = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix}, \\ K_1' = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix}, \quad K_2' = \begin{pmatrix} 0 & C\gamma_4\gamma_5 \\ C\gamma_5\gamma_4 & 0 \end{pmatrix}. \end{array}\right. \tag{2.5}$$

If we extract one of the eigenfunctions of X by

$$[(1 + X)/2]\Psi = \Psi_1, \tag{2.6}$$

and if we represent this Ψ_1 in the chiral representation given in Eq. (2.5), then Ψ_1 occupies the upper half of the eight-component spinor.

$$\Psi_1' = \begin{pmatrix} \psi' \\ 0 \end{pmatrix}. \tag{2.7}$$

As the usual fermion seems to be in an eigenstate of Λ, it is quite possible that there are in nature particles which are in an eigenstate of X. Suppose a particle can be expressed by Ψ_1, say in the right-handed co-ordinate system, then ψ' in Eq. (2.7) will obey exactly the same physical laws as a usual achiral particle, since the upper left quarter of each Γ_μ' is exactly the usual γ_μ. The only difference is that by space-inversion and by charge-conjugation with K_2 and by time-reversal with K_1, the condition $X\Psi = \Psi$ passes to $X\Psi = -\Psi$, which means the wave-function moves to the lower half. This situation is similar to the two-component neutrino theory, in which the two components

2052

by space-inversion pass to the other two components of the original four-component spinor. This means that the *form* of a law, such as Eq. (2.6) changes by space-inversion, but if one uses the opposite chirality for the same particle in the space-inverted co-ordinate system, one naturally obtains the same empirical prediction. As far as rotations are concerned, one can handle a chiral particle by the usual four-component formalism.

3. – Interaction terms.

Let us consider as an illustration tensorial quantities of the type:

$$Q = \bar{\Psi} \Gamma_4 O \Phi \,, \tag{3.1}$$

with

$$O = 1, \Gamma_5, \Gamma_\mu, \Gamma_\mu \Gamma_5 \quad \text{or} \quad \Gamma_\mu \Gamma_\nu \,. \tag{3.2}$$

These quantities have the same transformation properties as the corresponding quantities written in terms of the γ's instead of the Γ's in the usual theory.

If both Ψ and Φ represent the ordinary achiral particles ($\Lambda = 1$), we have

$$\Psi = [(1 + \Lambda)/2]\Psi \,, \qquad \Phi = [(1 + \Lambda)/2]\Phi \,, \tag{3.3}$$

and with the aid of the Λ-representation, Eq. (2.1), we can bring Q to its usual four-component counterpart. Similarly, if both Ψ and Φ belong to the same eigenvalue of X, we have for instance

$$\Psi = [(1 + X)/2]\Psi \,, \qquad \Phi = [(1 + X)/2]\Phi \,, \tag{3.4}$$

and with the help of the X-representation, Eq. (2.5), we can again express Q in the customary four-component form.

If, however, Ψ and Φ are eigenfunctions of X, but belonging to the opposite chiralities, Q will vanish. This means that it is impossible for a chiral particle to disappear giving birth in its place to another chiral particle of opposite chirality, as far as a term of the type (3.1) is concerned. Such a process would be possible if Λ would be inserted in the expression of Q.

Finally, if one of Ψ and Φ is an eigenstate of Λ, and the other is an eigenstate of X, we cannot use one representation for Ψ and another for Φ. Let us assume, for definiteness

$$\Psi = [(1 + \Lambda)/2]\Psi \,, \qquad \Phi = [(1 + X)/2]\Phi \,, \tag{3.5}$$

in the right-handed co-ordinate system. Then the quantity Q can be written as

$$Q = \bar{\Psi}\Gamma_4[(1+\Lambda)/4]O\Phi + \Psi\Gamma_4[(1+\Lambda)/4]OX\Phi\,. \tag{3.6}$$

If the first term is a certain tensorial quantity, then the second term is the corresponding pseudo-tensorial quantity. Note that P and X anticommute. This is exactly the same situation as in the two-component neutrino theory, where one has $(1-\gamma_5)/2$ at the place where $(1+X)/2$ stands in the present case.

One can re-state this situation with the help of the upper and lower halves of the wave-functions. If we use the Λ-representation, Ψ becomes the upper half:

$$\Psi = \begin{pmatrix} \psi \\ 0 \end{pmatrix}, \tag{3.7}$$

while Φ will occupy both halves:

$$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}. \tag{3.8}$$

In virtue of the assumption (3.5), these two parts are related to each other in the right-handed co-ordinate system by

$$\varphi_2 = \gamma_5\varphi_1\,. \tag{3.9}$$

If one uses the X-representation, one can write

$$\Phi' = \begin{pmatrix} \varphi' \\ 0 \end{pmatrix}. \tag{3.10}$$

This Φ' can be brought to the Λ-representation by the use of the transformation given in Eq. (2.4). By calculating $\Phi = T\Phi'$, it can easily be shown that $\varphi_1 \sim \varphi'$ and $\varphi_2 \sim \gamma_5\varphi'$, in agreement with Eq. (3.9).

Q now becomes in the Λ-representation

$$Q = \tfrac{1}{2}(\bar{\psi}\gamma_4 O\varphi_1 + \bar{\psi}\gamma_4 O\gamma_5\varphi_2)\,. \tag{3.11}$$

According to the expression of P in the Λ-representation, space inversion is effected by simply multiplying the upper and lower halves by γ_4. Thus, Q transforms as

$$Q \to \tfrac{1}{2}[\bar{\psi}\gamma_4(\gamma_4 O\gamma_4)\varphi_1 - \bar{\psi}\gamma_4(\gamma_4 O\gamma_4)\gamma_5\varphi_2]. \tag{3.12}$$

2054

This shows that the first and the second terms transform differently. In order to obtain the same prediction in the space-inverted co-ordinate system, we need not only transform the q's by the prescribed rule, but also change the connection between φ_1 and φ_2 given in Eq. (3.9) to

$$\varphi_2 = -\gamma_5\varphi_1 . \tag{3.13}$$

4. – Concluding remarks.

The fact alone that a fermion obeys the Dirac equation and allied laws does not guarantee that the particle is devoid of handed-ness. An actual test may be possible through an interaction between an achiral particle and a suspected chiral particle.

There seems to be some reason to believe that a particle with a finite « strangeness » has some kind of handed-ness [2]. It is thus a tempting idea to consider hyperons to be chiral spinor particles, although it is also quite possible that they are composite particles which cannot be described by spinors, including chiral spinors [2]. The formalism presented in the present paper might prove instrumental in analyzing the behavior of the interactions involving hyperons.

Appendix

Passage to the Salam-Lee-Yang-Landau type of two-component neutrino theory [1] can be done from either the A-representation or the X-representation. In either case, it is necessary only to take a representation in which γ_5 and $\gamma_4\gamma_a$ ($a = 1, 2, 3$) have diagonal forms when expressed in the double-2×2-matrix form. For instance, we can use

$$\gamma_a = \begin{pmatrix} 0 & i\sigma_a \\ -i\sigma_a & 0 \end{pmatrix}, \qquad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.1}$$

where σ_a ($a = 1, 2, 3$) are the Pauli matrices. Then the hamiltonian and the helicity become

$$H = \begin{pmatrix} \sigma_a p_a & m \\ m & -\sigma_a p_a \end{pmatrix}, \qquad \eta = \begin{pmatrix} \sigma_a p_a & 0 \\ 0 & \sigma_a p_a \end{pmatrix}. \tag{A.2}$$

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[2] S. Watanabe: *Phys. Rev.*, in press (The June 15 issue, 1957).

Since $m = 0$, the hamiltonian is diagonal in the present sense. As a result, the separation of the first two components and the last two components, which can be performed by γ_5, is a constant of motion.

The above separation can be done in either the A-representation or the X-representation. The only difference is that, in the former, the first two components by space-inversion pass to the third and fourth components while in the latter they pass to the fifth and sixth components. This can be easily seen by substituting γ_4 and γ_5 of Eq. (A.1) in P of Eq. (2.1) and P' of Eq. (2.5), respectively. It should be noted that all the transformation rules adopted in the present paper are such that they need not be altered when applied to the electron and proton.

RIASSUNTO (*)

È possibile formulare per una particella dotata di spin $\frac{1}{2}$, chiralità (orientamento) definita e massa finita una teoria spinorica a quattro componenti. L'interazione tra uno spinore achirale (non orientato) e uno spinore chirale richiede un'impostazione a otto componenti. Tale interazione non conserva la parità. Due formulazioni sono possibili: 1) si può rendere la teoria separatamente invariante per l'inversione del tempo e per la combinazione dell'inversione dello spazio e della coniugazione della carica, o 2) la si può rendere invariante per l'inversione dello spazio e del tempo. La presente teoria può essere applicata agli iperoni. Una teoria del neutrino di tipo Salam-Lee-Yang-Landau [1] si può derivare dal presente formalismo per il caso $m = 0$. (Vedi Appendice).

(*) *Traduzione a cura della Redazione.*

Electronic States of Diatomic Molecules: the Lithium Molecule (*).

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Summary. — The method of «atoms in molecules» is applied to the calculation of the electronic energy levels of Li_2. The molecule is treated as a two-electron system; only configurations involving $2s\sigma$, $2p\sigma$ and $2p\pi$ molecular orbitals are considered. The results for the ${}^1\Sigma_u^+ - {}^1\Sigma_g^+$ transition are in reasonable agreement with the experimental results; this is not the case for the ${}^1\Pi_u - {}^1\Sigma_g^+$ interaction, probably owing to the inaccurate value of the « core » interaction in the two-electron molecular model. The purely theoretical LCAO MO results obtained taking into account configuration interaction are in disagreement with the experimental results in both cases. Although no decisive conclusion regarding the accuracy of the method can be drawn, since inclusion of $1s$-electrons will possibly influence the energy spacings, one does not find the disappointing results obtained by others in a similar calculation.

1. – The problem of calculating the electronic energy levels of molecules can be solved, in principle, by expanding the molecular function as a complete set of Slater-determinants. Practically, of course, only a limited number of terms in the expansion can be considered, (configuration interaction method), hence the accuracy of the results depends on the slow or fast convergency of the chosen set. Two semi-empirical methods ([1-4]), bearing some analogy to each other, have been proposed some years ago to improve the convergence

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([1]) W. Moffitt: *Proc. Roy. Soc.*, A **210**, 224 (1951).
([2]) W. Moffitt: *Proc. Roy. Soc.*, A **210**, 245 (1951).
([3]) R. Pariser: *Journ. Chem. Phys.*, **21**, 568 (1953).
([4]) F. G. Fumi and R. G. Parr: *Journ. Chem. Phys.*, **21**, 1864 (1953).

2057

of the C I treatment. We are mainly interested in the method developed by MOFFIT. The calculation of the molecular energy is done as in a perturbative problem, the zero-order molecular function being a proper linear combination of functions built from the states of the infinitely separated atoms. The interaction energy of atoms is evaluated by means of the usual simple orbital approximations to the accurate atomic functions, while the atomic term values are obtained from atomic spectral data. The problem of atoms with its intrinsic difficulties, is hence separated from the more strictly molecular problem, and the elimination of the major source of errors — the theoretical calculation of atomic energies — should give a fairly good convergence with a small number of configurations.

This method has been applied with success to the study of oxygen ([1]), ethylene and benzene ([5]) molecules. A more detailed discussion of the H_2 molecule ([6]) has shown that the results of Moffitt's correction should be considered with some caution, furthermore an application to a simplified model of Li_2 molecule ([7]) has given very disappointing results.

In the present paper we have applied Moffitt's empirical correction to the calculation of the electronic energy levels of Lithium molecule, following substantially the simple scheme which Moffitt has used to treat O_2. Li_2 has been schematically considered as a two electron system, by supposing that the atomic K shell electrons may be treated as if they had shrunk into the nuclei; the wave functions have been constructed by means of the $2s\sigma$, $2p\sigma$ and $2p\pi$ MO. Such a simplified model has been adopted essentially because the only tabulated atomic integrals are those involving the AO which are linearly combined to build the MO we are using. Its limits, on which there has been some discussion ([8]), do not allow us to hope in a good agreement with the experiments. However we are interested to know whether the semi-empirical Moffitt results can be favorably compared with the results of the purely ASMO method.

2. – Table I gives the configurations which correspond to the three electronic states of the molecule, as well as the corresponding wave functions. The wave functions are anti-symmetrized molecular orbitals, constructed by the standard procedure. The application of the empirical correction requires

([5]) W. MOFFITT and J. SCANLAN: *Proc. Roy. Soc.*, A **218**, 464 (1953) and A **220**, 530 (1953).

([6]) C. W. SCHERR: *Journ. Chem. Phys.*, **22**, 149 (1954); A. C. HURLEY: *Proc. Phys. Soc.*, A **68**, 149 (1955); R. PAUNCZ: *Acta Physica Acad. Sci. Hungaricae*, **4**, 237 (1954).

([7]) A. RAHMAN: *Physica*, **20**, 623 (1954).

([8]) J. H. BARTLETT and W. H. FURRY: *Phys. Rev.*, **38**, 1615 (1931); H. M. JAMES: *Journ. Chem. Phys.*, **2**, 794 (1934); C. A. COULSON and W. E. DUNCANSON: *Proc. Roy. Soc.*, A **181**, 379 (1934).

2058

TABLE I. – *Wave functions for some electronic states of* Li_2.

State	Configuration	Wave function (*)
$^1\Sigma_g^+$	$A = s_g,\ s_g$	$\varphi_A = (s_g,\ \bar{s}_g)$
	$B = s_u,\ s_u$	$\varphi_B = (s_u,\ \bar{s}_u)$
	$C = \sigma_g\ \sigma_g$	$\varphi_C = (\sigma_g,\ \bar{\sigma}_g)$
	$D = \sigma_u,\ \sigma_u$	$\varphi_D = (\sigma_u,\ \bar{\sigma}_u)$
	$E = \pi_g,\ \pi_g$	$\varphi_E = 2^{-\frac{1}{2}}[(\pi_g^+,\ \bar{\pi}_g^-) - (\bar{\pi}_g^+,\ \pi_g^-)]$
	$F = \pi_u,\ \pi_u$	$\varphi_F = 2^{-\frac{1}{2}}[(\pi_u^+,\ \bar{\pi}_u^-) - (\bar{\pi}_u^+,\ \pi_u^-)]$
	$G = s_g,\ g_g$	$\varphi_G = 2^{-\frac{1}{2}}[(s_g,\ \bar{\sigma}_g) - (\bar{s}_g,\ \sigma_g)]$
	$H = s_u,\ \sigma_u$	$\varphi_H = 2^{-\frac{1}{2}}[(s_u,\ \bar{\sigma}_u) - (\bar{s}_u,\ \sigma_u)]$
$^1\Sigma_u^+$	$I = s_g,\ s_u$	$\varphi_I = 2^{-\frac{1}{2}}[(s_g,\ \bar{s}_u) - (\bar{s}_g,\ s_u)]$
	$L = \sigma_g,\ \sigma_u$	$\varphi_L = 2^{-\frac{1}{2}}[(\sigma_g,\ \bar{\sigma}_u) - (\bar{\sigma}_g,\ \sigma_u)]$
	$M = \pi_g,\ \pi_u$	$\varphi_M = 2^{-1}[(\pi_g^+,\ \bar{\pi}_u^-) - (\bar{\pi}_g^+,\ \pi_u^-) + (\pi_g^-,\ \bar{\pi}_u^+) - (\bar{\pi}_g^-,\ \pi_u^+)]$ $\varphi_{M'} = 2^{-1}[(\pi_g^+,\ \bar{\pi}_u^-) - (\bar{\pi}_g^+,\ \pi_u^-) - (\pi_g^-,\ \bar{\pi}_u^+) + (\bar{\pi}_g^-,\ \pi_u^+)]$
	$N = s_u,\ \sigma_g$	$\varphi_N = 2^{-\frac{1}{2}}[(s_u,\ \bar{\sigma}_g) - (\bar{s}_u,\ \sigma_g)]$
	$O = s_g,\ \sigma_u$	$\varphi_O = 2^{-\frac{1}{2}}[(s_g,\ \bar{\sigma}_u) - (\bar{s}_g,\ \sigma_u)]$
$^1\Pi_u$	$P = s_g,\ \pi_u$	$\varphi_P = 2^{-\frac{1}{2}}[(s_g,\ \bar{\pi}_u^+) - (\bar{s}_g,\ \pi_u^+)]$
	$Q = s_u,\ \pi_g$	$\varphi_Q = 2^{-\frac{1}{2}}[(s_u,\ \bar{\pi}_g^+) - (\bar{s}_u,\ \pi_g^+)]$
	$R = \sigma_g,\ \pi_u$	$\varphi_R = 2^{-\frac{1}{2}}[(\sigma_g,\ \bar{\pi}_u^+) - (\bar{\sigma}_g,\ \pi_u^+)]$
	$S = \sigma_u,\ \pi_g$	$\varphi_S = 2^{-\frac{1}{2}}[(\sigma_u,\ \bar{\pi}_g^+) - (\bar{\sigma}_u,\ \pi_g^+)]$

(*) The following abbreviations have been used for the molecular orbitals: $s_g; s_u, \sigma_g, \sigma_u, \pi_g, \pi_u$ for $2s\sigma_g$, $2s\sigma_u$; $2p\sigma_g$, $2p\sigma_u$; $2p\pi_g$, $2p\pi_u$, respectively. The superscripts + or − represent one electron MO having the value $+\hbar$ or $-\hbar$ for the component of the angular momentum on the molecular axes. The bar indicates that the spatial part of the one-electron function is associated with spin function β, while the absence of a bar indicates spin functions α. The round brackets represent normalized Slater determinants.

the knowledge of wave-functions of pure asymptotic behaviour, i.e. functions describing states whose dissociations products are either both ions or both neutral atoms. The asymptotic behaviour of the wave functions may be obtained by symmetry arguments or by considering directly the expression of the wave functions at infinite internuclear separation. This second way enables one to obtain the knowledge of the valence states of the dissociation products. Table II gives the wave functions of pure asymptotic behaviour. The last column lists, for every function, the valence states of the dissociation products; they can be obtained by comparing the function representing the dissociation products with the eigen-functions of the terms of the neutral atom and of the ions (9).

(9) In complex cases it may be convenient to analyse the composition of a valence state by means of the projection operator (P. O. LÖWDIN: *Phys. Rev.*, **97**, 1509 (1955)); R. FIESCHI and P. O. LÖWDIN: to be published).

2059

TABLE II. - *Scheme for the application of the « atoms in molecules » correction.*

State	Wave function	Dissociation products
$^1\Sigma_u^+$	$\psi_1 = 2^{\frac{1}{2}}(\varphi_A + \varphi_B)$	$Li^+(^1S) + Li^-(^1S)$
	$\psi_2 = 2^{-\frac{1}{2}}(\varphi_A - \varphi_B)$	$Li\ (^2S) + Li\ (^2S)$
	$\psi_3 = 2^{-\frac{1}{2}}(\varphi_C + \varphi_D)$	$Li^+(^1S) + Li^-[(\frac{2}{3})^{\frac{1}{2}}\ ^1D + (\frac{1}{3})^{\frac{1}{2}}\ ^1S']$ (*)
	$\psi_4 = 2^{\frac{1}{2}}(\varphi_C - \varphi_D)$	$Li\ (^2P) + Li\ (^2P)$
	$\psi_5 = 2^{-\frac{1}{2}}(\varphi_E + \varphi_F)$	$Li^+(^1S) + Li^-[(\frac{1}{3})^{\frac{1}{2}}\ ^1D - (\frac{2}{3})^{\frac{1}{2}}\ ^1S']$
	$\psi_6 = 2^{-\frac{1}{2}}(\varphi_F - \varphi_E)$	$Li\ (^2P) + Li\ (^2P)$
	$\psi_7 = 2^{-\frac{1}{2}}(\varphi_G + \varphi_H)$	$Li^+(^1S) + Li^-(^1P)$
	$\psi_8 = 2^{-\frac{1}{2}}(\varphi_G - \varphi_H)$	$Li\ (^2S) + Li\ (^2P)$
$^1\Sigma_u^+$	$\psi_9 = \varphi_I$	$Li^+(^1S) + Li^-(^1S)$
	$\psi_{10} = \varphi_L$	$Li^+(^1S) + Li^-[(\frac{2}{3})^{\frac{1}{2}}\ ^1D + (\frac{1}{3})^{\frac{1}{2}}\ ^1S']$
	$\psi_{11} = \varphi_M$	$Li^+(^1S) + Li^-[(\frac{1}{3})^{\frac{1}{2}}\ ^1D - (\frac{1}{3})^{\frac{1}{2}}\ ^1S']$
	$\psi_{12} = \varphi_{M'}$	$Li\ (^2P) + Li\ (^2P)$
	$\psi_{13} = 2^{-\frac{1}{2}}(\varphi_N + \varphi_O)$	$Li^+(^1S) + Li^-(^1P)$
	$\psi_{14} = 2^{-\frac{1}{2}}(\varphi_N - \varphi_O)$	$Li\ (^2S) + Li\ (^2P)$
$^1\Pi_u$	$\psi_{15} = 2^{-\frac{1}{2}}(\varphi_P + \varphi_Q)$	$Li^+(^1S) + Li^-(^1P)$
	$\psi_{16} = 2^{-\frac{1}{2}}(\varphi_P - \varphi_Q)$	$Li\ (^2S) + Li\ (^2P)$
	$\psi_{17} = 2^{-\frac{1}{2}}(\varphi_R + \varphi_S)$	$Li^+(^1S) + Li^-(^1D)$
	$\psi_{18} = 2^{-\frac{1}{2}}(\varphi_R - \varphi_S)$	$Li\ (^2P) + Li\ (^2P)$

(*) The coefficients of the symbols for the terms indicate the weight of each term in the valence state. $^1S'$ indicates an ^1S state originating from a higher configuration than ^1S.

The elements of the energy matrix are expressed as usual in terms of integrals over MO's, then in terms of integrals over AO's, by using the proper LCAO expression for the MO's. The values of the integrals over AO's are taken from the tables of KOTANI and others [10]. These tables give also the expression for the Slater AO's. For the effective charge the value $Z = 1$ has been chosen, instead of the Slater value $Z = 1.3$, following the indications of a calculation on F_2 [11]. The approximation of zero overlap and zero differential overlap, which has given a great simplification in the calculation of the electronic levels of O_2 ad O_2^+ [4,12] cannot be used here, since its justification holds only for π-electron systems. A simplification of the matrix follows from the symmetry properties of the MO with respect to exchange of the two nuclei (*).

[10] M. KOTANI, E. ISHIGURO, K. HIJIKATA, T. NAKAMURA and AMEMIYA: *Journ. Phys. Soc. of Japan*, **8**, 463 (1953).

[11] H. DINGER and INGA FISHER-HJALMARS: *Journ. Chem. Phys.*, **22**, 346 (1954).

[12] F. BASSANI, E. MONTALDI and F. FUMI: *Nuovo Cimento*, **3**, 893 (1956).

(*) A well known theorem states that the matrix of an operator H which commutes with another operator O is diagonal with respect to the eigenfunctions of O:

2(60)

3. – In order to apply Moffitt's empirical correction we follow essentially the scheme which has been used to treat O_2 [1], i.e. we correct the values of the energy matrix elements only on considerations of the asymptotic behaviour of the MO's. As a result of the correction one has, instead of the purely theoretical matrix elements H^{theor}, the new elements

$$H_{11}^{\text{emp}} = H_{11}^{\text{theor}} + [W_{11}(\infty) - H_{11}^{\text{theor}}(\infty)] ,$$

$$H_{12}^{\text{emp}} = H_{12}^{\text{theor}} - H_{12}^{\text{theor}}(\infty) ;$$

$W_{11}(\infty)$ indicates the experimental energies of the dissociation products of the wave functions ψ_1, $H_{11}^{\text{theor}}(\infty)$ and $H_{12}^{\text{theor}}(\infty)$ indicate the asymptotic theoretical values of the matrix elements. Table III gives the comparison between the spectroscopic values and the theoretical values (using Slater orbitals) for the atomic and ionic states of Li.

TABLE III. – *Energy of the states of interest of* Li^+, Li, Li^-.

Atomic or ionic state	Electronic configuration	Experimental energy (in a. u.)	Theoretical energy
$Li^+(S)$	(K shell electrons are omitted)	0	0
Li (^2S)	$2s$	— 0.198 1	$-Z/4$
Li (^2P)	$2p$	— 0.130 2	$-Z/4$
$Li^-(^1S)$	$(2s)^2$	— 0.198 1	$-0.636\,7\cdot Z/2$
$Li^-(^1P)$	$(2s, 2p)$	— 0.198 1	$-0.556\,4\cdot Z/2$
$Li^-(^1D)$	$(2p)^2$	— 0.198 1	$-0.629\,7\cdot Z/2$
$Li^-(^1S')$	$(2p)^2$	— 0.198 1	$-0.566\,4\cdot Z/2$

Spectral data and ionization potentials are taken from Landolt-Börnstein tables, vol. I, 1950; the energy of $Li^+(^1S)$ (= 7.280 a.u.) has been chosen as the arbitrary zero energy; the same zero point is implicit in the theoretical expressions, since in the present scheme the $1s$ electrons have merged into the nucleus. For the affinity of Li atom, and the affinity spectrum, following RAHMAN [7], the value zero has been chosen, although theoretical calculations [13] indicate for the affinity a value somewhat higher than — 0.5 eV.

Table IV gives the corrections for each matrix element; matrix elements which are not mentioned remain unchanged.

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the operator O $(a \leftrightarrow b)$ which exchanges the two nuclei, commutes with the energy operator H, hence the matrix elements connecting the eigenfunctions of O $(a \leftrightarrow b)$ with eigenvalue $+1$ (symmetric), to those with eigenvalue -1 (antisymmetric), vanish.

([13]) See, for instance, T. ARAI and T. ONISHI: *Journ. Chem. Phys.*, **26**, 70 (1957).

2061

TABLE IV. – *Matrix elements having the correct asymptotic energy.*

State $^1\Sigma_g^+$	State $^1\Sigma_u^+$	State $^1\Pi_u$
$H^e_{11} = H^t_{11} + 0.1202$	$H^e_{99} = H^t_{99} + 0.1202$	$H^e_{15\,15} = H^t_{15\,15} + 0.0801$
$H^e_{22} = H^t_{22} + 0.1037$	$H^e_{10\,10} = H^t_{10\,10} + 0.1062$	$H^e_{16\,16} = H^t_{16\,16} + 0.1717$
$H^e_{33} = H^t_{33} + 0.1062$	$H^e_{11\,11} = H^t_{11\,11} + 0.0956$	$H^e_{17\,17} = H^t_{17\,17} + 0.1167$
$H^e_{44} = H^t_{44} + 0.2396$	$H^e_{12\,12} = H^t_{12\,12} + 0.2396$	$H^e_{18\,18} = H^t_{18\,18} + 0.2396$
$H^e_{55} = H^t_{55} + 0.0956$	$H^e_{13\,13} = H^t_{13\,13} + 0.0801$	
$H^e_{66} = H^t_{66} + 0.2396$	$H^e_{14\,14} = H^t_{14\,14} + 0.1717$	
$H^e_{77} = H^t_{77} + 0.0801$	$H^e_{9\,10} = H^t_{9\,10} - 0.0401$	
$H^e_{88} = H^t_{88} + 0.1717$	$H^e_{9\,11} = H^t_{9\,11} - 0.0568$	
$H^e_{13} = H^t_{13} - 0.0401$	$H^e_{10\,11} = H^t_{10\,11} - 0.0149$	
$H^e_{15} = H^t_{15} - 0.0568$		
$H^e_{35} = H^t_{35} - 0.0149$		

4. – The electronic excitation energies from the ground state $^1\Sigma_g^+$ to the $^1\Sigma_u^+$ and $^1\Pi_u$ states are shown in Table V. The experimental energies are obtained using the Morse parameters quoted in Herzberg's table ([14]). The agreement between the experimental and the « Moffitt » degree of excitation for the

TABLE V. – *Electronic excitation energies in* Li_2 *(a. u.).*

		Internuclear distance (Å) (*)					
		2.117	2.382	2.646	2.911	3.176	3.705
$^1\Pi_u - {}^1\Sigma_g^+$	ASMO	0.1774	0.1678	0.1593	0.1520	0.1444	0.1287
	Moffitt	0.1768	0.1727	0.1691	0.1661	0.1636	0.1618
	experiment	0.1096	0.1002	0.0952	0.0918	0.0891	0.0841
$^1\Sigma_u^+ - {}^1\Sigma_u^+$	ASMO	0.1131	0.1166	0.1175	0.1166	0.1129	0.1014
	Moffitt	0.0497	0.0571	0.0638	0.0704	0.0770	0.0907
	experiment	0.0775	0.0737	0.0684	0.0634	0 0593	0.0544

(*) The equilibrium internuclear distance for the states $^1\Sigma_g^+$, $^1\Sigma_u^+$, $^1\Pi_u$ is 2.673 Å, 3.108 Å, 2.936 Å respectively.

$^1\Sigma_u^+ - {}^1\Sigma_g^+$ transition is satisfactory, close to the equilibrium distance, and there is a god improvement on the results obtained by the conventional ASMO theory; Moffitt results for the higher ($^1\Sigma_u - {}^1\Pi_g^+$) transition are still unsatisfactory, and slightly worse than the purely theoretical results. However

([14]) G. HERZBERG: *Spectra of Diatomic Molecules* (New York, 1950), p. 546.

the « Moffitt » energy curve for the $^1\Pi_u$ state shows a minimum, which is in good agreement with the experimental minimum, while the purely theoretical curve does not.

The negative result for the $^1\Pi_u - {}^1\Sigma_g^+$ transition is probably due to the two-electron approximation for Li_2. The consideration of the functions corresponding to the three electronic states of the molecule shows that the inclusion of the 1*s* electrons in the molecular core would give larger theoretical energy separation (hence a still worse agreement with the experimental values); the empirical correction, in that case, would work in the favourable direction for both separations, as one expects.

In this calculation no fitting of the « core » parameters to experimental excitation energies has been made, because of the high number of core integrals to be fitted. It is not surprising that the agreement with the experimental values is not so good as in previous calculations on diatomic molecules ([1,4,12]) where it was possible to fit a core integral to the experimental values of a molecular state. It should be pointed out that the method used by MOFFITT for the treatment of O_2, and followed here by us, in general does not correct all the theoretical atomic energy of a given molecular state. The correction is done for each molecular function from considerations on its dissociation products, and, in the LCAO MO scheme, the dissociation products do not describe completely the atomic contents of a molecular function.

It may be of interest to notice that the problem of the Lithium molecule could be also treated by the method of Fumi and Parr. Their method however should be modified, because of the high number of one-centre integrals. One could only try to determine, for these integrals, a common « empirical » effective charge.

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I would like to thank Prof. F. FUMI for suggesting this investigation and Dr. E. MONTALDI for useful discussions, and for checking the calculations.

RIASSUNTO (*)

Si applica il metodo degli « atomi nelle molecole » al calcolo dei livelli energetici degli elettroni del Li_2. Si tratta la molecola come un sistema di due elettroni; si considerano solo configurazioni con $2s\sigma$, $2p\sigma$ e $2p\pi$ come orbitali molecolari. I risultati per la transizione $^1\Sigma_u^+ - {}^1\Sigma_g^+$ sono in ragionevole accordo coi risultati sperimentali; ciò non avviene per l'interazione $^1\Pi_u$ $^1\Sigma_g^+$ probabilmente per effetto del valore poco esatto dell'interazione di « core » nel modello molecolare a due elettroni. I risultati LCAO MO puramente teorici ottenuti tenendo conto della interazione configurativa sono in disaccordo coi risultati sperimentali nei due casi. Benchè non si possa trarre alcuna conclusione decisiva riguardo all'esattezza del metodo, poichè l'inclusione di elettroni 1*s* potrebbe influenzare gli intervalli di energia, non si trovano, tuttavia, i risultati deludenti ottenuti da altri in calcoli simili.

(*) *Traduzione a cura della Redazione.*

On the Conservation of the Lepton Charge.

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(ricevuto il 14 Maggio 1957)

Summary. — The independence of the conservation law for light particles (« lepton charge ») on other invariance properties of the theory of weak interactions, including the two component theory of the neutrino, is shown. Using invariance properties of S-matrix elements with respect to canonical transformations which leave initial and final state unchanged, those expressions of the second degree in the coupling constants are found, which in the matrix elements for double-processes of negaton emission which would violate the conservation law, can alone occur. The two component neutrino theory is hereby a special case.

1. – The conservation law in question is often more inaccurately called the « conservation of light particles ». On the other hand, the concept of a lepton charge points to the fact that the quantity conserved according to this law is the sum of terms capable of both positive and negative signs, ascribed to the different kinds of light particles (« leptons ») involved in the process considered. The lepton charge does not coincide with the electromagnetic charge, but is related to it inasmuch as light fermions with opposite electric charges always have lepton charges different from zero, the signs of which are also opposite. For every electrically charged light fermion it must be investigated separately whether the lepton charge has the same or the opposite sign to the electric charge (a common sign factor for all lepton charges remaining conventional).

Under the assumption of the validity of such a conservation law one tried to extend this concept to neutrinos by ascribing to neutrinos emitted together with negatons the opposite sign of lepton charge to that ascribed to neutrinos emitted together with positons (let us say -1 and $+1$ respectively, so that

negatons and positons get the lepton charge $+1$ and -1 respectively). Such a conservation law has the important consequence that double-processes, where there is neither an electron nor a neutrino in the initial state, while two negatons and no neutrino are present in the final state, should be strictly forbidden.

Such a double-process has for instance been looked for by R. DAVIS [1], the first step being, in his case, the emission of neutrino and negaton in a β decay process of a pile, while the second step is the reabsorption of this same neutrino joined with the emission of a second negaton in the reaction $^{37}\mathrm{Cl}+\nu \rightarrow \rightarrow {}^{37}\mathrm{A}+\mathrm{e}^-$. The reaction in question was indeed absent and its cross-section was found to be smaller than $\frac{1}{3}$ of its theoretical maximum value (given the sum of the absolute squares of all coupling constants). We note already here that the well known coincidence of this theoretical maximum value with the cross-section obtained for Majorana-neutrinos is only true for a parity conserving interaction. Nor has the so-called double β decay without emission of a neutrino, equally forbidden by the conservation law of the lepton charge, ever been observed.

It is the purpose of this note to draw the attention to the theoretical possibility of mixed interactions for which the definition of « neutrino » and « anti-neutrino » is ambiguous and which give for the cross-section of these double-processes *any value between zero and the theoretical maximum value.* A direct check of the conservation law, which claims zero for the cross-sections in question, must therefore be considered as a rather difficult quantitative experimental problem, to be distinguished from a mere distinction between the two discrete values zero and the maximum. In this respect the theoretical situation is the same for parity conserving and parity violating interactions and in the latter case also holds for the two component theory.

An indirect check for the conservation law of the lepton charge in the case of the μ-meson decay would be possible, if the two component theory could here be proved experimentally. The conservation law would then forbid any admixture of the case of Michel parameter 0 to the simple case of Michel parameter $\frac{3}{4}$.

In the following we illustrate our statement in the more complicated case of the nucleon β decay.

2. – Let $\psi^c = C^{-1}\overline{\psi}$ with $\overline{\psi} = \psi^* \gamma_4$ be the charge conjugate solution of the Dirac-equation, where $\gamma_\mu^T = -C\gamma_\mu C^{-1}$, so that $\psi^c = -\psi C$ [2]. We discuss

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[1] R. DAVIS: *Phys. Rev.*, **97**, 766 (1955); *Bulletin of the Washington meeting* (1956), p. 219.

[2] In the corresponding equation on p. 46 of my paper in the volume « Niels Bohr and the development of physics », London 1955, there is unfortunately an error of sign.

2065

here a mixed interaction energy density which we write

$$(1) \qquad H_{\text{int}} = \sum_{i=1}^{5} (\overline{\psi}_{\text{n}} O_i \psi_{\text{p}})[g_{\text{I},i}(\overline{\psi}_{\nu} O_i \psi_{\text{e}}) - f_{\text{I},i}(\overline{\psi}_{\nu}\gamma_5 O_i \psi_{\text{e}}) + \\ + g_{\text{II},i}(\psi_{\nu} C O_i \psi_{\text{e}}) + f_{\text{II},i}(\psi_{\nu} C\gamma_5 O_i \psi_{\text{e}})] + \text{herm. conj.}$$

Here n, p, ν, e refer to neutron, proton, neutrino and electron respectively, ψ_{p} and ψ_{e} shall refer to the emission of a proton and a negaton or to the absorption of an antiproton and a positon. The matrix elements whose absolute squares determine double-processes, where two negatons are emitted, are therefore polynomials of the second degree in those coupling constants which do not contain explicitely their conjugate complex. The symbol O corresponds to the well known five possibilities of scalar, pseudoscalar, vector, pseudovector and tensor interaction, distinguished by an index i or j, which assumes five values.

We do not give in this note explicit expressions for the matrix elements of the double-processes considered ([3]), but restrict ourselves to demonstrating that for the important case of a *vanishing neutrino-restmass* ($m_{\nu} = 0$) considerable simplifications occur, as these matrix elements can only depend on certain combinations of the coupling constants. Indeed there exists a canonical transformation

$$(\text{I}) \qquad \psi' = a\psi + b\gamma_5\psi^c = a\psi + b\gamma_5 C^{-1}\overline{\psi} \qquad \overline{\psi}' = a^*\overline{\psi} - b^*(\overline{\psi^c})\gamma_5 = a^*\overline{\psi} + b^*\psi C\gamma_5$$

with the inverse

$$(\text{I}') \qquad \psi = a^*\psi' - b\gamma_5 C^{-1}\overline{\psi}' \qquad \overline{\psi} = a\overline{\psi}' - b^*\psi' C\gamma_5 ,$$

in which the c-numbers, a, b have to fulfil the single condition

$$(2) \qquad |a|^2 + |b|^2 = 1 .$$

For the canonical character (invariance of the anticommutators) of this transformation the factor γ_5 is essential. This factor has the consequence that in the Hamiltonian of the free particles (as usual written as a sum of commutators) the term with the restmass is not invariant under (I), while the other terms remain invariant. We therefore apply canonical transformations which explicitely depend on γ_5 only to the neutrino field ψ_{ν} for which we assume $m_{\nu} = 0$.

([3]) These are contained in a note by C. Enz: *Nuovo Cimento*, **6**, 250 (1957).

2066

Another simple canonical transformation, often used in the literature, is

$$\text{(II)} \qquad \begin{cases} \psi' = \exp[i\alpha\gamma_5]\psi = (\cos\alpha + i\gamma_5 \sin\alpha)\psi\,, \\ \overline{\psi}' = \overline{\psi}\exp[i\alpha\gamma_5] = \overline{\psi}(\cos\alpha + i\gamma_5 \sin\alpha)\,. \end{cases}$$

The transformations (I) and (II), which commute with each other, give combined the most general linear canonical transformation of the type

$$\psi' = a_1\psi + b_1\gamma_5 C^{-1}\overline{\psi} + a_2\gamma_5\psi + b_2 C^{-1}\overline{\psi}\,. \tag{3}$$

Due to the fact that $\gamma_5 C^{-1}$ and $C\gamma_5$ are skew-symmetric (use $\gamma_5^T = +\, C\gamma_5 C^{-1}$, $C^T = -\, C$), while $\gamma_4 C^{-1}$ and $C\gamma_4$ are symmetric, the conditions for the canonical character of (3) take the simple form [4]

$$\begin{aligned} &a_1 a_1^* + b_1 b_1^* + a_2 a_2^* + b_2 b_2^* = 1\,, \\ &a_1 b_2 - b_1 a_2 = 0\,, \\ &a_1 a_2^* + b_1 b_2^* + a_2 a_1^* + b_2 b_1^* = 0\,. \end{aligned}$$

These have the general solution

$$\text{(4)} \qquad \begin{cases} a_1 = \cos\alpha\cdot a\,, & b_1 = \cos\alpha\cdot b\,, \\ a_2 = i\sin\alpha\cdot a\,, & b_2 = i\sin\alpha\cdot b\,, \end{cases}$$

$$|a|^2 + |b|^2 = 1\,, \tag{2}$$

which just corresponds to the combination of (I) and (II).

In the case of the neutrino with vanishing restmass the transformations (I) and (II) are equivalent to very simple linear transformations of the coupling constants, namely for (I)

$$\text{(I}a\text{)} \qquad \begin{cases} g'_{\mathrm{I},i} = a g_{\mathrm{I},i} + b f_{\mathrm{II},i}\,; & g'_{\mathrm{II},i} = a^* g_{\mathrm{II},i} + b^* f_{\mathrm{I},i} \\ f'_{\mathrm{II},i} = -\, b^* g_{\mathrm{I},i} + a^* f_{\mathrm{II},i}\,; & f'_{\mathrm{I},i} = -\, b g_{\mathrm{II},i} + a f_{\mathrm{I},i}\,. \end{cases}$$

This means that g_{I}, f_{II} transform according to a unitary transformation with determinant 1 and tha. g_{II}, f_{I} transform as g_{I}^*, f_{II}^* or also as f_{II}, $-g_{\mathrm{I}}$.

[4] The calculation is a bit simpler in the Majorana-representation, where $C = \gamma_4$, and γ_4, γ_5 skew-symmetric and pure imaginary.

The transformation (II) is also equivalent to a linear transformation of the coupling constants, namely

$$(\mathrm{II}a)\quad \begin{cases} g'_{\mathrm{I},i} = g_{\mathrm{I},i}\cos\alpha + f_{\mathrm{I},i}\,i\sin\alpha\,; & g'_{\mathrm{II},i} = g_{\mathrm{II},i}\cos\alpha - f_{\mathrm{II},i}\,i\sin\alpha \\ f'_{\mathrm{I},i} = g_{\mathrm{I},i}\,i\sin\alpha + f_{\mathrm{I},i}\cos\alpha\,; & f'_{\mathrm{II},i} = -\,g_{\mathrm{II},i}\,i\sin\alpha + f_{\mathrm{II},i}\cos\alpha\,. \end{cases}$$

Here the pairs $(g_{\mathrm{I}}, f_{\mathrm{I}})$, $(g_{\mathrm{II}}, -f_{\mathrm{II}})$, $(g^*_{\mathrm{I}}, -f^*_{\mathrm{I}})$, $(g^*_{\mathrm{II}}, f^*_{\mathrm{II}})$ transform equally.

The transformation law is a bit simpler for the linear combinations

$$(5)\quad \begin{cases} F_{1,i} = g_{1,i} - f_{\mathrm{I},i}\,; & G_{1,i} = g_{1,i} + f_{\mathrm{I},i}\,, \\ F_{2,i} = g_{\mathrm{II},i} + f_{\mathrm{II},i}\,; & G_{2,i} = -\,g_{\mathrm{II},i} + f_{\mathrm{II},i}\,, \end{cases}$$

which transform for (I) both in the same way, namely

$$(6)\quad \begin{cases} F'_{1,i} = aF_{1,i} + bF_{2,i}\,; & G'_{1,i} = aG_{1,i} + bG_{2,i}\,, \\ F'_{2,i} = -\,b^*F_{1,i} + a^*F_{2,i}\,; & G'_{2,i} = -\,b^*G_{1,i} + a^*G_{2,i}\,, \end{cases}$$

while for the transformation (II) there is the simpler law

$$(7)\quad \begin{cases} F'_{1,i} = \exp[-i\alpha]F'_{1,i}\,; & G_{1,i} = \exp[i\alpha]G_{1,i}\,, \\ F'_{2,i} = \exp[-i\alpha]F_{2,i}\,; & G'_{2,i} = \exp[i\alpha]G_{2,i}\,. \end{cases}$$

The combined transformations (I) and (II) contain as a special case the transformation $\psi'_\nu = \gamma_5\psi_\nu$ which, for our choice of the sign of f_1 in (1), simply interchanges all f's with the corresponding g's, so that the invariants computed in the following will be symmetric in the f's and g's.

The physical significance of the transformations (I), (II) lies in the principle that—even if the Hamiltonian is not invariant for them—*the S-matrix elements of all processes must also stay invariant (up to an arbitrary phase factor), if the initial and the final states are left invariant* [5]. *In this case they can therefore only depend on the invariant forms of the coupling constants.* In particular

[5] The same principle was also applied by D. L. Pursey, independently of the present author, in a paper « *Invariance Properties of Fermi Interactions* ». Pursey only considers interactions in agreement with the conservation of the lepton charge and therefore only the transformation (II) (supplemented by multiplication of the ψ's with an arbitrary phase factor). I am indebted to Dr. Pursey for communicating me his paper prior to publication.

2068

this condition is fulfilled for the double-processes with emission of two negatons and absence of neutrinos in the initial and final states. These processes would violate the conservation law of the lepton charge. While the conjugate complex of the coupling constants do not enter explicitely into the matrix elements of these particular processes, this is the case for other processes with invariant initial and final states. This invariance holds indeed, for all states, in the definition of which a distinction between neutrino and antineutrino does not enter explicitely.

3. – In order to obtain the invariants of the second degree in the f's and g's or in the F's and G's, we consider first the case of a single operator O only, where we can omit the index i.

From (5) and (6) one immediately obtains the invariants

$$F_1F_1^* + F_2F_2^*\,, \qquad G_1G_1^* + G_2G_2^* \tag{8}$$

and

$$F_1G_2 - F_2G_1\,. \tag{9}$$

These invariants already characterise the linear group of four complex variables $(F_1, F_2;\ G_1, G_2)$ completely. Indeed, the most general transformation which leaves (8) and (9) invariant is just the combination of (6) and (7), which we write in matrix form

$$\begin{cases} (F_1', F_2') = (F_1, F_2)\exp[-i\alpha]\cdot\begin{pmatrix} a, & -b^* \\ b, & a^* \end{pmatrix} \\ (G_1', G_2') = (G_1, G_2)\exp[+i\alpha]\cdot\begin{pmatrix} a, & -b^* \\ b, & a^* \end{pmatrix}, \end{cases} \tag{10}$$

(corresponding to 4 real parameters).

To prove this, one notes first that the only transformations (which are assumed not to contain explicitely the conjugate complex of the variables) which leave (8) invariant, are separate unitary transformations of the F's and G's which we may write in matrix form

$$\begin{cases} (F_1', F_2') = (F_1, F_2)\exp[-i\alpha]\cdot\begin{pmatrix} a, & -b^* \\ b, & a^* \end{pmatrix}, \\ (G_1', G_2') = (G_1, G_2)\exp[-i\beta]\cdot\begin{pmatrix} c, & -d^* \\ d, & c^* \end{pmatrix}. \end{cases} \tag{11}$$

Discussing now the invariant (9), it is useful first to examine the special case where (F_1, F_2) remain unchanged. One sees immediately that the only unitary transformation of the G's which leaves (9) invariant is the identity. However, due to the group property of the unitary transformation the general case can be reduced to this particular

one by decomposing (10) into two steps

$$(F_1'', F_2'') = (F_1, F_2) \exp[-i\alpha] \cdot \begin{pmatrix} a, & -b^* \\ b, & a^* \end{pmatrix},$$

$$(G_1'', G_2'') = (G_1, G_2) \exp[+i\alpha] \cdot \begin{pmatrix} a, & -b^* \\ b, & a^* \end{pmatrix},$$

and

$$(F_1', F_2') = (F_1'', F_2''),$$

$$(G_1', G_2') = (G_1'', G_2'') \exp[-i\beta'] \cdot \begin{pmatrix} c', & -d'^* \\ d', & c'^* \end{pmatrix},$$

the latter quantities being determined by the condition

$$\alpha - \beta' = -\beta; \quad \begin{pmatrix} a, & -b^* \\ b, & a^* \end{pmatrix} \begin{pmatrix} c', & -d'^* \\ d', & c'^* \end{pmatrix} = \begin{pmatrix} c, & -d^* \\ d, & c^* \end{pmatrix}, \tag{12}$$

or

$$\beta' = \alpha + \beta; \quad \begin{pmatrix} c', & -d'^* \\ d', & c' \end{pmatrix} = \begin{pmatrix} a^*, & b^* \\ -b, & a \end{pmatrix} \begin{pmatrix} c, & -d^* \\ d, & c^* \end{pmatrix},$$

which expresses that the two steps combined give (11).

Now the first step certainly leaves the form (8) invariant since the expression for (G_1'', G_2'') can also be written

$$\begin{pmatrix} G_2'' \\ -G_1'' \end{pmatrix} = \begin{pmatrix} a^*, & b^* \\ -b, & a \end{pmatrix} \exp[i\alpha] \cdot \begin{pmatrix} G_2 \\ -G_1 \end{pmatrix}.$$

Therefore, the second step in which the F's are unchanged has to leave (8) invariant too. Hence, according to the previous result, it follows

$$\beta' = 0; \qquad \begin{pmatrix} c', & -d'^* \\ d', & c' \end{pmatrix} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix},$$

so that according to (11)

$$\beta = -\alpha; \qquad \begin{pmatrix} c, & -d^* \\ d, & c^* \end{pmatrix} = \begin{pmatrix} a, & -b^* \\ b, & a^* \end{pmatrix},$$

and *we are left with the transformations* (10), q.e.d.

Passing now to the general case of several operators O_i distinguished by the index i, one immediately obtains from (8) and (9)—for instance by substituting for the F's and for the G's arbitrary linear combinations of the F_i''s

2070

and G_i''s respectively and comparing the coefficients—the invariants

$$F_{1i}F_{1j}^* + F_{2i}F_{2j}^* ; \qquad G_{1i}G_{1j}^* + G_{2i}G_{2j}^* , \tag{8a}$$

$$F_{1i}G_{2j} - F_{2i}G_{1j} . \tag{9a}$$

Returning now to the $f_{\mathrm{I}i}$, $f_{\mathrm{II}i}$ and $g_{\mathrm{I}i}$, $g_{\mathrm{II}i}$ with the help of (5), taking the sum and the difference in the case of (8a) and splitting (9a) into the parts symmetric and antisymmetric in i and j, we obtain the final form of the invariants

$$K_{ij} = K_{ji}^* = g_{\mathrm{I}i}^* g_{\mathrm{I}j} + f_{\mathrm{I}i}^* f_{\mathrm{I}j} + g_{\mathrm{II}i}^* g_{\mathrm{II}j} + f_{\mathrm{II}i}^* f_{\mathrm{II}j} \tag{13}$$

$$L_{ij} = L_{ji}^* = g_{\mathrm{I}i}^* f_{\mathrm{I}j} + f_{\mathrm{I}i}^* g_{\mathrm{I}j} - g_{\mathrm{II}i}^* f_{\mathrm{II}j} - f_{\mathrm{II}i}^* g_{\mathrm{II}j} \tag{14}$$

and

$$I_{ij} = I_{ji} = g_{\mathrm{I}i} g_{\mathrm{II}j} + g_{\mathrm{I}j} g_{\mathrm{II}i} + f_{\mathrm{I}i} f_{\mathrm{II}j} + f_{\mathrm{I}j} f_{\mathrm{II}i} , \tag{15}$$

$$J_{ij} = -J_{ji} = g_{\mathrm{I}i} f_{\mathrm{II}j} - g_{\mathrm{I}j} f_{\mathrm{II}i} + f_{\mathrm{I}i} g_{\mathrm{II}j} - f_{\mathrm{I}j} g_{\mathrm{II}i} . \tag{16}$$

We note that for the case $i = j$ the maximum of $|I|^2$ with given K is $\frac{1}{4}K^2$, which is reached for

$$|f_{\mathrm{I}}|^2 = |f_{\mathrm{II}}|^2 ; \qquad |g_{\mathrm{I}}|^2 = |g_{\mathrm{II}}|^2$$

and

$$\frac{f_{\mathrm{I}} f_{\mathrm{II}}}{g_{\mathrm{I}} g_{\mathrm{II}}} = \frac{|f_{\mathrm{I}}|^2}{|g_{\mathrm{I}}|^2} = \frac{|f_{\mathrm{II}}|^2}{|g_{\mathrm{II}}|^2} .$$

We now consider expressions which are invariant apart from a phase factor only. The first of them, denoted by M_{ij}, which transforms for (10) according to

$$M_{ij}' = M_{ij} \exp[-2i\alpha] , \tag{17}$$

is given by

$$\begin{aligned} M_{ij} &= F_{1i}^* G_{1j} + F_{2i}^* G_{2j} = \\ &= g_{\mathrm{I}i}^* g_{\mathrm{I}j} - f_{\mathrm{I}i}^* f_{\mathrm{I}j} - g_{\mathrm{II}i}^* g_{\mathrm{II}j} + f_{\mathrm{II}i}^* f_{\mathrm{II}j} + g_{\mathrm{II}i}^* f_{\mathrm{I}j} - f_{\mathrm{I}i}^* g_{\mathrm{I}j} + g_{\mathrm{II}i}^* f_{\mathrm{II}j} - f_{\mathrm{II}i}^* g_{\mathrm{II}j} . \end{aligned} \tag{18}$$

The other two are

$$\begin{aligned} N_{\mathrm{I},ij} &= -N_{\mathrm{I},ji} = F_{1i} F_{2j} - F_{2i} F_{1j} = \\ &= g_{\mathrm{I}i} g_{\mathrm{II}j} - g_{\mathrm{I}j} g_{\mathrm{II}i} - f_{\mathrm{I}i} f_{\mathrm{II}j} + f_{\mathrm{I}j} f_{\mathrm{II}i} + g_{\mathrm{I}i} f_{\mathrm{II}j} - g_{\mathrm{I}j} f_{\mathrm{II}i} - f_{\mathrm{I}i} g_{\mathrm{II}j} + f_{\mathrm{I}j} g_{\mathrm{II}i} \end{aligned} \tag{18a}$$

and

$$(18b)\quad N_{\mathrm{II},ij} = -N_{\mathrm{II},ji} = G_{1i}G_{2j} - G_{2i}G_{1j} =$$
$$= -g_{\mathrm{I}i}g_{\mathrm{II}j} + g_{\mathrm{I}j}g_{\mathrm{II}i} + f_{\mathrm{I}i}f_{\mathrm{II}j} - f_{\mathrm{I}j}f_{\mathrm{II}i} + g_{\mathrm{I}i}f_{\mathrm{II}j} - g_{\mathrm{I}j}f_{\mathrm{II}i} - f_{\mathrm{I}i}g_{\mathrm{II}j} + f_{\mathrm{I}j}g_{\mathrm{II}i}$$

which transform according to

$$(17a)\qquad N'_{\mathrm{I},ij} = N_{\mathrm{I},ij}\exp[-2i\alpha]\,; \qquad N'_{\mathrm{II},ij} = N_{\mathrm{II},ij}\exp[+2i\alpha]\,.$$

The application of our result to the double-processes of emission of two negatons, in which we are here particularly interested, is therefore *that its S-matrix elements can only depend on the invariants* (15) *and* (16) (or possibly either (18*a*) or (18*b*) without the others).

The conservation law for the lepton charge on the other hand requires the vanishing of these invariants (15), (16) *and* (18*a*), (18*b*). If this is the case, one can, by suitable transformation, (I) and (II), always reach the normal form $g_{\mathrm{II}i} = f_{\mathrm{II}i} = 0$, which corresponds to the gauge group $\psi_\nu \to \psi_\nu e^{i\alpha}$, $\psi_e \to \psi_e e^{i\alpha}$. (One is first left with $b = 0$, $a = e^{i\alpha}$, but the phase shift of g_{I}, f_{I} can be replaced by a phase shift of ψ_e).

Regarding parity we mention briefly that it is sufficient for parity conservation, if either all f_{II}, g_{I} or all f_{I}, g_{II} or all f_{I}, f_{II} or all g_{I}, g_{II} are zero. It is necessary for parity conservation that $L_{ij} = 0$ ([6]).

4. – For the discussion of the two component theory without the premise of the conservation law for the lepton charge it is convenient to use this theory in the form of the Majorana-theory, for which the reality condition

$$(19)\qquad \psi_\nu^c \equiv C^{-1}\overline{\psi}_\nu = \psi_\nu$$

is characteristic ([7]). Although (in the q-number form of a commutator) the current vector is identically zero in such a theory, there exists a pseudovector

$$(20)\qquad J_\mu^{(\nu)} = -\tfrac{1}{2}i\overline{\psi}_\nu\gamma_5\gamma_\mu\psi_\nu\,,$$

([6]) This condition agrees with the condition of PURSEY, l.c. for the particular case $g_{\mathrm{II}} = f_{\mathrm{II}} = 0$, which he treated.

([7]) The equivalence of both theories has been shown by J. SERPE: *Physica*, **18**, 295 (1952). Independently of this paper my attention was drawn to this equivalence by Prof. M. FIERZ, whom I would like to thank very much. It was also discovered by MCLENNAN. Note that what is called space reflection by Majorana corresponds to the operation CP in the notation of OEHME, YANG and LEE. For γ_5 diagonal, the reality condition (11) takes the simple form $\psi_3 = -\psi_2^*$, $\psi_4 = \psi_1^*$. Compare also K. M. CASE: *Phys. Rev.* (in press).

2072

which, according to (19), already has the form of a commutator and which, due to the vanishing of the neutrino restmass fulfils a continuity equation for the free neutrino. When a general conservation law holds (8) it is therefore still possible in the particular case $m_\nu = 0$, to define in the Majorana theory « neutrinos » and « antineutrinos » with the help of the 4-component of the pseudovector $J_\mu^{(\nu)}$.

For our problem it is decisive that the reality condition (19) essentially restricts the transformation group (I). Inserting (19) into (I) we obtain indeed

$$\psi' = (a + b\gamma_5)\psi \; ; \qquad \overline{\psi}' = \overline{\psi}(a^* - b^*\gamma_5) \, ,$$

which is only a canonical transformation if $ab^* + a^*b = 0$ or a/b is pure imaginary. Apart from an arbitrary common phase factor $e^{i\beta}$, the old condition (2) leads back, in this case, to the transformation (II) with

$$a = \cos\alpha \; ; \qquad b = i \sin\alpha \qquad (\alpha \text{ real}). \tag{21}$$

For the following it will be sufficient and convenient (although not essential) to normalize the phase factor to unity.

Due to the Majorana-condition (19) the terms multiplied with g_{II} and f_{II} in the expression (1) for the interaction energy can be omitted, so that it now reads

$$H_{\text{int}} = \sum_{i=1}^{5} (\overline{\psi}_{\text{n}} O_i \psi_{\text{p}})[g_i(\overline{\psi}_\nu O_i \psi_{\text{e}}) - f_i(\overline{\psi}_\nu \gamma_5 O_i \psi_{\text{e}})] + \text{herm. conj.} \tag{22}$$

Both from the equivalent of (IIa), namely

$$\text{(III}a\text{)} \qquad \begin{cases} g_i' = g_i \cos\alpha \; + f_i i \sin\alpha \\ f_i' = g_i i \sin\alpha + f_i \cos\alpha \end{cases}$$

and from the circumstance that g_{II}, f_{II} transform now like g_{I}, $-f_{\text{I}}$, one checks, by substituting the latter for the former in (15) and (16), the invariant character of

$$I_{ij} = I_{ji} = g_i g_j - f_i f_j \tag{23}$$

(8) Both postulates, Majorana theory and conservation law together, can only be fulfilled if the interaction violates parity. Indeed the total current which fulfils a continuity equation, is then the sum of the pseudovector for the neutrino and vectors for the other light particles. Compare B. Touschek: *Nuovo Cimento* 5, 1281 (1957).

2073

and

$$J_{ij} = -J_{ji} = f_i g_j - f_j g_i \,, \tag{24}$$

with respect to the transformation group (II). The matrix elements for double-processes violating the conservation law for the lepton charge, in the two component theory, can only depend on these combinations of the coupling constants.

If this conservation law holds, the invariants must vanish, leading to the two possibilities

$$g_i = f_i \quad \text{or} \quad g_i = -f_i \,, \tag{25}$$

with the same sign for all values of the index i. In this case (IIIa) gives

$$g_i' = g_i \exp[i\alpha] \quad \text{or} \quad g_i' = g_i \exp[-i\alpha] \,,$$

which change of the coupling constants can again be replaced by a change of the phase of ψ_e, namely

$$\psi_e' = \psi_e \exp[-i\alpha] \quad \text{or} \quad \psi_e' = \psi_e \exp[i\alpha] \,, \tag{26}$$

which besides

$$\psi_\nu = \exp[i\alpha\gamma_5]\psi_\nu$$

is a special case of the gauge group given by TOUSCHEK (9) for the conservation law.

Inserting (25) into the expression for H_{int} one obtains the well known result that ψ_ν only occurs in the combination $\overline{\psi}_\nu(1-\gamma_5)$ and $(1+\gamma_5)\psi_\nu$ or $\overline{\psi}_\nu(1+\gamma_5)$ and $(1-\gamma_5)\psi_\nu$ in the Hamiltonian.

It was, however, the purpose of this paper to illustrate the independent position of the conservation law for the lepton charge in all existing neutrino theories which will need also independent quantitative methods to check it empirically.

* * *

I am indebted to Dr. C. ENZ (Zürich) and to Dr. D. L. PURSEY (Edinburgh) for stimulating discussions.

(9) B. TOUSCHEK: *Nuovo Cimento*, l. c.

RIASSUNTO (*)

Si dimostra l'indipendenza della legge di conservazione delle particelle leggere (carica dei leptoni) da altre proprietà di invarianza della teoria delle interazioni deboli, compresa la teoria a due componenti del neutrino. Servendosi di proprietà di invarianza degli elementi della matrice S rispetto alle trasformazioni canoniche che lasciano inalterati gli stati iniziali e finali, si trovano espressioni del genere di secondo grado nelle costanti d'accoppiamento, che sono le sole che possono presentarsi negli elementi di matrice nei processi doppi per l'emissione di negatoni che violerebbero la legge di conservazione. La teoria del neutrino a due componenti ne diventa un caso speciale.

(*) *Traduzione a cura della Redazione.*

2075

The Response of a Liquid Scintillation Counter to Extensive Air Showers of Low Density.

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(ricevuto il 23 Aprile 1957)

Summary. — A large area liquid scintillation counter suitable for experiments on extensive cosmic ray air showers is described. Pulse height distributions are given for vertically incident penetrating particles, the sea level cosmic ray flux and air shower particles. It is found that the scintillator is a more sensitive detector of the particles in cosmic ray air showers than is a tray of Geiger counters of the same area. The greater sensitivity of the scintillator is explained in terms of its greater detection efficiency for the low energy γ-rays present in an air shower. It is found that a value of 2.1 for the ratio of the density of γ-rays (as detected by the scintillator) to the density of electrons (as detected by the Geiger counter) is required to account for the increased sensitivity of the scintillation counter.

1. – Introduction.

The extensive air showers of cosmic rays have been studied in the past using Geiger counters, ion chambers and cloud chambers, and more recently using scintillation counters ([1,2]). From such investigations it has been possible to estimate the total number of particles in a shower and to locate the position of the core.

(*) Also supported by the Nuclear Research Foundation within the University of Sydney.

([1]) *The Oxford Conference on Extensive Air Showers.* Ed. by T. E. Cranshaw (Harwell, Berks, England, 1956).

([2]) G. W. Clark, F. Scherb and W. B. Smith: Technical Report No. 69 (1956) (Massachusetts Institute of Technology, Cambridge, Mass.).

For such a purpose a large area liquid scintillation counter has been developed ([3]). The advantages gained by using such a counter are the low cost of installation and maintenance, high reliability and a linear response to a wide range of values of energy loss in the scintillator.

However the use of a detector which gives a pulse whose height is proportional to the energy loss in a dense medium poses some difficulties in interpretation.

The major contribution to the energy lost in the scintillator by the particles in an extensive air shower comes from electrons and photons. Since the energy distribution and relative densities of these particles is not accurately known, the pulse height distribution from the scintillator for air shower particles cannot be deduced from the pulse height distribution for single relativistic particles.

The investigation reported here was carried out in order to determine the relation between the response to low density extensive air showers of the scintillation counter and of trays of Geiger counters, which have negligible detection efficiency for photons.

2. – The scintillation counter.

2·1. *Description.* – If the scintillation counter is to be useful at low densities, the response to energy loss must be uniform over the sensitive area. Several designs were tried in which the photomultiplier was near to, or in contact with, the scintillator. This construction suffers from poor uniformity of response. Also, the performance characteristics change markedly as the scintillating liquid evaporates. This design was abandoned in favour of a more satisfactory one, in which the photomultiplier views the liquid from a distance comparable with the diameter of the sensitive area. Several detectors of this type have been in continuous use for over a year without any noticeable deterioration in their performance.

The detector consists of 23 litres of p-terphenyl plus diphenyl-hexatriene in xylene in an enamelled dish, 55 cm in diameter. A low concentration of p-terphenyl (3.5 g/l) is used to prevent precipitation during low temperature periods. The scintillations are detected by a single photomultiplier (E.M.I. type 6099B with 11.5 cm diameter photocathode) located 55 cm above the liquid. The dish and photomultiplier are housed in a light-tight galvanised iron right-cylinder whose internal surfaces are lined with white cardboard, to which several coats of white paint have been applied.

The housing is fitted with a breather tube to allow for expansion and contraction of the xylene vapour during the daily temperature cycle. After several months, the small amount of xylene which has been lost by this process is replaced, and for this purpose a filling tube is incorporated in the housing.

The photomultiplier high tension is supplied from a well-regulated radio-frequency supply. Pulses from the photomultiplier are amplified by a single-stage amplifier, inverted and fed through 110 Ω coaxial line to the recording equipment.

([3]) M. H. Brennan and K. Landecker: *Rev. Sci. Instr.*, **27**, 112 (1956).

2·2. *Light collection.* – The use of diphenylhexatriene as a wavelength shifter is essential in this case since the reflection coefficients of the enamel and the white paint drop sharply below 4 000 Å. In the region of the peak of the diphenylhexatriene emission spectrum both the enamel and the white paint have reflection coefficients of 0.80 ± 0.01.

Measurements on the light collection efficiency as a function of radial distance of a source placed 5 cm from the bottom of the enamel dish were made using the scintillations produced by α-particles from a Polonium source and by a collimated beam of γ-rays directed from below the scintillator. Measurements were also made using a small incandescent source.

The results obtained using the three sources were identical, within the experimental uncertainties, and are presented in Table I. Those labelled « direct light » were obtained with the inside of the housing lined with black cloth, whilst those labelled « total » were obtained with the inside walls lined with the white cardboard and the enamel dish in position. Values are all relative to « direct light » and zero radial displacement.

TABLE I.

Distance from centre (cm)	0	8	15	22	27
« Direct Light »	1.00	0.90	0.80	0.73	0.66
« Total Light »	4.50	4.36	4.28	4.09	3.64

It will be observed that the reflecting walls and dish improve the light collection efficiency by a factor 4.5, in reasonable agreement with the relation

$$\eta = \frac{\eta_0}{1 - r}, \tag{1}$$

where η is the collection efficiency with reflector, η_0 the collection efficiency without reflector and where r, the reflection coefficient of the walls and dish, has a value of 0.80.

The nearly uniform efficiency over most of the collecting area is the result of removing the photomultiplier to a distance from the source comparable with the diameter of the dish.

We may now obtain an estimate of the number of photoelectrons ejected from the photocathode when a relativistic singly-charged particle passes vertically through the centre of the dish. We take for the most probable energy loss in 10 cm of xylene the value 16.3 MeV [4]. We further assume that, on the average, 200 eV is required to produce one visible photon [5] and that the average photocathode efficiency is of order 0.1 [6]. Using these figures

[4] R. BASKIN and J. R. WINCKLER: *Phys. Rev.*, **92**, 464 (1953).

[5] F. REINES C. L. COWAN jr., F. B. HARRISON and D. S. CARTER: *Rev. Sci. Instr.*, **25**, 1061 (1954).

[6] J. B. BIRKS: *Scintillation Counters* (London, 1953).

2078

and the experimentally determined ratio of total to direct collection efficiency, we estimate that approximately 100 photoelectrons will be ejected from an 11.5 cm diameter photocathode when a relativistic singly-charged particle passes vertically through the centre of the dish. We neglect absorption of visible photons in the liquid, since their mean free path is much greater than the average path length in the liquid.

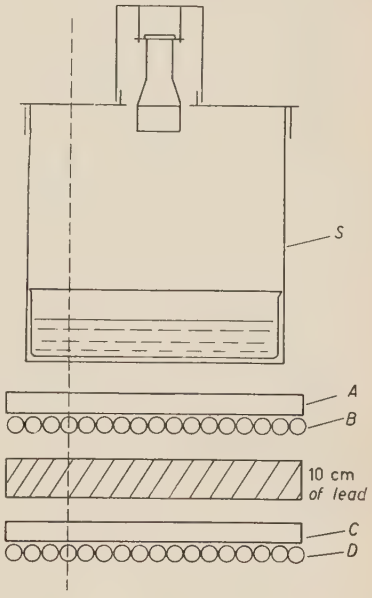

Fig. 1. - Scintillation counter and Geiger counter telescope.

3. – Pulse height distribution for single particles.

If a detector of the type described is to be used to « count » the number of particles passing simultaneously through it, as is the case in air shower experiments, its response to single particles must be known. Accordingly, the following experiment was performed.

3·1. *Experimental arrangement.* – The scintillator counter (S) was placed on top of a large area Geiger telescope ($ABCD$) as shown in Fig. 1. Each tray consisted of 25 counters. The counters of trays A and B and also of C and D were at right-angles to each other.

The associated circuitry was arranged so that a master pulse was obtained from the Geiger trays only if a particle passed vertically through the apparatus. 10 cm of lead placed between the two sets of trays ensured that only relativistic particles were detected. Pulses from the scintillation counter were fed to

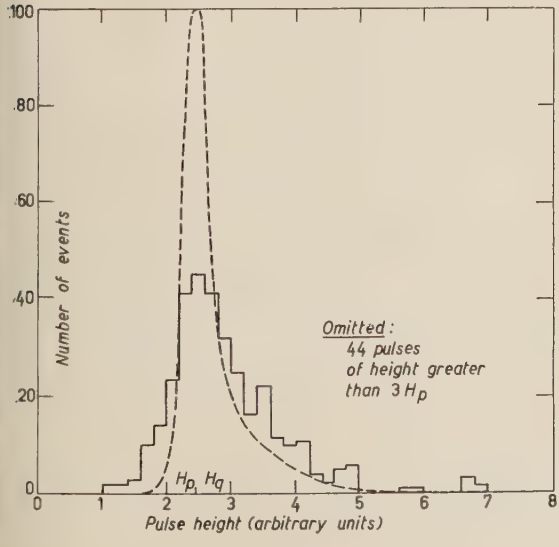

Fig. 2. - Pulse height distribution for single relativistic particles. Also shown are H_P and H_A, the most probable and average pulse heights for single particles. The curve shown is the energy loss distribution for single particles, calculated from the theory of Symon.

the vertical deflection plates of a Tektronix type 531A oscilloscope, which was triggered by a master pulse from the Geiger counter telescope. The trace was photographed on a slowly moving film.

3·2. *Results.* – Fig. 2 shows a histogram of the pulse height distribution obtained, together with a curve, based on the energy loss calculations of Symon (7) normalized to the most probable pulse height, H_P, and to the total number of events, 339, with pulse height less than $3H_P$.

Pulses of height greater than $3H_P$ (44 out of a total of 383 events) are assumed to be caused by cosmic ray showers produced in the laboratory roof and in the scintillator housing. The contribution to such pulses from the tail of the single particle pulse height distribution should be negligible. A number of events with zero pulse height from the scintillator were also recorded. Their frequency was entirely consistent with the relative areas of the scintillator and the larger area Geiger telescope and they are not included in the distribution of Fig. 2.

We notice that although the theoretical energy loss distribution has a half-width of $\sim 15\%$, the light pulse distribution has a half-width of $\sim 25\%$. The additional spread can be fully accounted for by the spread due to photoelectron statistics, to non-uniformity in the light collection efficiency, and to some contribution from knock-on electrons produced in the housing accompanying the penetrating particle through the scintillator.

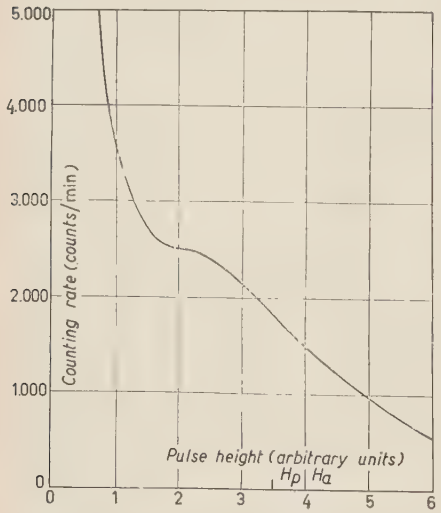

Fig. 3. – Integral-bias curve for cosmic ray particles.

3·3. *Response to single cosmic ray particles.* – The integral pulse height distribution without the requirement of a coincidence with the Geiger counter telescope is shown in Fig. 3. This distribution was obtained with the scintillator on the open roof of the School of Physics. The most probable pulse height (H_P) and the average pulse height (H_A) for single penetrating particles (obtained as described in Sect. 3·1) are also shown.

The distribution (*) is assumed to be the sum of a low energy distribution due to photons and particles which stop in the scintillator and a single particle distribution, similar to that described in Sect. 3·2, with an average pulse height H_A. In the region of H_A the counting rate is a sensitive measure of the gain of the scintillation counter. Accordingly a counting rate of 1500 counts

(7) B. Rossi: *High Energy Particles* (New York, 1952).

(*) The distribution is similar to that obtained by F. B. Harrison, C. L. Cowan jr. and F. Reines: *Nucleonics*, **12**, No. 3, 44 (1954).

20580

per min, which occurs at a bias level equal to H_A, can be used as a means of determining the « single particle level ». The counting rate of the scintillator is regularly checked at this bias level and the photomultiplier gain adjusted, if necessary, to give the above rate.

4. – Low density air showers.

Having established a reliable method for determining the « single particle level », we may now compare the response to low density air showers of the scintillation counter with that of a tray of Geiger counters.

4·1. *Experimental arrangement.* – The air shower detector consisted of three trays of individually hodoscoped counters. Each tray had a sensitive area of 0.96 m² and comprised 24 counters of equal area. These three trays, which were being used to investigate the density spectrum of showers of very low density (+), provided a coincidence pulse when at least one counter from each tray was struck in coincidence. The hodoscope results were recorded on punched cards.

As in the experiment described in Sect. 3, the pulse from the scintillator was taken to the vertical deflection plates of an oscilloscope whose sweep was triggered by the three-fold coincidence pulse from the Geiger counter trays. The pulse height was again recorded on continuously moving film. The rate of accidental coincidences between triggering event and scintillator pulses of height greater than that corresponding to 1 MeV energy loss in the scintillator was negligible.

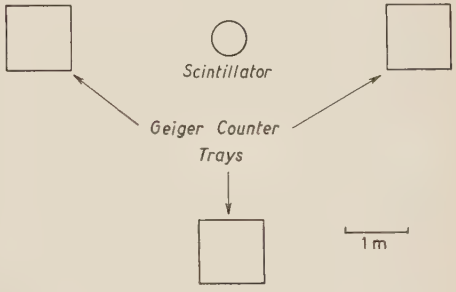

Fig. 4. – Low density air shower array

The arrangement of scintillator and three Geiger trays is shown in Fig. 4. During the course of the experiments on the density spectrum of showers, a separate run was made with a fourth tray of individually hodoscoped Geiger counters placed on the circle passing through the other three trays. The area of the fourth tray was 0.23 m², equal to that of the scintillator. The pulse height from the scintillator was not recorded during this run.

4·2. *Results.* – A total number of 1 587 showers were recorded with the scintillator. Of these, 714 $((45 \pm 2)\%)$ gave a pulse equal to or greater than that corresponding to 1 MeV energy loss.

In the run with the fourth tray of Geiger counters, in position, a total number of 1 121 showers were recorded. Of these 404 $((36 \pm 2)\%)$ set off one or more of the Geiger counters in this tray. The corresponding proportion of

(+) The author is indebted to H. D. Rathgeber and H. S. Murdoch for the use of this equipment, also for access to their unpublished results.

such coincidences calculated on the assumption of constant exponent 1.45 in the integral density spectrum of air showers is 34% which agrees with the experimental value.

A possible explanation of the discrepancy between the detection efficiency of the scintillator and of the Geiger counter tray is that the density of particles is not uniform over the array. The scintillator may have been in general in a region of higher particle density than the Geiger trays. This would be the case for example if the showers detected were predominantly small ones whose cores fell within the triangle defined by the three triggering trays. Accordingly, evidence for such a bias was sought in the hodoscope records; it was found, however, that those counters close to the centre of the array did not fire any more frequently than the ones near the outer extremities of the trays. The absence of any evidence for this bias agrees with the results of calculations similar to those made by SINGER ([8]).

The discrepancy between the counting rates of the scintillator and the fourth tray of Geiger counters must therefore be due to the greater sensitivity of the scintillator to the particles in an extensive air shower. We shall show later that this higher sensitivity may be explained in terms of the scintillator's response to low energy γ-rays.

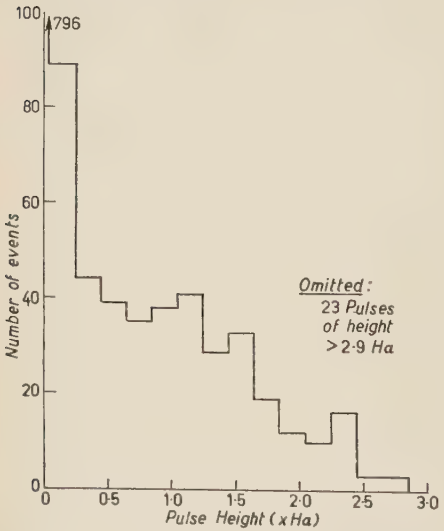

Fig. 5. – Pulse height distribution from scintillator for showers for which the electron density lies in the range 0.3 m^{-2} to 3 m^{-2}.

Again, as in the experiments reported in Sect. 3, a pulse height distribution was obtained from the scintillator. In doing so, attention was restricted to showers of very low density for which the probability of more than one particle traversing the scintillator is expected to be small. These were showers for which not more than 10 Geiger counters of the 72 in the three triggering trays were fired. The major contribution to events of this type comes from showers whose calculated densities are in the range 0.3 m^{-2} to 3 m^{-2}. The pulse height distribution from the scintillator for 1230 such showers is shown in Fig. 5.

The same selection criterion was applied to the run with the fourth tray of Geiger counters with the following results:

Total numbers of showers	927	
Number in which no Geiger counter in the fourth tray was fired	677	(73%)
Number in which one Geiger counter was fired	185	(20%)
Number in which 2 or more counters were fired	65	(7%)

([8]) S. F. SINGER: *Phys. Rev.*, **81**, 579 (1951).

2082

Unlike the single particle distribution of Fig. 2, the pulse height distribution for the particles in a low density air shower, shown in Fig. 5, exhibits a large number of events in which the energy lost is a small fraction of H_A, the average energy loss for a single straight through particle. A contribution of this type would be expected from the interactions in the scintillator of low energy γ-rays. Again, comparing the results obtained on the multiplicity of counters fired in the fourth tray with the pulse height distribution of Fig. 5 it may be readily found that a bias level corresponding to 8 MeV energy loss would be required for the scintillator sensitivity to be equal to that of the Geiger counter tray in which at least one counter must be struck. The corresponding minimum energy loss to be accepted if the sensitivity is to equal that of the Geiger tray in which at least two counters are to be struck is of order 30 MeV, or somewhat less than twice the average energy loss for straight through particles.

It should be noted that the above conclusions apply only where the triggering arrangement is similar to that used in this investigation. Other triggering requirements detecting different shower densities could be expected to lead to an energy loss distribution different from that of Fig. 5.

4·3. *Discussion.* – The scintillation counter gives a measure of the total energy loss by particles traversing it; in this it differs from the Geiger counter which merely indicates the presence or absence of a traversal. A further point of difference lies in the sensitivity of the scintillator to γ-rays compared with the corresponding sensitivity of the Geiger counter.

Considering first the sensitivity to electrons of the two types of detector which are here compared, we may note that little quantitative work has been done on the energy dependence of the efficiency of Geiger counters as used in air shower work. In the present experiment the total shielding between the outside air and the sensitive volume of the Geiger counters was 0.71 gm cm^{-2}, comprising aluminium roof, glass counter wall and copper cathode; the corresponding shielding in the case of the scintillator consisted of 1.2 gm cm^{-2} iron. The attenuation of incident electrons traversing this shielding will therefore be greater for the case of the scintillator than for the Geiger counters. The fact that the iron roof of the scintillator housing is farther from the scintillator liquid than is an aluminium roof above the Geiger trays (this last distance is of order 15 cm) will increase this difference because of scattering. From the electron range-energy relation of Johns, Cunningham and Katz [9] the cut-off energy for the scintillator is 4.5 MeV, if we consider only pulses greater than 1 MeV, and for the counter trays, 1.5 MeV. These are of course lower limits; without knowledge of the energy distribution of low energy electrons in air showers and of the attenuation of electrons as a function of energy the *effective* cut-off energies cannot be determined.

Although the scintillator is less sensitive to incident low energy electrons than is the Geiger counter tray, it is a considerably more efficient detector of low energy γ-rays present in the extensive air showers. Table II shows the calculated efficiency of the xylene scintillator for the detection of photons of the energies listed through the production of at least one electron which has

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[9] H. E. Johns, J. R. Cunningham and L. Katz: *Phys. Rev.*, **83**, 952 (1951).

an energy of 1 MeV or greater by the two processes of Compton scattering and pair production. The probability that such an electron will be produced in the iron roof and will be detected is smaller — of order 0.03 for photons in the energy range 5 to 50 MeV. The efficiency of detection of photons by the Geiger counter tray on the other hand is only of order 0.01.

TABLE II.

Photon Energy (MeV)	1.2	1.5	2	4	10	20	30-250
Probability for production of electrons with energy > 1 MeV in scintillator	0	0.13	0.20	0.21	0.15	0.12	0.1

It is reasonable therefore to attribute the greater sensitivity of the scintillation counter to extensive air showers to its greater efficiency in detecting low energy γ-rays. If we assume a γ-ray energy spectrum of the form proposed by RICHARDS and NORDHEIM ([10]) and detection probabilities as listed in Table II, we can calculate the contribution of such a spectrum to the counting rate of the scintillator in coincidence with the three Geiger trays. If this is done we find that the increased sensitivity of the scintillator can be explained if the ratio of the density of photons (as detected by the scintillator) to the density of electrons (as detected by the Geiger counters) is 2.1. This result is of the same order of magnitude as that predicted by the theoretical calculations of Richards and Nordheim.

In obtaining this result we have not taken account of the fact that the scintillator is *less efficient* than the Geiger counter in detecting low energy electrons; the effective cut-off energy for electrons is considerably higher for the scintillator. If this effect were taken into account the result would be a slight *increase* in the value for the ratio of the density of photons to the density of electrons.

In principle if the electron and photon energy spectra at low energies were known, and if the attenuation of the electrons in the absorber above the scintillator were also known the pulse height distribution of Fig. 5 could be calculated theoretically. In the absence of sufficient information this has not been attempted. It can be said however that this pulse height distribution cannot be accounted for on the simple assumption that all electrons give a pulse height distribution similar to that of Fig. 2 for straight through particles and that added to this is the pulse height distribution to be expected from a photon spectrum of the Richards-Nordheim form. There are too many pulses of height less than H_A, indicating the presence of electrons of less than 16 MeV.

Further experiments are now in progress, designed to compare the responses of scintillation counters and Geiger trays to showers of higher particle density.

([10]) J. A. RICHARDS and L. W. NORDHEIM: *Phys. Rev.*, **74**, 1106 (1948). The differential spectrum approximates to $1/E$ at low energies and to $1/E^2$ at high energies.

20844

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The author acknowledges the helpful criticism of Dr. D. D. MILLAR in discussions on this work. He is also indebted to Mr. C. S. WALLACE for his assistance in the work described in Sect. **4**. His thanks are also due to Professor H. MESSEL and the Nuclear Research Foundation within the University of Sydney for the excellent facilities made available to him, and to the Australian Atomic Energy Commission for a Research Studentship.

RIASSUNTO (*)

Si descrive un contatore a scintillazione a liquido di grande superficie adatto per esperienze su sciami estesi dell'atmosfera. Si danno le distribuzioni d'intensità d'impulsi per particelle penetranti incidenti verticalmente, per il flusso di raggi cosmici a livello del mare e per le particelle degli sciami atmosferici. Si trova che lo scintillatore è un rivelatore più sensibile delle particelle degli sciami atmosferici che non un telescopio di contatori Geiger della stessa superficie. La maggior sensibilità dello scintillatore si spiega in termini della sua maggior efficacia di rivelazione per i raggi γ di bassa energia presenti in uno sciame dell'aria. Si trova che è necessario un valore 2.1 per il rapporto della densità di raggi γ (rivelati dallo scintillatore) alla densità di elettroni (rivelati dal Geiger) per render conto della maggior sensibilità del contatore a scintillazione.

(*) *Traduzione a cura della Redazione.*

(*La responsabilità scientifica degli scritti inseriti in questa rubrica è completamente lasciata dalla Direzione del periodico ai singoli autori*)

Unified Description of Ordinary and Isotopic Space.

D. IVANENKO and H. SOKOLIK

Physical Department of the Moscow University

(ricevuto il 26 Gennaio 1957)

The main purpose of this paper is to develop a mathematical formalism used for the unification of habitual and isotopic space. As the description of new internal degrees of freedom of elementary particles by means of iso-space proved to be very successful, it seems necessary to include isotopic variable terms in the equations governing the behavior of wave functions. The previous attempts published in papers ([1-3]) seem unsatisfactory as they lead to the equations which consist of two parts, each part operating on corresponding variables, e.g. 7-dimensional space being thus reduced to a direct sum of 4 and 3-dimensional subspaces. In our recent paper ([3]) a general theory of such equations was developed, according to which equations describing particles of arbitrary space and isotopic spin have a form:

$$L_i \frac{\partial \psi}{\partial x_i} + \varkappa \psi + (\sigma_j I_j / \Lambda)\psi = 0 \qquad \begin{matrix}(i = 1, 2, 3, 4)\\ (j = 1, 2, 3)\end{matrix} \tag{1}$$

where L_i and σ_j satisfy conditions of Lorentz and isotopic invariance. In ([3]) we managed to obtain a classification of (1) according to irreducible representations of the Lorentz and isotopic groups defined by numbers K_0 K_1 and l respectively, after which (x, ω) is transformed. When these representations are defined by $(-\frac{1}{2}\ \frac{3}{2}) = (\frac{1}{2}\ \frac{3}{2})$ and $\frac{1}{2}$, we have the equation proposed by A. PAIS ([1]). We believe it more reasonable to admit rotations between 4-space and iso-space. Bearing in mind this possible fusion of spaces we shall treat in this paper the mathematical problem of the classification of n-dimensional equations, which have the form:

$$\Gamma_i \frac{\partial \psi}{\partial x_i} + \varkappa \psi = 0 \qquad (i = 1, 2, \ldots n) \tag{2}$$

according to representations of an n-dimensional group of rotations and inversions.

([1]) A. PAIS: *Physica*, **59**, 8, 69 (1953).
([2]) I. TAMM and V. GINSBURG: *Žu. Eksper. Teor. Fiz.* (*J. Exper. Theoret. Phys.*), **17** (1947).
([3]) D. IVANENKO and H. SOKOLIK: *Dokl. Ak. Nauk SSSR* (*C. R. Ac. Sci. USSR*), **97**, no. 4 (1954).

As it was shown by E. CARTAN (4), such a problem can be formulated in the following way: one must find irreducible representations of the Lee-group, to which the direct product $\partial\psi_i/\partial x_j$ is reduced; ψ_i is known to transform after an arbitrary representation of the same group (n-gradient is $\partial/\partial x_j$). Invariance conditions for the $2k+2$-case have the form:

$$[[\Gamma_{2K+2} I_{2K+2,2K+1}] I_{2K+2,2K+1}] = \Gamma_{2K+2}\,, \tag{3}$$

where $I_{2K+2\,2K+1}$ is the generator of the $2k+k$-orthogonal group (5), other Γ_i being expressed by means of relations:

$$[\Gamma_{2K+2}, I_{2K+2,2K}] = \Gamma_{2K} \dots [\Gamma_{2K+2}, I_{2K+2,1}] = \Gamma_1\,. \tag{2'}$$

According to (3), basic vectors of the representation are defined by numbers satisfying following unequalities;

$$m_{2P+1,i+1} \leqslant m_{2P,i} \leqslant m_{2P+1,j}\,, \quad m_{2P,i+1} \leqslant m_{2P-1,j} \leqslant m_{2P,i}\,, \quad -m_{2P,P} \leqslant m_{2P-1,P} \leqslant m_{2P,P}\,, \tag{4}$$
$$(i = 1, 2, \dots, P)\,.$$

Then using a Lemma of Shur one gets Γ_{2K+2} in the following form:

$$\Gamma_{2K+2}(\alpha)_\tau = \sum_{\tau'} C_{\tau\tau'}^{m_{2K,1} \dots m_{2K,K}} \xi(\alpha)_{\tau'}\,, \tag{5}$$

where τ stands for irreducible representation. Substituting (5) in (3) we have a system of n homogeneous algebraic equations for $C_{\tau\tau'}^{m_{2K,1}\dots m_{2K,K}}$, where $n = 13K!/2!(K-2)!$ $K \neq 1$ and $K=1$, $n=4$. The necessary and sufficient condition for $C_{\tau\tau'}^{m_{2K,1}\dots m_{2K,K}}$ being different from zero is «linking» τ and τ', which means that τ' is one of the irreducible representations to which $\tau\cdot\tau_0$ is reduced, where τ_0 is a $2k+2$-vector representation defined by (1, 0, ..., 0). Thus we obtain for $2k+2$-case for the direct product:

$$[\tau\times\tau_0] = (m_{2K+1,1}+1 \dots m_{2K+1,K+1}) \dotplus (m_{2K+1,1}-1 \dots m_{2K+1,K+1}) \dotplus \dots + \tag{6}$$
$$\dotplus (m_{2K+1,1} \dots m_{2K+1,K+1}+1) \dotplus (m_{2K+1,1} \dots m_{2K+1,K+1}-1)\,.$$

Respective expression for the $2k+1$-case have following form:

$$[\tau\times\tau_0] = (m_{2K,1}+1 \dots m_{2K,K}) \dotplus (m_{2K,1}-1 \dots m_{2K,K}) \dotplus \dots + \tag{7}$$
$$\dotplus (m_{2K,1} \dots m_{2K,K}+1) \dotplus (m_{2K,1} \dots m_{2K,K}-1) \dotplus (m_{2K,1} \dots m_{2K,K}) \Delta m_{2K,K}\,;$$
$$\Delta_{2K,K} = \begin{cases} 1 & m_{2K,K} \neq 0 \\ 0 & m_{2K,K} = 0 \end{cases}.$$

(4) E. CARTAN: *Leçons sur la theorie des spineurs* (1938).
(5) I. GEL'FAND and M. ZETLIN: *Dokl. Ak. Nauk SSSR* (*C. R. Ac. Sci. USSR*), **71**, no. 6 (1950).

Zentralblatt

Now we can obtain an effective formula for Γ_{2K+2}:

$$C_{\tau\tau}^{m_{2K,1}\dots m_{2K,K}} = C_{\tau\tau'} \prod_{1\leqslant j\leqslant K} \sqrt{(m_{2K,j} + m_{2K-1,j} + 2K - i - j + 2)(m_{2K,j} - m_{2K+1,j} + i - j - 1)}$$

$$m'_{2K-1,i} = m_{2K+1,i} + 1\,, \qquad m'_{2K+1,j} = m_{2K+1,j}, \qquad i \neq j. \tag{8}$$

Similiar treatment gives in $2k+1$-case:

$$\text{if} \quad m'_{2K,i} = m_{2K,i} + 1\,, \qquad m'_{2K,j} = m_{2K,j}\,, \qquad j \neq i \tag{9}$$

$$C_{\nu\nu'}^{m_{2K-1,1}\dots m_{2K-1,K}} = C_{\nu\nu'} \prod_{1\leqslant j\leqslant K} \sqrt{(m_{2K,i} + m_{2K-1,i} + 2K - j - i + 1)(m_{2K,i} - m_{2K-1,j} - j + i - 1)}$$

and

$$m'_{2K,i} = m_{2K,i} \qquad (i = 1, 2, \dots, K),$$

if

$$C_{\nu\nu'}^{m_{2K-1,1}\dots m_{2K-1,K}} = C_{\nu\nu'} \prod_{1\leqslant j\leqslant K} m_{2K-1,j}\,,$$

$C_{\tau\tau'}$ and $C_{\nu\nu'}$ being arbitrary complex numbers. (ν is an irreducible representation of the $2k+1$-group).

One sees that Γ_{2K+2} and L_{2K+1} are built of blocks $C_{\tau\tau'}^{m_{2K,1}\dots m_{2K,K}}$ and $C_{\nu\nu'}^{m_{2K-1,1}\dots m_{2K-1,K}}$, numbers $m_{2K,1}\dots m_{2K,K}$ and $m_{2K-1,1}\dots m_{2K-1,K}$ standing for values of space spin, isotopic spin and other invariants of isotopic space, in case these blocks have diagonal form with eigenvalues different from zero. For instance, in 8-dimensional case we have three quantum numbers corresponding to ordinary spin, isotopic spin and new number proposed in [6] which can be connected with strangeness of GELL-MANN and NISHIJIMA. Thus knowledge of the representation after which ψ is transformed gives us full information about Γ_i.

Condition of the inversion invariance

$$[\Gamma_n T] = 0\,, \tag{10}$$

brings further restriction on $C_{\tau\tau'}$ and $C_{\nu\nu'}$. T being defined as generator of the inversion group: $x'_n = -x_n$; $x'_i = x_i$, where x_n is the imaginary variable corresponding to the time. Under inversion $\tau = (m_{2K+1,1}\dots m_{2K+1,K+1})$ change into a representation $\dot{\tau} = (m_{2K+1,1}\dots -m_{2K+1,K+1})$; in the case when $m_{2K+1,K+1} = 0$ τ and τ' are identical.

We come now to the following form for T:
in $2K+1$, $K+1$ case

$$T = \begin{pmatrix} 0 & (-1)^{\Sigma\lambda_i} \\ (-1)^{\Sigma\lambda_i} & 0 \end{pmatrix} \quad \lambda_i = m_{2K,i} \ \text{ or } \ m_{2K,i} + 1 \qquad (i = 1, 2, \dots, K). \tag{11}$$

T is applied to vector $\begin{pmatrix} \xi(\alpha)_\tau \\ \xi(\alpha)_{\dot{\tau}} \end{pmatrix}$.

[6] A. SALAM and I. C. POLKINGHORNE: *Nuovo Cimento*, **2**, 685 (1955).

In $\lambda_{2K+1,K+1}=0$ case

$$T=(-1)^{\Sigma\lambda_i} \qquad \lambda_i=m_{2K,i} \text{ or } m_{2K,i}+1 \qquad (i=1,2,\ldots,K). \tag{12}$$

In $2k+1$-case τ and $\dot{\tau}$ are always identical.

Thus the conditions of inversion invariance in $2k+2$-case are brought to the form:

$$\begin{cases} C_{\tau\tau'}=\pm C_{\tau\dot{\tau}'} & \tau=\dot{\tau} \qquad \tau'\neq\dot{\tau}' \\ C_{\tau\tau'}=\pm C_{\dot{\tau}\tau'} & \tau\neq\dot{\tau} \qquad \tau'=\dot{\tau}' \\ C_{\tau\tau'}=C_{\dot{\tau}\dot{\tau}'} & \tau=\dot{\tau} \qquad \tau'=\dot{\tau}' . \end{cases} \tag{13}$$

In $2K+1$ case similar relations can be obtained in the same way.

As one of the possible applications of new formalism we can point out that expressions (11) and (12) make it possible to treat non-conservation of K-meson parity [7] by admitting that parity corresponds to inversion in 8-space. In that case alternation of λ_1; $m_{2K,1}\to m_{2K,1}+1$ connected with non-conservation of space parity, is compensated by shifting isotopic parity λ_2 or λ_3.

* * *

We would like to thank M. Zetlin and A. Brodsky for valuable discussions.

(7) T. D. Lee and C. N. Yang: *Phys. Rev.*, **102**, no. 1, 290 (1956).

2089

Nature of « p » in Bonnor's Unified Field Theory.

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(ricevuto il 30 Marzo 1957)

In 1953, Einstein [1] proposed the following set of field equations:

$$\text{(I)} \qquad \begin{cases} \text{(1.1)} & \underset{+-}{g_{ik:l}} = 0 \\ \text{(1.2)} & \underset{\vee}{\Gamma^s_{is}} = 0 \\ \text{(1.3)} & \underline{R_{ik}} = 0 \\ \text{(1.4)} & \underset{\vee}{R_{[ik,l]}} = \underset{\vee}{R_{ik,l}} + \underset{\vee}{R_{kl,i}} + \underset{\vee}{R_{li,k}} = 0 , \end{cases}$$

where R_{ik} is the tensor given by

$$R_{ik} = \Gamma^s_{ik,s} - \tfrac{1}{2}(\Gamma^s_{is,k} + \Gamma^s_{ks,i}) - \Gamma^s_{it}\Gamma^t_{sk} + \Gamma^s_{ik}\underline{\Gamma^t_{st}} . \tag{2}$$

He derived this set of field equations by the variation

$$\delta \int_{\cdot}^{\cdot} \mathfrak{H}\, \mathrm{d}T = 0 . \tag{3}$$

the Hamiltonian $\mathfrak{H}$, being given by

$$\mathfrak{H} = \mathfrak{g}^{ik} R^{**}_{ik} , \tag{3.1}$$

where

$$\mathfrak{g}^{ik} = \omega g^{ik} , \qquad (\omega = \sqrt{-\det g_{ik}}),$$

[1] A. Einstein: *The Meaning of Relativity* (Princeton, 1953); Appendix II (4th Ed.).

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and

$$R^{**}_{i\,k} = \Gamma^{s}_{ik,s} - \Gamma^{t}_{is}\Gamma^{s}_{tk} - (\log\omega)_{,ik} + (\log\omega)_{,t}\Gamma^{t}_{ik}\,.$$

It may be noted that (3) is varied subject to the condition

$$\mathfrak{g}^{is}_{\vee\,,s} = 0\,, \tag{3.2}$$

which is equivalent to (1.2).

Bonnor, in 1954 [2] obtained a modified form of the field equations (I), by taking the Hamiltonian

$$\mathfrak{H}^{*} = \mathfrak{H} + p^{2}\mathfrak{g}^{ik}g_{\underset{\vee}{ki}}\,, \tag{4}$$

$\mathfrak{H}$ being given by (3.1) and p being an arbitrary real or imaginary constant, and by varying

$$\delta\int\mathfrak{H}^{*}\,\mathrm{d}T = 0\,,$$

under the condition $g^{is}_{\vee\,,s} = 0$.

These modified field equations are:

$$\text{(II)}\qquad \begin{cases} (5.1) & g_{\underset{+-}{ik;l}} = 0 \\ (5.2) & \Gamma^{s}_{\underset{\vee}{is}} = 0 \\ (5.3) & R_{\underline{ik}} + p^{2}U_{\underline{ik}} = 0 \\ (5.4) & R_{\underset{\vee}{[ik,l]}} + p^{2}U_{\underset{\vee}{[ik,l]}} = 0\,, \end{cases}$$

where

$$U_{ik} = g_{\underset{\vee}{ki}} - g^{mn}_{\vee}g_{im}g_{nk} + \tfrac{1}{2}g^{mn}_{\vee}g_{nm}g_{ik}\,. \tag{6}$$

By putting $g_{ik} = \delta_{ik} + \gamma_{ik}$ and neglecting the products of the V_{ik} and their derivatives, the set (II), in the linearized form becomes [3]:

$$\begin{cases} (7.1) & T^{l}_{ik} = \tfrac{1}{2}(-\gamma_{ki,l} + \gamma_{lk,i} + \gamma_{il,k}) \\ (7.2) & \gamma_{\underset{\vee}{is},s} = 0 \\ (7.3) & \gamma_{\underline{is},sk} + \gamma_{\underline{sk},si} - \gamma_{\underline{ki},ss} - \gamma_{ss,ki} = 0 \\ (7.4) & \gamma_{\underset{\vee}{[ik,l],ss}} = 4p^{2}\gamma_{\underset{\vee}{[ik,l]}}\,. \end{cases}$$

[2] W. B. Bonnor: *Proceedings of the Royal Society of London*, A **226**, 366 (1954).

[3] For example: A. Einstein and E. G. Strauss: *Annals of Mathematics*, **47**, 731 (1946).

It may be noted that BONNOR (1954) gave,

$$\underset{\vee}{\gamma}_{[ik,l],ss} = -4p^2\underset{\vee}{\gamma}_{[ik,l]},$$

instead of (7.4). But by direct calculation we have

$$\underset{/}{U}_{ik} \sim -2\underset{\vee}{\gamma}_{ik} \qquad \text{and} \qquad \underset{\vee}{R}_{ik} \sim \tfrac{1}{2}\gamma_{ik,ss},$$

and therefore (5.4) yields (7.4)

Defining electric current density by

$$I_{ikl} = \underset{\vee}{g}_{[ik,l]}, \tag{8}$$

the equation (7.4) takes up the form

$$I_{ikl,ss} = 4p^2 I_{ikl}. \tag{9}$$

The set (II) therefore implies a restriction on the current density, different from that implied by the set (I), which can be obtained by putting $p = 0$ in the result (9).

For a spherically symmetric particle at rest, the equation (9) takes up the form

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\varrho}{\mathrm{d}r}\right) = 4p^2\varrho, \tag{10}$$

ϱ being the charge density and r the radial co-ordinate. The most general solution of this equation is of the form

$$\varrho = \frac{1}{r}\left(A\exp[2pr] + B\exp[-2pr]\right), \tag{11}$$

where A and B are arbitrary constants.

We will now discuss the solution (11) according to the nature of the constant p.

a) Let p be real and positive.

Applying the boundary condition that ϱ tends to a finite constant as $r \to \infty$, we have that $A = 0$, so that the solution (11) takes the form

$$\varrho = \frac{B}{r}\exp[-2pr]. \tag{11.1}$$

If we suppose that the solution (11.1) refers to a particle, it will not apply near $r = 0$ as the equation (9) and consequently the solution (11.1) is true only for weak fields. The solution (11.1), therefore, gives the approximate charge distribution for large r, say $r > R$.

b) Let now p be real and negative so that we write $p = -p_1$ where p_1 is

positive. Then the solution (11) takes the form

$$\varrho = \frac{1}{r}\left(B \exp\left[2p_1 r\right] + A \exp\left[-2p_1 r\right]\right).$$

Applying again the boundary condition, we have (as in the case *a*)) $B = 0$ so that the solution takes up the form

$$\varrho = \frac{A}{r} \exp\left[-2p_1 r\right].$$

This solution is exactly the same as (11.1).

Thus, when p is real, we get

$$\varrho = \frac{A}{r} \exp\left[-2p_1 r\right], \tag{12}$$

as a solution, where $p = p_1$ if p is positive and $p = -p_1$ if p is negative.

The solution, corresponding to (12), in the field given by the set (1) is obtained, by solving (10) after putting $p = 0$, as

$$\varrho = \frac{A}{r} + B. \tag{13}$$

The charge density decreases more rapidly according to (12) than according to (13). Also the integral

$$4\pi \int_R^\infty \varrho r^2 \, \mathrm{d}r,$$

giving the total charge outside the sphere of radius R, converges for (12) but not for (13).

c) Finally let us take p to be imaginary. If $p = ip_2$ where p_2 is real, then (11) takes the form

$$\varrho = \frac{1}{r}\left(A_2 \cos 2p_2 r + B_2 \sin 2p_2 r\right), \tag{14}$$

where A_2 and B_2 are arbitrary constants. Here ϱ is not of the same sign throughout and $\varrho \to 0$ as $r \to \infty$. Also it is interesting to note that the integral

$$4\pi \int_R^\infty r^2 \varrho \, \mathrm{d}r,$$

oscillates infinitely, between $-\infty$ and $+\infty$.

2003

Thus, it is suggested that the arbitrary constant « p » should preferably be taken as real.

* * *

I wish to express my respectful thanks to Dr. R. S. Mishra for his kind help and guidance.

2104

The Elastic and Inelastic Scattering of Polarized Protons by Carbon.

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(ricevuto il 6 Aprile 1957)

The inelastic scattering of 95 MeV protons by carbon was first studied by Strauch [1] who showed that at large scattering angles (30° to 80°) the cross-sections for scattering from the 4.43 MeV level [2] and for elastic scattering differed by less than a factor of two. In double scattering experiments the polarization produced by scattering from this level has been measured at 173 MeV [3] and the asymmetry at 155 MeV [4] and 220 MeV [4]. We have now measured the cross-section and the asymmetry for scattering from the 4.43 MeV and the 9.61 MeV levels of carbon at proton energies of 135 and 95 MeV. In an inelastic scattering process the asymmetry produced by scattering a 100% polarized beam is unlikely to differ greatly from the polarization produced by the scattering of an unpolarized beam [Skyrme and Squires [6]]. We will assume here that these two quantities are equal.

1. – Apparatus.

A counter telescope consisting of 3 thin plastic scintillators in triple coincidence followed by a sodium iodide crystal (1.75 inch diam. by 2.5 inch long) was used to detect the scattered protons and measure their energy. The energy spectrum of the 140 MeV polarized proton beam normally has a full width at half intensity of 7 MeV as measured with this telescope. By selecting protons in a 20 μs interval of the 250 μs long beam pulse this energy resolution was reduced to 3.4 MeV. This gated beam had an intensity of 15% of the total beam intensity and a polarization of $(72 \pm 3)\%$. A 100 MeV beam was obtained by slowing down the 140 MeV protons in an aluminium absorber. The angular resolution (half width at half height) of the counter telescope was 1.6°

(1) K. Strauch and F. Titus: *Phys. Rev.*, **103**, 200 (1956).

(2) F. Ajzenberg and T. Lauritsen: *Rev. Mod. Phys.*, **27**, 77 (1955).

(3) H. Tyrén, P. Hillman and A. Johansson: *Nucl. Phys.*, **3**, 336 (1957).

(4) R. Alphonce, A. Johansson and G. Tibell: *Nucl. Phys.*, **3**, 185 (1957).

(5) W. G. Chesnut, E. M. Hafner and A. Roberts: *Phys. Rev.*, **104**, 449 (1956).

(6) T. H. R. Skyrme and E. J. Squires: private communication.

2005

at 135 MeV (mean energy in the target) and 3.2° at 95 MeV. At angles of scattering less than 15° a range telescope resolutions were then 1° at 135 MeV and 1.8° at 95 MeV.

The energy spectra observed with

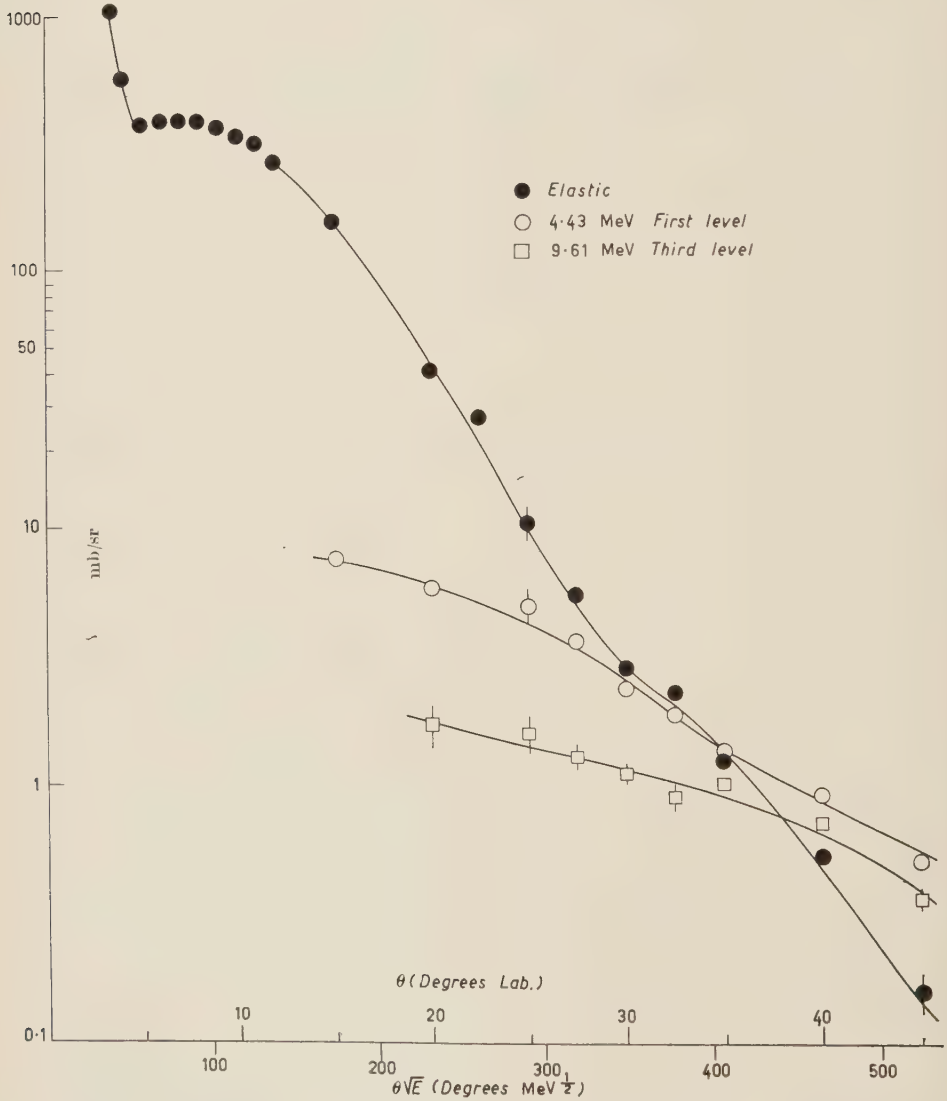

Fig. 1. - Differential cross-section for the scattering of 135 MeV protons by carbon.

was used together with the ungated beam, since here the inelastic cross-sections are very small compared with the elastic cross-section. The angular

the sodium iodide telescope had peaks corresponding to elastically scattered protons and to protons scattered from the 4.43 and 9.61 MeV levels. A broad

peak due to a group of levels around 21 MeV was also observed together with a continuum produced by other levels. and 9.61 MeV levels of carbon. As with measurements at other energies ([1,7,8]) the cross-section for scattering from the

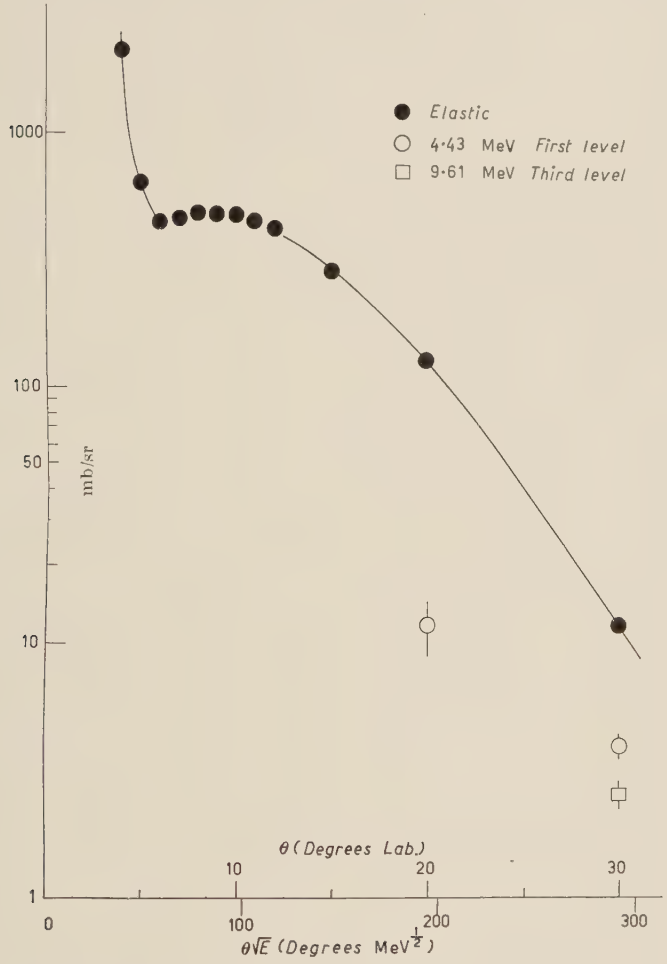

Fig. 2. – Differential cross-section for the scattering of 95 MeV protons by carbon.

A more detailed description of the apparatus and the method of analysing the spectra will be given in a later paper.

2. – Results.

Figs. 1 and 2 and Table I show the cross-sections for elastic scattering and for inelastic scattering from the 4.43 MeV 7.65 MeV level was small and poorly determined. Errors shown in the table are due to counting statistics only and an absolute error of $\pm 7\%$ at 135 MeV and $\pm 10\%$ at 95 MeV should be applied

([7]) H. TYRÉN and TH. A. J. MARIS: *Nucl. Phys.*, **3**, 52 (1957).

([8]) TH. A. J. MARIS and H. TYRÉN: *Nucl. Phys.*, **3**, 35 (1957).

TABLE I. – *The differential cross-section and the polarization of* 135 MeV *and* 95 MeV *protons scattered by carbon.*

θ (Lab.) Degrees	$\theta\sqrt{E}$ Degrees $(\text{MeV})^{\frac{1}{2}}$	Elastic		4.43 MeV Inelastic		9.61 MeV Inelastic	
		Cross-section mb/sr	Polarization %	Cross-section mb/sr	Polarization %	Cross-section mb/sr	Polarization %
135 MeV							
3.5	41	1037 ±18	6 ± 2				
4	47	560 ± 9	25 ± 2				
5	58	372 ± 6	42 ± 2				
6	70	379 ± 6	45 ± 2				
7	81	378 ± 7	45 ± 2				
8	93	375 ± 5	47 ± 2				
9	104	346 ± 7	46 ± 2				
10	116	329 ± 6	51 ± 2				
11	128	311 ± 6	50 ± 3				
12	140	255 ± 5	55 ± 3				
15	175	139 ± 4	63 ± 1	7.45 ± .70	23 ± 11		
20	233	40.2 ± 1.0	87 ± 2	5.8 ± .4	64 ± 6	1.71 ± .32	44 ± 20
22.5	262	21.1 ± .8	96 ± 3				
25	291	10.2 ± 1.5	93 ± 2	4.85 ± .70	82 ± 9	1.55 ± .29	74 ± 15
27.5	320	5.4 ± .2	74 ± 3	3.58 ± .20	64 ± 5	1.27 ± .15	64 ± 12
30	349	2.82 ± .12	44 ± 4	2.31 ± .12	63 ± 5	1.09 ± .09	72 ± 8
32.5	378	2.23 ± .11	32 ± 6	1.85 ± .18	51 ± 10	.90 ± .09	28 ± 13
35	407	1.16 ± .05	10 ± 4	1.34 ± .06	22 ± 4	.99 ± .05	34 ± 5
40	465	.525 ± .027	66 ± 6	.92 ± .04	−22 ± 5	.71 ± .04	−20 ± 7
45	523	.161 ± .026	64 ± 24	.56 ± .03	−36 ± 6	.36 ± .03	−44 ± 12
95 MeV							
4	39	2089 ±32	9 ± 2				
5	49	633 ±12	25 ± 3				
6	59	448 ± 8	26 ± 2				
7	68	460 ± 8	25 ± 2				
8	78	477 ± 6	25 ± 2				
9	88	478 ± 8	25 ± 2				
10	98	465 ± 8	24 ± 2				
11	107	449 ± 8	24 ± 2				
12	117	418 ± 7	27 ± 2				
15	146	280 ± 9	29 ± 3				
20	195	125 ± 5	47 ± 3	11.6 ±2.6	−35 ± 31		
30	293	11.4 ± .5	58 ± 4	3.90 ± .34	29 ± 12	2.56 ± .29	19 ± 15

2048

to the cross-section values, owing mostly to uncertainties in the beam intensity monitoring.

sults (9) at 130 MeV differ from the present elastic scattering results at the three largest angles, since they were

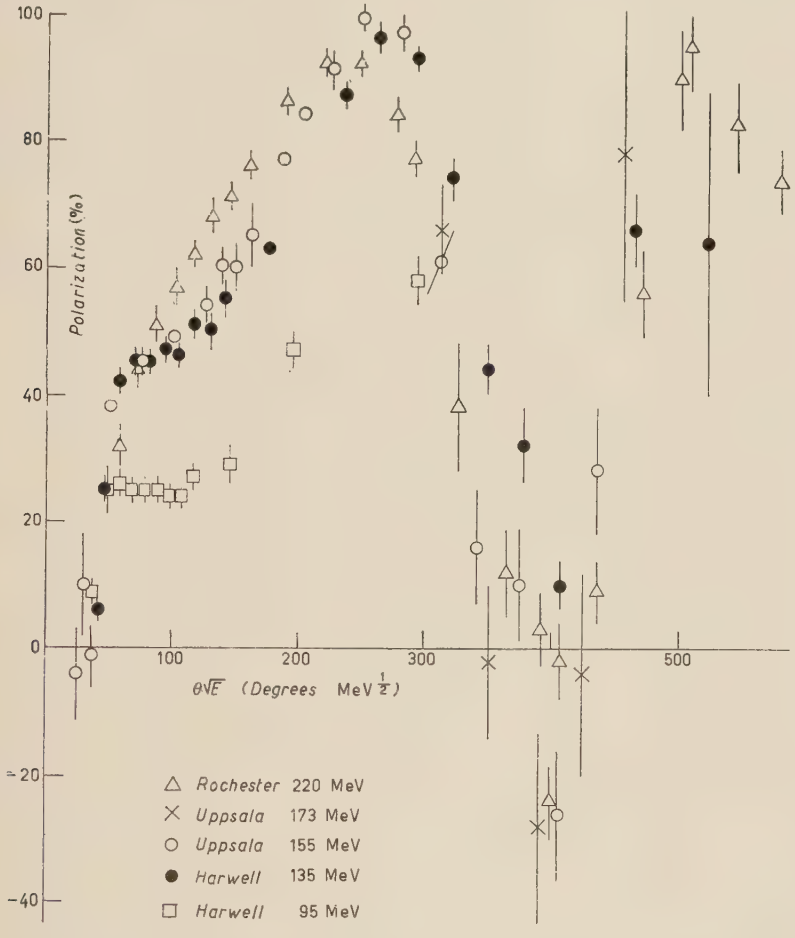

Fig. 3. – Polarization of protons elastically scattered by carbon.

Table I also gives the polarization results and these are compared with higher energy measurements in Figs. 3 and 4. The minimum in the elastic polarization is associated with the expected diffraction minimum of the elastic cross-section, so in order to compare results at different energies the angular scale has been multiplied by the square root of the energy. Our previous results obtained with a detector having a threshold 10 MeV below the energy for elastic scattering and hence they included some inelastic scattering.

The 4.43 MeV level inelastic polarization which is plotted in Fig. 4 shows very little energy variation between

(9) J. M. Dickson, B. Rose and D. C. Salter: *Proc. Phys. Soc.*, **1**, 68, 361 (1955).

20019

135 and 220 MeV except perhaps at the smallest angles. There is, however, a considerable difference between the 135 and 95 MeV results. The 9.61 MeV inelastic polarization (Fig. 5) shows no significant difference from that for the 4.43 MeV level either at 135 or 95 MeV. This is consistent with the prediction of MARIS ([10]) that in first Born approximation elastic scattering and all collective excitations of the nucleus will

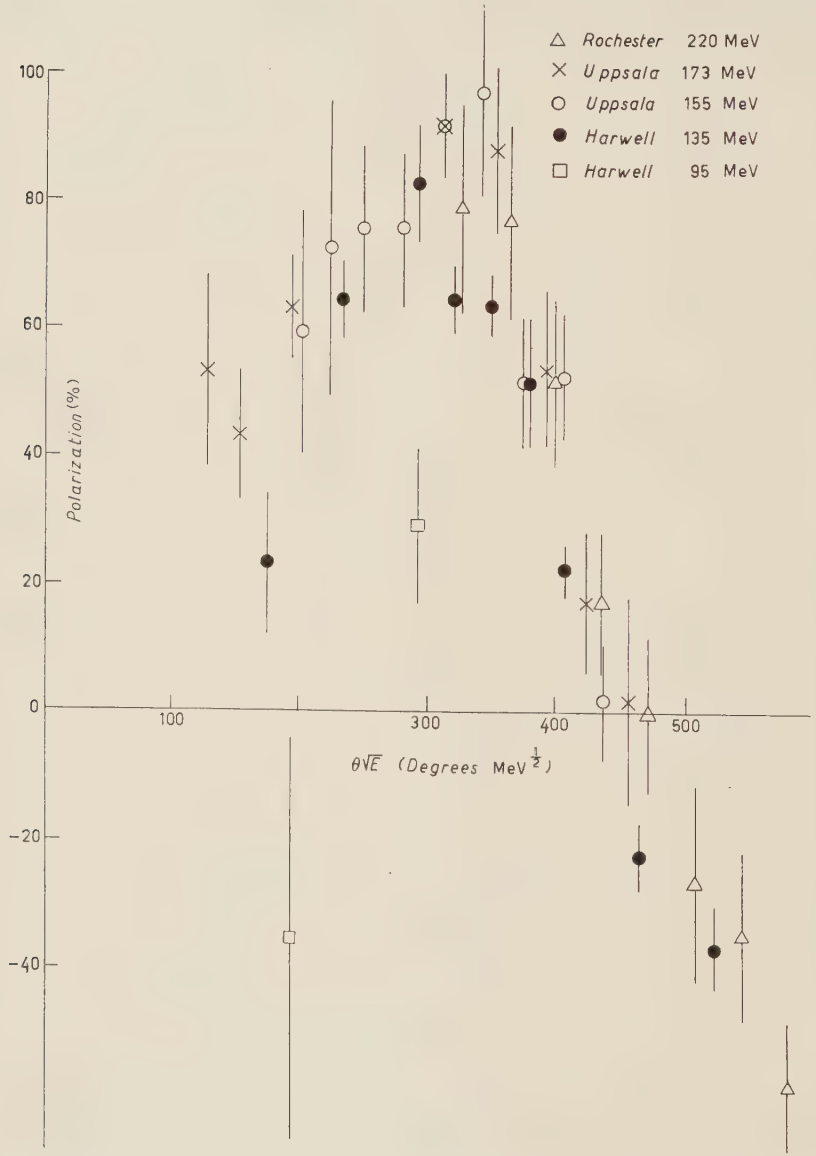

Fig. 4. – Polarization of protons inelastically scattered from the 4.43 MeV level of carbon.

([10]) TH. A. J. MARIS: *Nucl. Phys.*, **3**, 213 (1957).

2100

produce the same polarization. In the case of elastic scattering the rapid decrease of the polarization below about 130 MeV can be explained by the increase in the real part of the central potential with decreasing energy [STERNHEIMER (11)]. It seems reasonable to suggest, therefore, that the energy variation of the inelastic polarization may be attributable to the same cause.

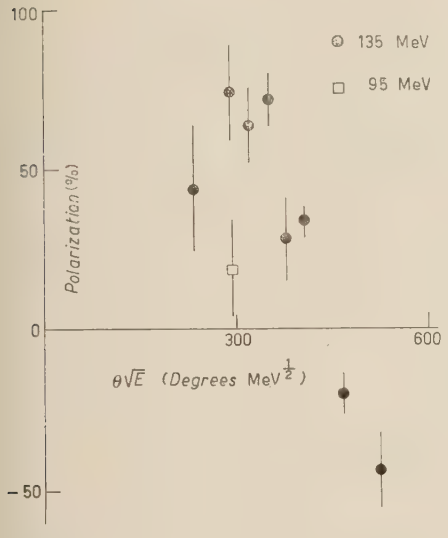

Fig. 5. – Polarization of protons inelastically scattered from the 9.61 MeV level of carbon.

The spacing of the higher levels is too close to allow us to examine the individual levels with the present apparatus. At some scattering angles the energy of the peak at about 21 MeV was not the same for left and for right scattering. This may indicate that the angular dependence of the polarization of the levels near 21 MeV is not the same for all levels.

* * *

We wish to thank R. B. OWEN for providing the sodium iodide crystal, R. H. THOMAS and R. C. CARTER for their help in taking data and the cyclotron operating crew for their cooperation during the experiment.

(11) R. M. STERNHEIMER: *Phys. Rev.*, **100**, 886 (1955).

Formal Model of Magnetic Hysteresis.

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(ricevuto il 12 Aprile 1957)

The possibility of using the Preisach model for hysteresis in the high field region is beeing investigated. A similar research being in progress at the T. H. of Stuttgart (1), it seems useful to report some of our preliminary results. The detailed theory of the model, and more experimental data will be available shortly. Preisach (2) has shown that magnetic hysteresis can be formally explained assuming that the ferromagnetic behavior of a volume element in the material is simply a rectangular loop, in general not symmetric with respect to the I-axes; so that the four sides of the loop are represented by

$$I = \pm I_s ,$$
$$H = a ,$$
$$H = b ,$$

where a is in general different from $-b$ (Fig. 1). More recently, Néel (3) and Llibutry (4) have used this model in describing magnetic phenomena in the low field region.

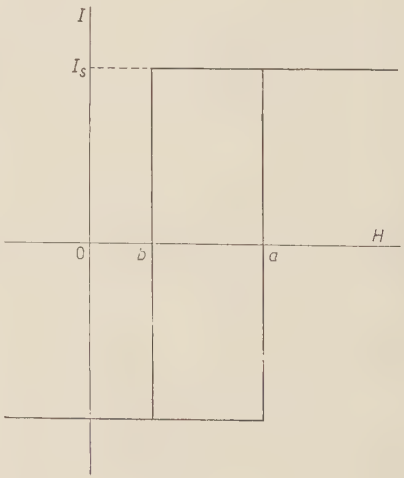

Fig. 1.

The model can be used also in describing magnetic properties at high fields, provided a distribution function

(*) Present address of D. P.: I.T.I. Amedeo Avogadro, Torino.

(1) R. Feldtkeller and H. Wilde: *Elektrotechnische Zeits.*, A **77**, 449 (1956).

(2) F. Preisach: *Zeits. Phys.*, **91**, 277 (1935).

(3) L. Néel: *Cahiers de Physique*, No. 12 (1942).

(4) L. Llibutry: *Thèses* 17, Université de Grenoble (1950).

$\varphi(a, b)$ is introduced. The function $\varphi(a, b)$ can be defined as follows: the probability of finding a loop having one side falling between a and $a+\mathrm{d}a$, and the other side between b and $b+\mathrm{d}b$ is

$$\mathrm{d}P = \varphi(a, b)\cdot \mathrm{d}a\,\mathrm{d}b\ .$$

In most cases it seems hopeless to derive the function $\varphi(a, b)$ on the basis of the knowledge of the structure of the material. Therefore we have investigated the possibility of determining $\varphi(a, b)$ by a few experiments.

Once $\varphi(a, b)$ is known, every path of I, when H is varied in an arbitrary way, can be theoretically computed. We have found that $\varphi(a, b)$ can be deduced numerically from the magnetization curve and the largest hysteresis loop experimentally measured, provided we assume that

$$\varphi(a, b) = \varphi_1(a)\cdot\varphi_2(b)\ .$$

This assumption means that the event that a volume element has the right side of the loop at a and the event that the same volume has the left side of the loop at b are independent. This seems resonable, and consistent with the theories of magnetization processes. The function $\varphi(a, b)$ does not account for possible reversible processes, since the elemental loop is irreversible. On the other hand, the results are in good agreement with experiments, even if reversible processes are neglected.

Following the method outlined we have determined the function $\varphi(a, b)$ for a silicon-iron alloy, with residual induction of 0.24 Wb/m² and coercive force

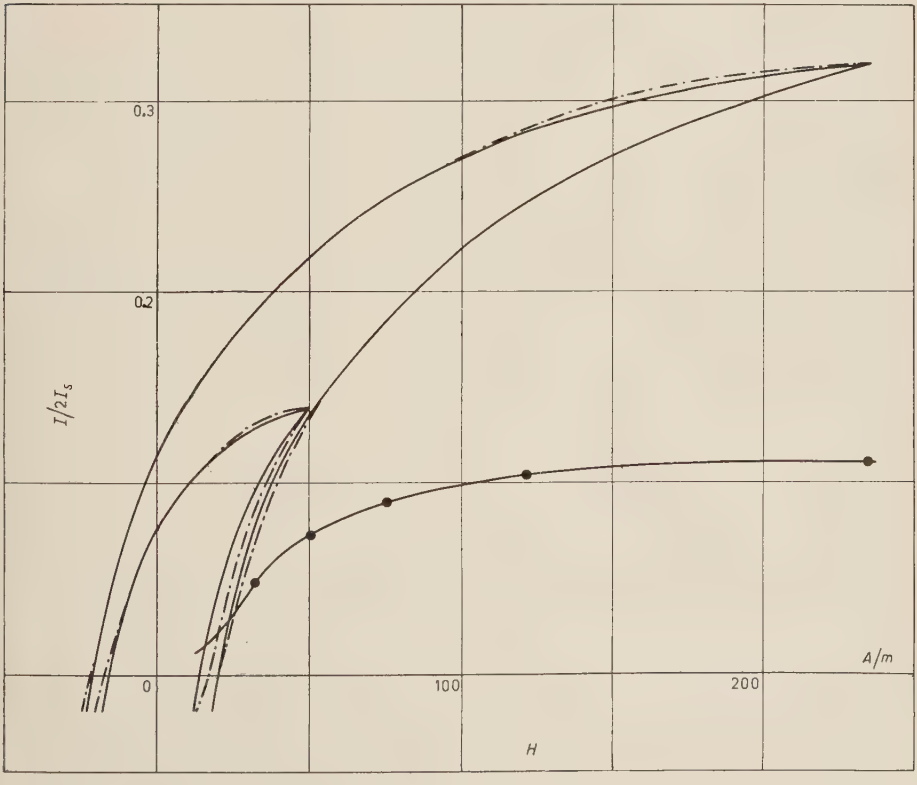

Fig. 2.

of 25 A/m. From $\varphi(a, b)$ we have computed two symmetrical loops, with maximum magnetization of the order of $\frac{1}{3}$ and $\frac{2}{3}$ of the saturation magnetization. Also, the curve of the residual induction of symmetrical loops as a function of their maximum field has been computed. No correction for reversible magnetization has been made.

Fig. 2 shows the loops measured (solid line) and computed (dotted line), and also the computed curve of residual induction. On the latter curve the experimental data are the solid circles.

The agreement is quite satisfactory, and the error is often of the order of the uncertainty in the measure of magnetic curves.

2104

IL NUOVO CIMENTO Vol. VI, N. 1 1° Luglio 1957

Multiple Production of Particles and Hydrodynamical Aspect of Quantum-Theory of Field (*).

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(ricevuto il 18 Aprile 1957)

Among many theories on the multiple production of particles in high energy nucleon-nucleon collisions, it seems that, by introducing the hydrodynamical model ([1-3]) for the high energy meson-nucleon cloud, Landau has most successfully interpreted the qualitative behaviors of experimental results except for the problem of inelasticity ([4]). According to him, these complicated phenomena are to be understood in the following three steps. In the *first period* directly after the collision of primary particles, the meson-nucleon cloud surrounding them is suddenly excited into a furious thermal motion and, because of strong interactions in high density, goes rapidly into a local equilibrium. The *next period* is chara terized by the hydrodynamical expansion of the cloud, in which Landau has used the assumption of perfect fluid. In the *final period* the cloud (which is cold and dilute) displays the particle character and separates into free particles. In his theory the final interactions among particles in the process of expansion are taken into account through the hydrodynamical equation which, together with the simple initial boundary conditions to be considered to represent the effects of interactions in the first period, allow us to calculate the angular and energy distributions of the produced particles.

One may say that the above analysis has straightened the complicated processes out and grasped the essentials of the phenomena. Nevertheless, Landau's theory is only a phenomenological one, that is, it cannot present the procedure for examination of the quantum theory of field from the experiments. One may ask (i) the

(*) This article was read at the *Meeting of the Physical Society of Japan*, Kyoto University, Oct. 1956.

([1]) L. D. Landau: *Izv. Akad. Nauk SSSR*, **17**, 51 (1953).

([2]) S. Z. Belenikij and L. D. Landau: *Usp. Fiz. Nauk SSSR*, **56**, 309 (1955).

([3]) S. Amai, H. Fukuda, C. Iso and M. Sato: *Prog. Theor. Phys.*, **17**, 241 (1957). The other literatures are cited in this paper.

([4]) G. T. Zatsepin: *Reports of Oxford Conference on the extensive air showers* (1956), p. 8.

question whether the initial boundary conditions in his theory are acceptable or not. (ii) the question whether the quantum theory of field permits the hydrodynamical description for motions of the meson-nucleon cloud, and (iii) what kind of phenomenological equation in hydrodynamics one should assume, i.e., perfect fluid or more complicated viscous one. Furthermore, though it is the case, we must examine the validity of the assumptions about rapid formation of the local equilibrium. In the present note, we shall study the above problems (except the first) in order to establish the theoretical method to investigate increasing high energy experiments.

The macroscopic theory of relativistic hydrodynamics is described by an energy-momentum density tensor $T_{\mu\nu}$, obeying the equation $\nabla_\nu T_{\mu\nu} = 0$, and a matter flow J_μ, obeying the equation $\nabla_\mu J_\mu = J$. Here ∇_ν stands for the differential operator and J denotes the rate of increase (or decrease) of matter. By introducing the local four-velocity $U_\mu = J_\mu/\sqrt{-J_\mu^2}$, the energy-momentum density tensor $T_{\mu\nu}$ is decomposed as follows (5):

$$T_{\mu\nu} = \varepsilon U_\mu U_\nu + U_\mu q_\nu + U_\nu q_\mu + t_{\mu\nu}\,, \tag{1}$$

in which $\varepsilon = U_\mu T_{\mu\nu} U_\nu$, $q_\mu = -\Delta_{\mu\nu} T_{\nu\lambda} U_\lambda$ and $t_{\mu\nu} = \Delta_{\mu\gamma} T_{\gamma\lambda} \Delta_{\lambda\nu}$ stand for the invariant rest energy, the heat four-current and the stress tensor, respectively, $\Delta_{\mu\nu} (= \delta_{\mu\nu} + U_\mu U_\nu)$ being a projection tensor to the space-like direction perpendicular to U_μ. The hydrodynamical pressure p of the fluid is defined by

$$p = \tfrac{1}{3}\,\mathrm{Tr}\,(t_{\mu\nu}) = \tfrac{1}{3}\,\varepsilon + \tfrac{1}{3}\,\mathrm{Tr}\,(T_{\mu\nu})\,, \tag{2}$$

which becomes the equation of state in equilibrium. If $t_{\mu\nu}$ is decomposed in the hydrostatic components $p_s \Delta_{\mu\nu}$ and the viscous one $-P_{\mu\nu}$ dependent of deformation velocities, the component of $T_{\mu\nu}$ independent of $P_{\mu\nu}$ and q_μ, of course, coincides with the energy-momentum tensor of perfect fluid. For practice, it is of importance to use the phenomenological equation to first order of $\nabla_\mu U_\nu$ for $P_{\alpha\beta}$

$$P_{\alpha\beta} = \eta_{(s)} [\Delta_{\alpha\nu} \Delta_{\beta\zeta} (\nabla_\zeta U_\nu + \nabla_\nu U_\zeta) - \tfrac{2}{3} \Delta_{\alpha\beta} \Delta_{\nu\zeta} \nabla_\zeta U_\nu] + \eta_{(v)} \Delta_{\alpha\beta} \varrho D v + \sum_i L^{(i)} \Delta_{\alpha\beta} A^{(i)}\,, \tag{3}$$

where $\eta_{(s)}$, $\eta_{(v)}$, $L^{(i)}$ and $A^{(i)}$ denote the coefficients of shear viscosity, bulk viscosity, visco-chemical effect and the chemical affinity of the i-th constituent (*), and $\varrho = 1/v = \sqrt{-J_\mu^2}$ and $D = U_\alpha \nabla_\alpha$. Thus the pressure p is expressed in the form

$$p = p_s - \eta_{(v)} \varrho D v - \sum_i L^{(i)} A^{(i)}\,, \tag{4}$$

where it is noted that the terms except p_s vanish in equilibrium but not in local equilibrium.

Now we go to define $T_{\mu\nu}$ in terms of the quantum theory of field with an energy-momentum density tensor $\mathcal{T}_{\mu\nu}$ (which is an operator). In a way analogous to

(*) For a mixed fluid, $A^{(i)} = -\mu^{(i)} \nabla$ where $\mu^{(i)}$ is the chemical potential of the i-th constituent. In case of the fluid with (11) the chemical potentials could be disregarded at extreme high energy.

(5) C. ECKART: *Phys. Rev.*, **58**, 919 (1940); G. A. KLUITENBERG, S. R. DEGROOT and P. MAZUR: *Physica*, **19**, 689, 1079 (1953). Their (and the present authors') standpoint in the definition of local velocity is different from that of the following text books; L. D. LANDAU and E. M. LIFŠIC: *Mehanika Splošnyksred* (Moskva, 1954); C. MØLLER: *Theory of Relativity* (Oxford, 1952).

the classical theory given by KIRKWOOD ([6]), it is most favourable to define $T_{\mu\nu}$ with the equation

$$T_{\mu\nu} = \langle \mathcal{C}_{\mu\nu} \rangle - \langle \mathcal{C}_{\mu\nu} \rangle_{\text{vac}} , \tag{5}$$

where the symbol $\langle B \rangle$ stands for the quantum-statistical average of an operator B in the corresponding transport process and $\langle B \rangle_{\text{vac}}$ for the expectation value of it in vacuum. The matter flow J_μ in case of a neutral scalar (or pseudoscalar) meson field should be defined by

$$J_\mu(x) = \langle \varphi^+(x;\sigma) m \overleftrightarrow{p_\mu} \varphi^-(x;\sigma) \rangle_{x \text{ on } \sigma} , \tag{6}$$

in which $\overleftrightarrow{p_\mu} = (1/2i)(\overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu)$ and m is the meson mass. $\varphi^{\pm}(x;\sigma)$ is the component with plus or minus frequencies of the operator defined by YANG and FELDMAN. This definition of J_μ can be easily extended for the nucleon field and interacting fields.

For Schiff's type of meson field with the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left\{ (\nabla_\mu \varphi)^2 - m^2\varphi^2 + \frac{\lambda}{2}\varphi^4 \right\} , \tag{7}$$

Eq. (5) becomes, suppressing the subtraction of the vacuum term,

$$T_{\mu\nu} = \left\langle \varphi \overleftrightarrow{P_\mu} \overleftrightarrow{P_\nu} \varphi + \delta_{\mu\nu} \frac{\lambda}{4} \varphi^2) \varphi \right\rangle , \tag{8}$$

which yields the equation of state for meson fluid

$$3p = \varepsilon - m^2 \langle \varphi^2 \rangle . \tag{9}$$

For the field with the pseudoscalar (scalar) coupling between nucleon and pseudoscalar (scalar) meson, the corresponding equation becomes

$$3p = \varepsilon - m^2 \langle \varphi^2 \rangle - M \langle \bar{\psi}\psi \rangle , \tag{10}$$

in which ψ stands for the nucleon field with mass M. From the dimension-analytical point of view, Eqs. (9) and (10) approach the equation assumed by LANDAU

$$3p = \varepsilon \tag{11}$$

at extreme high energy. Furthermore, one might expect that this equation would hold even in the case of the presence of higher non-linearities of the meson field in the above interactions (without derivatives). However, the situations are quite different for interactions with derivatives. For instance, in case of the pseudovector (vector) coupling between nucleon and pseudoscalar (scalar) meson, the equation

([6]) J. G. KIRKWOOD: *Journ. Chem. Phys.*, **14**, 180 (1946); J. H. IRVING and J. G. KIRKWOOD: *Journ. Chem. Phys.*, **18**, 817 (1950); J. H. IRVING and R. W. ZWANZIG: *Journ. Chem. Phys.*, **19**, 1173 (1951).

of state turns to the form

$$3p = \varepsilon - m^2\langle\varphi^2\rangle - M\langle\bar{\psi}\psi\rangle - g\langle\bar{\psi}O_\mu\psi\nabla_\mu\varphi\rangle, \tag{12}$$

where g denotes the coupling constant and O_μ's are appropriate Dirac matrices. The last term in the right-hand side of Eq. (12) will have the order of magnitude of ε at extreme high energy. Consequently one may conclude that the equation of state in the region of extreme high energy becomes of the type (11) for perfect fluid or not according as the interaction terms contain derivatives or not. Then it might be suggested that accurate experiments (if possible) on multiple production would determine the type of coupling between nucleon and meson.

More detailed discussions are required about the viscosity coefficients and so on, in order to study the relation between the hydrodynamics of the meson-nucleon cloud and the quantum theory of field. In particular, it is of importance to calculate the relaxation time in the sense of the statistical mechanics in the framework of the quantum theory of field (The inverse of the relaxation time is readily expected to be closely related to the effective number of collisions. Roughly speaking, it has the same tendency as the product of the coupling constant and the density.) Such calculations have been performed by means of the methods prepared by KUBO, NAKANO, NAKAJIMA and others (7). For example, the viscosity coefficients will be calculated through the formulas

$$\begin{cases} \eta_{(s)} = \dfrac{1}{2}\displaystyle\int_0^\infty \mathrm{d}\tau \int_0^\beta \mathrm{d}\lambda \left\langle \int t_{12}^{\mathrm{op}}\,\mathrm{d}^3x,\ t_{12}^{\mathrm{op}}(\tau + i\lambda) \right\rangle_0, \\ \eta_{(v)} = \dfrac{1}{3}\displaystyle\int_0^\infty \mathrm{d}\tau \int_0^\beta \mathrm{d}\lambda \left\langle \int t_{11}^{\mathrm{op}}\,\mathrm{d}^3x,\ t_{11}^{\mathrm{op}}(\tau + i\lambda) \right\rangle_0. \end{cases} \tag{13}$$

where $\langle\ldots\rangle_0$ means the average in thermal equilibrium. Detailed reports will be published in the future issue of the Progress of Theoretical Physics.

Note added in proof.

In the derivations of the viscosity coefficients Eq. (13), the assumption of the constant deformation velocities has been used. The phenomenological equation for the stress tensor is, in general, to be written in the form

$$\boldsymbol{t}^{\mathrm{op}}(\ ) = p_s\mathbf{1} - \int \mathrm{d}^3\boldsymbol{x}' Y_{(s)}(\boldsymbol{x}-\boldsymbol{x}') \left\{ \mathrm{Sym}\nabla'\boldsymbol{u}(\boldsymbol{x}') - \frac{2}{3}\mathbf{1}\nabla'\cdot\boldsymbol{u}(\boldsymbol{x}') \right.$$

$$\left. \int \mathrm{d}^3\boldsymbol{x}' Y_{(v)}(\boldsymbol{x}-\boldsymbol{x}')\,\mathbf{1}\{\nabla'\cdot\boldsymbol{u}(\boldsymbol{x}')\} + \ldots \right.$$

(7) R. KUBO and K. TOMITA: *Journ. Phys. Soc. Japan*, **9**, 880 (1954); H. NAKANO: *Busseiron Kenkyu* (in Japanese), **84**, 25 (1955); **88**, 53 (1955) and **104**, 11, 20 (1957); *Prog. Theor. Phys.*, **17**, 145

2108

in the local rest system, where

$$Y_{(s)}(\boldsymbol{x}-\boldsymbol{x}') = \frac{1}{2}\int_0^\infty \mathrm{d}\tau \int_0^\beta \mathrm{d}\lambda \langle i_{12}^{\mathrm{op}}(x'),\quad i_{12}^{\mathrm{op}}(r,\tau+i\lambda)\rangle_0$$

$$Y_{(v)}(\boldsymbol{x}-\boldsymbol{x}') = \frac{1}{3}\int_0^\infty \mathrm{d}\tau \int_0^\beta \mathrm{d}\lambda \langle t_{11}^{\mathrm{op}}(x'),\quad t_{11}^{\mathrm{op}}(r,\tau+i\lambda)\rangle_0 .$$

(1957); S. Nakajima: *Busseiron Kenkyu* (in Japanese), **88**, 45 (1955) and **102**, 24 (1956); R. Kubo: *Busseiron Kenkyu* (in Japanese), **89**, 72, 79 (1955); R. Kubo and M. Yokota: *Busseiron Kenkyu* (in Japanese), **89**, 99 (1955); R. Kubo, N. Hashizume and M. Yokota: *Busseiron Kenkyu* (in Japanese), **91**, 32 (1955).

2109

Fermi Interaction with Non-Conservation of «Lepton Charge» and of Parity.

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(ricevuto il 14 Maggio 1957)

In a preceeding paper Pauli [1] investigates a Fermi-interaction which violates the conservation law for the number of light particles [2], called the « lepton charge » and for which the distinction of neutrinos and antineutrinos is therefore ambiguous. In particular he uses canonical transformations of the neutrino field which, for vanishing neutrino mass m_ν, leave the Hamiltonian of the free particles invariant.

In the present note some calculations with Pauli's Hamiltonian, which is

$$(1) \qquad H_{\text{int}} = \sum_i (\bar{\psi}_n O_i \psi_p)[g_{Ii}(\bar{\psi}_\nu O_i \psi_e) - f_{Ii}(\bar{\psi}_\nu \gamma_5 O_i \psi_e) + g_{IIi}(\psi_\nu C O_i \psi_e) + f_{IIi}(\psi_I C \gamma_5 O_i \psi_e)] + \text{herm. conj.},$$

are presented in more detail. Since we are not only interested in the case where $m_\nu = 0$ no use is made here of the canonical transformations. (Pauli's notation is used and $\hbar = c = 1$).

1. – As a consequence of the non-conservation of lepton charge exhibited in (1) the processes $n \to p + e^- + \nu$, $p \to n + e^+ + \nu$ lead to a superposition of neutrino and antineutrino in the final state, of the form

$$(2) \qquad \alpha|\bar{\nu}\rangle + \beta|\nu\rangle .$$

The amplitudes α, β are determined to first order by the matrix elements into the states $|\bar{\nu}\rangle$ and $|\nu\rangle$ respectively, that is (for β^--decay) by

$$(3) \qquad \begin{cases} \sum_i (\bar{u}_n O_i u_p)(\bar{v}_\nu [g_{Ii} - f_{Ii}\gamma_5] O_i u_e) = \xi\alpha , \\ \sum_i (\bar{u}_n O_i u_p)(u_\nu C [g_{IIi} + f_{IIi}\gamma_5] O_i u_e) = \xi\beta , \end{cases}$$

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[1] W. Pauli: *Nuovo Cimento* (this issue).

[2] Such an interaction has previously been considered to study the double β-decay by B. Touschek: *Zeits. Phys.*, **125**, 108 (1948), who used a very special form, and probably also by Sakata and by Maddox, see H. Umezawa: *Quantum Field Theory* (Amsterdam, 1956), footnote 1, p. 50.

and the normalization condition

$$|\alpha|^2 + |\beta|^2 = 1 \,. \tag{4}$$

u and v are the spinor functions in momentum space for particles and antiparticles respectively.

The matrix element into the final state (2) is then equal to ξ and the summation of $|\xi|^2$ over electron and neutrino spin leads to a factor

$$1 - c\,\frac{m_e m_\nu}{E_e E_\nu} + a\,\frac{\boldsymbol{p}_e \boldsymbol{p}_\nu}{E_e E_\nu} \pm b\left(\frac{m_e}{E_e} - d\,\frac{m_\nu}{E_\nu}\right). \tag{5}$$

in the energy-angle distribution of an allowed β^--decay. The coefficients a, b, c, d (no Coulomb correction is applied) are given by

$$a = \frac{\frac{1}{3}(K_{TT} - K_{AA})\,|M_{GT}|^2 - (K_{SS} - K_{VV})\,|M_F|^2}{(K_{SS} + K_{VV})\,|M_F|^2 + (K_{TT} + K_{AA})\,|M_{GT}|^2}, \tag{6a}$$

$$b = \frac{(K_{SV} + K_{VS})\,|M_F|^2 + (K_{TA} + K_{AT})\,|M_{GT}|^2}{(K_{SS} + K_{VV})\,|M_F|^2 + (K_{TT} + K_{AA})\,|M_{GT}|^2}, \tag{6b}$$

$$c = \frac{(K'_{SS} + K'_{VV})\,|M_F|^2 + (K'_{TT} + K'_{AA})\,|M_{GT}|^2}{(K_{SS} + K_{VV})\,|M_F|^2 + (K_{TT} + K_{AA})\,|M_{GT}|^2}, \tag{6c}$$

$$d = \frac{(K'_{SV} + K'_{VS})\,|M_F|^2 + (K'_{TA} + K'_{AT})\,|M_{GT}|^2}{(K_{SV} + K_{VS})\,|M_F|^2 + (K_{TA} + K_{AT})\,|M_{GT}|^2}, \tag{6d}$$

where

$$K_{ij} = K_{ji}^* = g_{\mathrm{I}i}^* g_{\mathrm{I}j} + g_{\mathrm{II}i}^* g_{\mathrm{II}j} + f_{\mathrm{I}i}^* f_{\mathrm{I}j} + f_{\mathrm{II}i}^* f_{\mathrm{II}j} \,, \tag{7}$$

$$K'_{ij} = K'^*_{ji} = g_{\mathrm{I}i}^* g_{\mathrm{I}j} + g_{\mathrm{II}i}^* g_{\mathrm{II}j} - f_{\mathrm{I}i}^* f_{\mathrm{I}j} - f_{\mathrm{II}i}^* f_{\mathrm{II}j} \,. \tag{7'}$$

In the case of the conventional β-interaction Hamiltonian ($q_{\mathrm{II}i} = f_{\mathrm{I}i} = f_{\mathrm{II}i} = 0$) the expressions for a and b are seen to reduce to those given by Rose [3]. The quantities K_{ij} are the same as in Pauli's paper and are thus invariants in the case $m_\nu = 0$, while the K'_{ij} are not. In view of this it is natural that the coefficients (a, b) of the terms of (5) which are independent of m_ν depend on K_{ij} only while the others (c, d) also depend on K'_{ij}.

Neglecting the Fierz terms (proportional to b) and integrating over the angle between electron and neutrino the correction factor (5) reduces to

$$1 \quad c\,\frac{m_e m_\nu}{E_e E_\nu} \,. \tag{5'}$$

[3] M. E. Rose, in K. Siegbahn: *Beta- and Gamma-Ray Spectroscopy* (Amsterdam, 1955), p. 280, formula (9a, b), but with $\gamma = 1$.

This factor, with the value $c = 1$ which follows from the conventional form of H_{int}, was first introduced by PRUETT [4] who emphasized its importance in the determination of an upper limit for m_ν. The form (5′) with the additional parameter c is due to KOFOED-HANSEN [5] who claimed that c may have any value between -1 and $+1$. Since this statement was in no way clear on the ground of the conventional form of H_{int} it seemed to us of interest to show the origin for such a parameter c. In fact, it follows from the expressions (6c), (7) and (7′) that

$$-1 \leqslant c \leqslant +1 .$$

Furthermore it is seen that a value $|c| < 1$ is possible also in the case where the matrix elements for double-processes calculated in part 2 only contain terms proportional to m_ν, i.e. when $I_{ij} = J_{ij} = 0$ [see formulae (9), (10) below]. This is the case, for example, with $f_{\text{II}i} = \lambda g_{\text{I}i}$, $g_{\text{II}i} = -\lambda f_{\text{I}i}$ which implies

$$K_{ij} = (1 + |\lambda|^2)(g^*_{\text{I}i} g_{\text{I}j} + f^*_{\text{I}i} f_{\text{I}j}) ,$$

$$K'_{ij} = (1 - |\lambda|^2)(g^*_{\text{I}i} g_{\text{I}j} - f^*_{\text{I}i} f_{\text{I}j}) ,$$

or also with $f_{\text{I}S} = -g_{\text{I}S}$, $f_{\text{I}T} = -g_{\text{I}T}$, $f_{\text{II}T} = g_{\text{II}T}$ (all other g's and f's zero) from which a value $c = 0$ follows [6]. On the other hand $|c| = 1$ is equivalent to the vanishing of all g's or all f's, in which case parity is conserved. Thus parity conservation is a necessary condition for $|c| = 1$, but it is not a sufficient one. Since parity is also conserved when all g_{I}'s and f_{II}'s or all g_{II}'s and f_{I}'s are zero [7].

2. – Two processes have so far been considered to decide experimentally whether the lepton charge is conserved in β-decay, viz. (*a*) the double β-decay $n_1 + n_2 \to p_1 + e_1^- + \nu + n_2 \to p_1 + p_2 + e_1^- + e_2^-$ and (*b*) the reaction $^{37}\text{Cl} + \nu \to {}^{37}\text{A} + e^-$ studied by DAVIS [8], for which the neutrino flux of a pile is used.

It is useful to consider case (*b*) as a process in two steps similar to case (*a*), the first and second steps being the process $n \to p + e^- + \nu$ in the pile nuclei and in ^{37}Cl respectively. Due to the non-conservation of lepton charge in H_{int} the neutrino of the first step is emitted into a state (2), which is also the initial state for the second step, the amplitudes α and β being functions of the momentum and spin variables of the particles involved in the first step. Thus the two steps are linked in a coherent way and it is appropriate to calculate the matrix element for the double-process by summing first over the neutrino variables of the intermediate state, and not to pass directly to the cross-section for the second step. The only

(4) J. R. PRUETT: *Phys. Rev.*, **73**, 1219 (1948).

(5) O. KOFOED-HANSEN: *Phil. Mag.*, **42**, 1448 (1951); also *Physica*, **18**, 1287 (1952).

(6) The first choice also implies the vanishing of the « relative invariants » $N_{\text{I},ij}$; $N_{\text{II},ij}$ introduced by PAULI so that, according to his results, lepton charge is strictly conserved (with arbitrary λ). The second choice gives $N_{\text{I},ST} = 4 g_{\text{I}S} g_{\text{II}T} \neq 0$ and so lepton charge is only approximately conserved through the non-existence (for $m_\nu = 0$) of the double-processes in the lowest order of perturbation theory, considered here.

(7) The two cases of parity conservation are distinguished by the choice of the phase factor η_{P} in the parity transformation of Lee, Oehme and Yang, *Phys. Rev.* **106**,340 (1957), formula (2): all g's or all f's zero has $\eta_{\text{P}} = \pm 1$, which is Racah's choice, all g_{I}'s, f_{II}'s or all g_{II}'s, f_{I}'s zero has $\eta_{\text{P}} = \pm i$.

(8) R. DAVIS: *Phys. Rev.*, **97**, 766 (1955); *Bulletin of the Washington meeting* (1956), p. 219.

difference of the double-process (b) as compared to (a) then is that, due to the spatial separation of the two steps, the intermediate states are *real* states.

Instead of introducing the intermediate states explicitely the S-matrix formalism has been used to calculate the matrix elements for the double-processes (a) and (b). Formally the spatial separation in case (b) then converts $\delta_+(p_1^2+m_\nu^2)$ into $\delta(p_1^2+m_\nu^2)$ from which only the emission part $(1/2\omega_\nu)\cdot\delta(p_{10}-\omega_\nu)$ survives (for the meaning of p_1 and ω_ν see below) exhibiting the fact that energy is separately conserved in the two steps of (b). The result is, in second order of perturbation theory,

$$(8a)\qquad \langle \mathrm{p_1 p_2 e_1^- e_2^-}\,|\,S_2\,|\,\mathrm{n_1 n_2}\rangle_{(a)} = -(2\pi)^{-4}\,\delta^4(p_1+p_2)\,\delta_+(p_1^2+m_\nu^2)\cdot T\,,$$

$$(8b)\qquad \langle \mathrm{p_1 p_2 e_1^- e_2^-}\,|\,S_2\,|\,\mathrm{n_1 n_2}\rangle_{(b)} = -(2\pi)^{-4}\,\delta^4(p_1+p_2)\cdot\frac{1}{2\omega_\nu}\cdot\delta(p_{10}-\omega_\nu)\cdot T\,,$$

p_1, p_2 and ω_ν are the four-momenta and neutrino energy

$$p_i = p_{\mathrm{n}_i} - p_{\mathrm{p}_i} - p_{\mathrm{e}_i}; \qquad \omega_\nu^2 = \boldsymbol{p}_1^2 + m_\nu^2\,,$$

respectively and

$$(9)\qquad T = \sum_{ij} Q_{ij}\{I_{ij}A_{ij} + J_{ij}B_{ij} - m_\nu\cdot(I'_{ij}C_{ij} + J'_{ij}D_{ij})\}\,.$$

The quantities occuring in (9) are

$$\begin{aligned} Q_{ij} &= (\bar{u}_{\mathrm{p}_1}O_i u_{\mathrm{p}_1})(\bar{u}_{\mathrm{n}_2}O_j u_{\mathrm{p}_2})\,,\\ A_{ij} &= (u_{\mathrm{e}_1}O_i^T C i(\gamma p_1)O_j u_{\mathrm{e}_2})\,,\\ B_{ij} &= (u_{\mathrm{e}_1}O_i^T C i(\gamma p_1)\gamma_5 O_j u_{\mathrm{e}_2})\,,\\ C_{ij} &= (u_{\mathrm{e}_1}O_i^T C O_j u_{\mathrm{e}_2})\,,\\ D_{ij} &= (u_{\mathrm{e}_1}O_i^T C\gamma_5 O_j u_{\mathrm{e}_2})\,, \end{aligned}$$

and

$$(10)\qquad \begin{cases} I_{ij} = g_{\mathrm{I}i}g_{\mathrm{II}j} + g_{\mathrm{I}j}g_{\mathrm{II}i} + f_{\mathrm{I}i}f_{\mathrm{II}j} + f_{\mathrm{I}j}f_{\mathrm{II}i}\,,\\ J_{ij} = g_{\mathrm{I}i}f_{\mathrm{II}j} - g_{\mathrm{I}j}f_{\mathrm{II}i} + f_{\mathrm{I}i}g_{\mathrm{II}j} - f_{\mathrm{I}j}g_{\mathrm{II}i}\,, \end{cases}$$

$$(10')\qquad \begin{cases} I'_{ij} = g_{\mathrm{I}i}g_{\mathrm{II}j} + g_{\mathrm{I}j}g_{\mathrm{II}i} - f_{\mathrm{I}i}f_{\mathrm{II}j} - f_{\mathrm{I}j}f_{\mathrm{II}i}\,,\\ J'_{ij} = g_{\mathrm{I}i}f_{\mathrm{II}j} + g_{\mathrm{I}j}f_{\mathrm{II}i} - f_{\mathrm{I}i}g_{\mathrm{II}j} - f_{\mathrm{I}j}g_{\mathrm{II}i}\,. \end{cases}$$

I_{ij} and J_{ij} are the invariants introduced by PAULI. They are in fact the only expressions in the coupling constants on which the matrix elements for processes of type (a) or (b) can depend if the neutrino mass is zero.

In the case of the Majorana neutrino, in which (1) can be replaced by

$$(1')\qquad H_{\mathrm{int}} = \sum_i (\bar{\psi}_{\mathrm{n}}O_i\psi_{\mathrm{p}})\{g_i(\bar{\varphi}_\nu O_i\psi_{\mathrm{e}}) - f_i(\bar{\varphi}_\nu\gamma_5 O_i\psi_{\mathrm{e}})\} + \text{herm. conj.}\,,$$

and the reality condition

$$\bar{\varphi}_\nu = C\varphi_\nu = -\varphi_\nu C\,,$$

holds, the matrix elements for the double-processes (a) and (b) are again given by (8a), (8b) and (9) but now with

$$\text{(11)} \qquad \begin{cases} I_{ij} = -g_i g_j + f_i f_j\,, \\ J_{ij} = \quad g_i f_j - f_i g_j\,, \end{cases}$$

$$\text{(11')} \qquad \begin{cases} I'_{ij} = -g_i g_j - f_i f_j\,, \\ J'_{ij} = \quad g_i f_j + g_j f_i\,. \end{cases}$$

These expressions follow from (10) and (10') if one puts there

$$\text{(12)} \qquad \begin{cases} g_{\mathrm{I}j} = -g_{\mathrm{II}i} = \dfrac{1}{\sqrt{2}}\, g_i\,, \\ f_{\mathrm{I}i} = \quad f_{\mathrm{II}i} = \dfrac{1}{\sqrt{2}}\, f_i\,. \end{cases}$$

This has to be so because (12) transforms (1) into (1') if one makes the well known identification

$$\varphi_\nu = \frac{1}{\sqrt{2}}\,(\psi_\nu + C^{-1}\bar{\psi}_\nu)\,,$$

$$\bar{\varphi}_\nu = \frac{1}{\sqrt{2}}\,(\bar{\psi}_\nu - \psi_\nu C_\nu)\,.$$

* * *

I wish to thank Prof. W. Pauli for his valuable suggestions and for illuminating discussions.

2111

Note on the Gamma Decay of Neutral Pions (*).

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(ricevuto il 31 Maggio 1957)

We have recently proposed (1) a theory of the strong interactions of K- and π-mesons with barions which is completely symmetrical in all barions and in all mesons. In this theory, which is formulated in a generalized seven dimensional isotopic spin space (2,3), the strong interactions are described by the following term in the Lagrangean density:

$$ig\bar{\psi}\gamma_5\Gamma_r\psi\varphi_r\,, \tag{1}$$

where repeated « r » indices mean summation from $r = 1$ to 7. The hermitian field φ_r describe the four K-mesons and the three π-mesons. The 8-component isospinor describes the 8 kinds of barions (P, N, Ξ_0, Ξ_-, Σ_+, Σ_-, Σ_0, Λ). The matrices Γ_r are seven anticommuting matrices operating on the 8-isospinor ψ, defined in reference (2). The theory at this stage is completely invariant under exchange of all barions and of all mesons and thus implies that the barion mass multiplet as well as the meson mass multiplet are completely degenerate. The introduction of the electromagnetic interaction as well as of a further interaction with K-mesons of the same strength of the electromagnetic one (both of them being not invariant under rotations in the generalized isotopic spin space) are shown to produce mass splittings both for the barion and meson multiplets of the correct order of magnitude.

In the present Note we wish to concentrate our attention on the strong π-mesons interactions and to consider the consequences for the lifetime of π_0-mesons for disintegration into two photons.

The terms of (1) which describe the interactions of π-mesons (*) take the

(*) Work done under the auspices of the Conselho Nacional de Pesquisas.

(1) Communication to the Seventh International Conference on High Energy Nuclear Physics (Rochester, April 1957). A more detailed analysis of this theory will be published elsewhere.

(2) J. Tiomno: *On the Theory of Hyperons and K-Mesons*, in *Nuovo Cimento* (in press).

(3) A similar theory of a 7-dimensional isotopic spin space, starting from somewhat different considerations, has been developed independently by M. Neuman whom we acknowledge for a private communication.

(*) The terms of interactions of barions with K-mesons in (1) have been analized in detail in reference (2) and shown to be equivalent to the interaction used in Schwinger's earlier theory [J. Schwinger: *Phys. Rev.*, **104**, 254 (1956)].

following form in the usual notation:

$$(2)\quad ig[N\gamma_5\boldsymbol{\tau}N + \Xi\gamma_5\boldsymbol{\tau}\Xi]\cdot\boldsymbol{\pi} - ig\left[\frac{1}{i}\boldsymbol{\Sigma}\Lambda\bar{\Sigma} + i(\Lambda\bar{\Sigma} - \bar{\Sigma}\Lambda)\right]\cdot\boldsymbol{\pi}\,,$$

where:

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \qquad \Xi = \begin{pmatrix} \Xi_0 \\ \Xi^- \end{pmatrix}.$$

Gell-Mann [4] and Schwinger [5] have developed, independently, a theory in which the π-mesons also interact symmetrically with all barions. Their interaction Lagrangean density seem however to differ from expression (2) in the signs of the terms in $\boldsymbol{\Sigma}$ which are opposite to ours. The main difference, however, between their theory and ours is that their interactions with K-mesons are not symmetrical in all barions and have a smaller strength then the interactions of π-mesons.

Now that we have well defined theories of the interactions of π-mesons with barions it is tempting to see which are the consequences for problems such as the decay of π_0-meson and, in particular, to find which of the two above referred theories is in better agreement with the observed value for the lifetime. The consequences of the usual meson theory to the γ'-instability of π_0-mesons first pointed out by Sakata and Tamikawa (before the discovery of π-mesons) have been examined by Finkelstein, Steinberger and others [6]. A value much smaller then the experimental one [7], has been, however, found as given by

$$(3)\qquad \frac{1}{\tau} = A\,\frac{g^2}{4\pi}\left(\frac{\mu}{M}\right)^2,$$

with

$$(4)\qquad A = \left(\frac{\alpha}{4\pi}\right)^2\mu\,, \qquad (\hbar = c = 1)\,,$$

where α is the fine structure constant, g is the strength of the coupling of pion and nucleon fields (PS-PS) and μ and M are the masses of π_0 and nucleons, respectively. The experimental value of $g^2/4\pi = 13$ leads to a lifetime $\tau = 5\cdot10^{-17}$ s to be compared with the experimental value $5\cdot10^{-15}$ s of reference [7].

Kinoshita [8] suggested that if the π_0-meson interacted also with a hyperon with the same strength as with protons a partial cancellation might occur and we could have, instead of (3):

$$(5)\qquad \frac{1}{\tau} = A\,\frac{g^2}{4\pi}\left(\frac{\mu}{M} - \frac{\mu}{M'}\right).$$

Actually if we assume interaction (2) (or Gell-Mann - Schwinger's one) we find:

$$(6)\qquad \frac{1}{\tau} = A\,\frac{g^2}{4\pi}\cdot\left[\frac{\mu}{M_N} - \frac{\mu}{M_\Xi} \mp \left(\frac{\mu}{M_\Sigma} - \frac{\mu}{M_{\Sigma_-}}\right)\right]^2,$$

where the signs $-$ or $+$ correspond respectively to our interaction (2) and to the Gell-Mann-Schwinger interaction. It is seen that we cannot distinguish between these theories as $M_{\Sigma_-} - M_{\Sigma_+} \ll M_\Sigma$. On the other hand we find from (6) for the lifetime of π_0 the value

$$(7)\qquad \tau = 6\cdot10^{-16}\ \text{s}\,,$$

(4) M. Gell-Mann: communications to the Rochester Conference (April 1957).

(5) J. Schwinger: communications to the Rochester Conference (April 1957).

(6) S. Sakata and Y. Tamikawa: *Phys. Rev.*, **57**, 548 (1940); R. J. Finkelstein: *Phys. Rev.*, **72**, 415 (1947); J. Steinberger: *Phys. Rev.*, **76**, 1180 (1949); H. Fukuda and Y. Miyamoto: *Progr. Theor. Phys. Japan*, **4**, 347 (1949).

(7) B. M. Anand: *Proc. Roy. Soc.* A **220**, 183 (1953).

(8) T. Kinoshita: *Phys. Rev.*, **94**, 1384 (1954).

2116

which should be compared with the more recent experimental determination (9)

$$\tau_{\text{exp.}} \cong 4 \cdot 10^{-16} \text{ s} . \tag{8}$$

Thus we see that the agreement of the theory with experiment has improved and this may be considered an argument in favour of the symmetrical interaction (2), at least in what refers to nucleons and cascade particles. The question of the sign of the terms in Σ and Λ (and thus the choice between the two theories) should be decided from other experimental results, in particular, from the scattering of π by protons.

* * *

We should express our acknowledgements to the U. S. Office of Naval Research who made possible our attendance to the Rochester Conference.

(9) J. OREAR as referred by L. W. ALVAREZ in the Rochester Conference (April 1957) and private communication.

EMENDATION

R. WEINER – **Nuclear Isomeric Shift on Spectral Lines,** *Nuovo Cimento*, **4**, 1587 (1956).

Some mistakes are contained in the above paper and they should be emended as follows:

page 1587 4th line from the bottom	$= e$	should read	$= -e$
page 1588 14th line	$= e$	» »	$= -e$
page 1588 17th line	the last factor r^{2n_r+l+1}	» »	r^{2n_r+2l+1}
page 1589 2nd and 3rd lines	... state;	» »	... state). and delete the rest of the paragraph.
page 1589 12th line	≈ -5	should read	$\approx +5$

LIBRI RICEVUTI E RECENSIONI

Reports on Progress in Physic – Volume XIX (1956); a cura di A. C. STICKLAND – Ed. The Physical Society, London pagg. 367.

Il contenuto del 19° volume dei *Reports on Progress in Physics*, pubblicato nel 1956, è assai vario nella scelta dei soggetti dei nove lavori che lo compongono; anch'esso, come i precedenti della stessa serie, contiene ampie sintesi che, sia sugli argomenti teorici che su quelli a carattere sperimentale e applicativo apportano un contributo reale alla messa a punto dei problemi.

La teoria cinetica dei liquidi monoatomici a temperatura ordinaria è illustrata da G. H. A. COLE, in una rassegna critica, sia per sistemi in equilibrio che per sistemi non in equilibrio. Vi sono discusse le teorie dello stato liquido, sviluppate nell'ultimo decennio sino al punto di rendere possibili soddisfacenti risultati quantitativi per i liquidi monoatomici e semi quantitativi per quelli poliatomici. Secondo l'A. si può affermare che queste teorie hanno finalmente aperto la giusta strada per la costruzione di una teoria completa dello stato liquido.

L'articolo di R. O. DAVIES che tratta della termodinamica dei fenomeni irreversibili o « teoria macroscopica della irreversibilità » è diviso in due parti; la prima riguarda i sistemi non uniformi che possono considerarsi in equilibrio locale e le cui proprietà possono essere perciò localmente descritte mediante le variabili caratteristiche degli stati di equilibrio. Una applicazione di questa teoria si ha ad esempio al moto dei fluidi viscosi in un campo di forze. La seconda parte invece studia i sistemi incompleti in condizioni uniformi; per descrivere lo stato locale del sistema occorrono qui una o più variabili oltre a quelle dell'equilibrio; queste considerazioni trovano applicazione ai fenomeni di rilassamento come l'assorbimento degli ultrasuoni e le proprietà viscoelastiche.

Sotto il nome di « Aero-termoacustica, generazione di suono per effetto di turbolenza e di processi termici » O. K. MAWARDI svolge un argomento di alto interesse applicativo. Si tratta della generazione di suoni durante il moto di un fluido viscoso compressibile, studiata partendo dalla equazione generale. Queste considerazioni permettono di risolvere alcuni dei problemi relativi ai suoni prodotti dagli aerei in volo ed altre questioni di carattere più generale. La prima parte del lavoro si occupa degli effetti acustici connessi con il moto turbolento, la seconda esamina il caso in cui nella corrente siano presenti sorgenti di calore.

J. H. TAIT scrive su alcuni argomenti della teoria della diffusione dei neutroni ed in particolare sul calcolo dello spettro

in un mezzo moderatore illimitato con una distribuzione omogenea di sorgenti, sulla distribuzione spaziale di neutroni rallentati e sul calcolo delle dimensioni critiche.

Sempre sul piano teorico è poi il lavoro di J. A. Ratcliffe riguardante alcuni aspetti della teoria della diffrazione e le loro applicazioni allo studio della ionosfera (scintillazione delle radio stelle, onde riflesse dalla ionosfera, ecc.).

La rassegna ad opera di H. Y. Fan sull'assorbimento infrarosso nei semiconduttori illustra sia gli aspetti teorici che quelli sperimentali distinguendo quattro meccanismi di assorbimento: 1) assorbimento intrinseco dovuto alla eccitazione degli elettroni che vengono portati nella banda di conduzione; 2) assorbimento dovuto ai portatori di carica liberi; 3) assorbimento legato all'impurità e difetti reticolari (accompagnato talvolta da fotoconduttività); 4) assorbimento connesso con vibrazioni reticolari. I risultati sperimentali si riferiscono al germanio, al silicio, al tellurio e all'antimoniuro di indio.

J. P. Blewett, del laboratorio di Brookhaven, si occupa dei sincrotroni a protoni descrivendo, dopo una introduzione generale, le caratteristiche dei tre apparecchi in funzione per energie superiori ad 1 GeV: il sincrotrone a protoni di Birmingham (1 GeV) il cosmotrone di Brookhaven (3 GeV) e il bevatrone di Berkeley (6 GeV). Dopo un paragone tra i pregi ed i difetti di queste macchine vengono descritti i progetti di altri acceleratori, tra i quali quello del CERN (25 GeV).

La determinazione dell'età geologiche con metodo radioattivo fa oggetto dell'articolo di L. H. Ahrens: oltre ai classici metodi basati sui dosaggi del piombo e dell'elio si sono sviluppati alcuni metodi nuovi che permettono un controllo e un miglioramento dei risultati precedentemente conseguiti. I più promettenti sono quelli basati sulla radioattività del rubidio e del potassio. Anche i metodi basati sul contenuto di piombo sono stati perfezionati colla determinazione dei rapporti di concentrazione tra i vari isotopi del piombo stesso e tra questi e gli isotopi dell'Uranio. Con questi mezzi è oggi possibile risalire con le età accertate sino a circa tre miliardi di anni fa. Si osservi però che le età determinate per varie vie non sono sempre coincidenti: forse i calcoli classici hanno portato a stime troppo basse e si potrà rendere necessaria una espansione della intera scala delle ere geologiche.

Ricordiamo infine il lavoro in cui C. G. Wynne descrive alcuni sistemi ottici recentemente messi a punto e l'uso di nuovi materiali e di nuove tecniche nella lavorazione ottica. La maggior parte dei sistemi presi in considerazione sono formati da diottri sferici ma non manca un cenno a sistemi asferici ed a sistemi riflettenti.

F. A. Levi

C. J. Gorter editor – *Progress in Low Temperature Physics*, Vol. 2, North-Holland Publishing Co., Amsterdmm, 1957; pag. 480.

Si tratta del secondo volume di questa raccolta di lavori riassuntivi nel campo della fisica delle Basse Temperature. Una volta si intendeva come tale quel gruppo di fenomeni, quali la superfluidità dell'elio, la superconduttività e poche altre proprietà della materia, che venivano studiate da quei pochi fisici che possedevano l'apparecchiatura necessaria per le basse temperature. Ma con il diffondersi di questa tecnica, non ha forse più molto senso mantenere uniti argomenti diversi e ormai specializzati.

Perciò è assai difficile recensire questo libro, perchè non credo ci siano persone sufficientemente preparate tanto sulla teoria dell'elio liquido che sull'orientamento dei nuclei, sui semiconduttori, sul rilassamento paramagnetico e così via.

Cio premesso, vogliamo segnalare gli articoli riguardanti l'elio, che abbiamo effettivamente letto, e che ci sono sembrati ottimi. Specialmente il primo articolo di J. De Boer: *Quantum effects and exchange effects on the thermodynamic properties of liquid helium*, è un autentico capolavoro di chiarezza (anche se tira un po' l'acqua al suo mulino). Gli altri articoli sull'elio sono di H. C. Kramers, di P. Winkel e D. H. Wansink, di K. R. Atkins, di C. Domb e J. Dugdale. Gli elettroni nei metalli sono trattati da B. T. Matthias, E. H. Sondheimer, U. A. Johnson e K. Lark-Horovitz, D. Shoenberg; i nomi sono tanto noti che non occorre aggiungere l'argomento specifico. Assai interessanti ancora il lavoro di C. J. Gorter sul rilassamento paramagnetico, di M. J. Steenland ed H. A. Tolhoek sull'orientamento dei nuclei, di D. Bijl sui calori specifici dei solidi, di H. Van Dijk e M. Durieux sulla scala di temperature nella zona dell'elio.

Infine un lungo articolo di rassegna sulle proprietà fisiche dei metalli rari, da parte di F. H. Spedding, S. Legvold, A. H. Daane e L. D. Jennings.

Non c'è dubbio che questo libro debba stare in tutte le biblioteche di fisica; ma gli articoli hanno un valore solo per gli specialisti.

G. Careri

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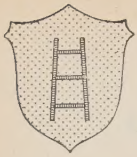

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